Monitoring The Level Of Students' GPAs Over Time

Saad T. Bakir, Alabama State University, USA
Bob McNeal, Alabama State University, USA

ABSTRACT

A nonparametric (or distribution-free) statistical quality control chart is used to monitor the cumulative grade point averages (GPAs) of students over time. The chart is designed to detect any statistically significant positive or negative shifts in student GPAs from a desired target level. This nonparametric control chart is based on the signed-ranks of the GPAs of the sampled students. The exact false alarm rate and the in-control average run length of the proposed chart can be computed exactly and are independent of the underlying probability distribution of GPAs. The traditional Shewhart X-bar control chart for monitoring the mean of a process is based on the assumption that data follows a normal distribution. However, student GPAs may differ significantly from the normal distribution. As a result, using a traditional control chart to monitor the GPAs of students may lead to incorrectly specifying the control limits and the average run length and/or the false alarm rate of the chart. A test study was conducted at the College of Business Administration at Alabama State University. The study monitored the median cumulative GPAs of management majors during the period Spring 2005 through Spring 2009. The study revealed that the GPAs of students were stable at a median level of 2.6 over the period of the study.

Keywords: Non parametric chart, Signed rank, control chart, student performance, continuous improvement, Shewhart, non normal distribution, probability distribution

INTRODUCTION

Student Cumulative Grade Point Averages (GPA) have been used throughout the educational literature to assess student progression and achievement (Milton, Pollio, and Eison 1986). Recent examples include: 1) the correlation of emotional intelligence and GPA (Rode, et al 2007), 2) a linear regression analysis of cumulative GPA and years to graduation (Hall, Smith and Chia 2008), and 3) predicting academic success (Ridgell and Lounsbury (2004). However, the vast majority of the literature focuses on prediction of student success rather than monitoring of a student's current performance. Routine monitoring of student performance could identify problems, similar to the way quality control measures (i.e., charts) identify problems in production processes; that is, before they become uncontrollable. Student progression can also be viewed as a process. For example, Jenkins (1997) advocated the application of quality control to the classroom as a means of improving student learning. The development of a graphical technique that can be applied, and interpreted by, most educators could identify significant negative shifts, which could be then followed by intervention attempts to improve student performance. Deming (1986), in his well known work *Out of the Crisis,* advocated statistical quality control charts as the best tool for monitoring the stability of a process over time.

A quality control chart is computed from sampled data over time and illustrates a statistically generated Upper Control Limit (UCL) and Lower Control Limit (LCL). An efficient control chart must continue sampling (i.e., sampling instance) as long as the process is stable (in-control) and must give an out-of-control signal as soon as the sampled data exceeds the UCL or LCL. When used to monitor student performance, in terms of GPAs, the out-of-control signal would occur when the data falls below the LCL. Exceeding the UCL would be viewed as an improvement in performance. The run length is the number of sampling instances taken before the control chart indicates an out-of-control alarm. The most common efficiency criterion of a control chart is the *Average Run Length (ARL)*, which is the long-term average of the run length.

It is desirable that the *ARL* of a control chart be large if the process is in-control (or stable) and small if the process is out-of-control (or unstable). The *false alarm rate* is the probability that the control chart gives an out-of-control signal when in fact the process is in-control. Traditional control charts are distribution-based procedures in the sense that the observations of the process output are assumed to follow a specified probability distribution. These distributions are either normal for continuous measurements or binomial (Poisson distribution) for attribute data. Montgomery (2001) provides a comprehensive reference on distribution-based control charts.

However, the widespread adoption of control charts as a basic tool in quality and Six-Sigma management has resulted in their application to processes where the data is significantly non-normal (Brassard et al. 2002). Applications of a traditional control chart to non-normal processes can result in mis-specification of the control limits, average run lengths and/or false alarm rates.

This analysis proves that a nonparametric, or distribution-free, quality control chart based on signed-ranks can be used to monitor the GPAs of students over time. Since GPAs are measured on a scale of 0 to 4, they do not conform to the assumptions that apply to a normal distribution. However, the chart is distribution-free in the sense that the control limits are the same, regardless of the underlying distribution of the sampled GPAs. As a result, the sample does not have to be normally distributed. In addition, the control chart used is outlier resistant because outliers do not affect the ranks and/or the signs of the observations.

According to Chakraborti, et al. (2001) and Chakraborti and Graham (2007), nonparametric control charts have a number of advantages over traditional control charts. The advantages of distribution-free control charts include applicability to markedly non-normal data, robustness with respect to outliers, constant in-control average run lengths, and false alarm rates, irrespective of the process underlying distribution.

THE NONPARAMETRIC MONITORING SCHEME

This section presents the steps required for the construction of a nonparametric scheme for the cumulative GPAs of students over time. The scheme is designed to check the stability of GPAs around a desired target level, θ_0, and to detect any statistically significant negative or positive shifts from a desired target value. The process design uses a Shewhart-type control chart based on the Wilcoxon signed-rank test statistic. Bakir (2004) developed the theoretical details that support this application, including the statistical tables for the signed-rank control chart. The procedure for applying this concept to student GPAs is as follows:

Step 1: Characteristic to be Monitored

This procedure is designed to monitor and detect statistically significant shifts (positive or negative) of the median (or average) cumulative GPA of a population of students from a given desired target value, θ_0.

Step 2: Sampling Plan.

At each semester $t, t = 1, 2, \cdots,$ **collect** the cumulative GPAs $(x_{t1}, x_{t2}, \cdots, x_{tn})$ of a sample of size $n > 1$ students selected randomly from the population of students under study.

Step 3: Theoretical Assumptions

The GPAs of students are assumed to be independent, have a continuous probability distribution and to be symmetric about a known desired target median θ_0. If the underlying probability distribution of the GPAs is markedly non-symmetric, a nonparametric control chart based on signs should be used. (Amin, Reynolds and Bakir 1995).

Step 4: Calculating the Charting (or Control Sequence) Statistics

At each semester, calculate the absolute deviation, $\left|x_{tj} - \theta_0\right|$, of each GPA from the desired target value, θ_0. Assign rank *1* to the least absolute deviation, rank *2* to the second least, and rank *n* to the largest absolute deviation. Mathematically, these ranks (denoted by R_{tj}^{+}) can be expressed by

$$R_{tj}^{+} = 1 + \sum_{i=1}^{n} I\left(\left|x_{ti} - \theta_0\right| < \left|x_{tj} - \theta_0\right|\right),\ t = 1, 2, \cdots;\ j = 1, 2, \cdots, n \qquad \text{(Eq. 1)}$$

where $I(u) = 1, 0$ if u is true or false. Then record the sign = -1, 0, 1 of each respective deviation $\left(x_{tj} - \theta_0\right) < 0, = 0, > 0$. Finally multiply the signs by their corresponding absolute ranks (as defined in Eq. 1), and sum the sampled students in a given semester. The resulting charting statistic, ψ_t, can be expressed by

$$\psi_t = \sum_{j=1}^{n} sign\left(x_{tj} - \theta_0\right) R_{tj}^{+},\ t = 1,\ 2,\ \cdots. \qquad \text{(Eq. 2)}$$

where $sign(u) = -1,\ 0,\ 1\ if\ u < 0, = 0, > 0.$

The charting statistic in Eq. 2 is linearly related to the Wilcoxon signed-rank statistic W^{+} (the sum of absolute ranks of positive observations) through the formula $\psi_t = 2W^{+} - n(n+1)/2$, (Gibbons and Chakraborti 2003).

Step 5: Control Limits

The corresponding upper and lower control limits, based on the sample size *n* and the desired false alarm rate, can be found in table 1 and 2 in Bakir's (2004) work. In most common applications *LCL = - UCL.*

Step 6: Signaling Rule

A statistically significant positive shift in the GPAs is indicated when the charting statistic falls above the upper control limit, i.e., $\psi_t \geq UCL$ .or below the lower control limit, i.e., $\psi_t \leq UCL$. No statistically significant shift from the desired target GPA is indicated when the charting statistics of all semesters fall within the upper and lower control limits.

Step 7: Continuous Improvement

First, identify the causes underlying the charting statistics that are less than the lower control limit, which indicate~~s~~ a negative shift in students GPAs. Second, identify the reasons for charting statistics falling above the upper control limit, which indicates a positive shift in students GPAs. The Pareto chart and the cause and effect (fishbone) diagram are useful diagnostic quality tools for both cases (Ritter 1992 and Brassard et al 2002). Then, devise solutions to correct poor student performance and implement factors that result in improved student performance. If the charting statistics of all semesters fall within the control limits, then students have maintained the desired target median GPA value.

A CASE STUDY

Development of the Charting Statistics

The nonparametric scheme discussed above was used to monitor the median level of cumulative GPAs of management majors in the Department of Business Administration at Alabama State University. All cohorts had earned between 50 and 60 semester credit hours between the Spring 2005 and Spring 2009 semesters and represent the population that has completed half of their college curriculum. Corrective action~~s~~ can be taken when the procedure indicates that their cumulative GPA has declined. The cumulative GPAs of a random sample of size $n = 10$ students were recorded for each semester. The collected cumulative GPAs (on a 1-4 score scale) of the sampled students are shown in Table 1.

Table 1: Cumulative GPAs of Randomly Selected Management Majors

SP 05	FA 05	SP 06	FA 06	SP 07	FA 07	SP 08	FA 08	SP 09
2.329	2.788	2.597	2.216	2.400	2.880	2.669	2.436	2.481
2.690	2.356	3.563	2.629	2.313	2.382	2.593	2.690	2.685
2.409	2.491	2.538	2.079	2.458	2.530	2.200	3.022	3.052
2.556	2.429	3.134	2.229	2.838	2.907	2.280	2.690	2.500
2.415	2.638	2.479	2.842	2.606	2.326	2.398	2.843	2.456
2.636	2.520	2.724	2.419	2.667	2.383	2.670	2.282	2.774
2.200	2.328	3.638	2.833	2.671	2.710	2.640	2.278	2.784
2.636	2.495	2.509	2.255	3.437	2.600	2.922	2.333	3.425
2.539	3.561	2.570	2.677	3.000	2.574	2.680	2.274	3.015
2.541	2.516	2.694	2.204	2.920	2.321	2.554	2.848	2.529

The desired target median GPA level was set at $\theta_0 = 2.600$ which is 65% of the maximum 4.0 and equates to a letter grade between C and B. As an example, Table 2 shows the results of the calculations for the Spring 05 sample of the first charting statistic ψ_1 as defined in Eq. (2).

Table 2: Calculations of the Signed-Rank Statistic for Spring 05 GPAs with Target Median GPA of 2.6

SP 05 X	X-Med	ABS(X-MED)	RANK ABS	SIGN	(SIGN)(RANK)
2.329	-0.271	0.271	9	-1	-9
2.690	0.090	0.090	6	1	6
2.409	-0.191	0.191	8	-1	-8
2.556	-0.044	0.044	3	-1	-3
2.415	-0.185	0.185	7	-1	-7
2.636	0.036	0.036	1	1	1
2.200	-0.400	0.400	10	-1	-10
2.636	0.036	0.036	1	1	1
2.539	-0.061	0.061	5	-1	-5
2.541	-0.059	0.059	4	-1	-4
			Sum(Sign)(Rank)		**-38**

The value of the charting statistic to be plotted for Spring 05 is **(-38)**. Calculations for the charting statistics, with lower and upper control limits of *-43* and *43* for the remaining semesters (i.e., Fall 05 through Spring 09), are displayed in Table 3. The control limits of ± 43 were obtained from Bakir's (2004) Table 2 and correspond to a false alarm rate of 0.02734, approximately the same as the traditionally assumed 0.02700 false alarm rate of the $\pm 3\sigma_{\bar{x}}$ limits for a Shewhart X-bar control chart. The false alarm rate of the nonparametric ± 43.000 limits control chart stays the same at 0.02734 irrespective of the underlying probability distribution of the GPAs. On the other hand, the false alarm rate of the traditional $\pm 3\sigma_{\bar{x}}$ limits on Shewhart X-bar control chart would be significantly different from the assumed 0.02700 if the underlying distribution of the GPAs is markedly non-normal (Amin, Reynolds and Bakir, 1995).

Table 3: GPA Control Chart Data with a Target Median GPA of 2.6

All Management Majors for Nine Semesters			
Semester	**Sum (Sign)(Rank)**	**LCL**	**UCL**
SP 05	-38.0	-43	43
FA 05	-19.0	-43	43
SP 06	23.0	-43	43
FA 06	-31.0	-43	43
SP 07	23.0	-43	43
FA 07	-8.0	-43	43
SP 08	-3.0	-43	43
FA 08	-12.0	-43	43
SP 09	29.0	-43	43

Quality Control Charts

Figure 1 demonstrates that the student GPAs are within the control limits and centered around the target median level of 2.6. Spring 05 was the lowest at -38; however, calculations for Spring 07 through Spring 09 indicate an upward shift in the GPAs. The scheme was also applied to the same data when the target GPA median was set at 2.5 producing Figure 2 and at 2.8 producing Figure 3.

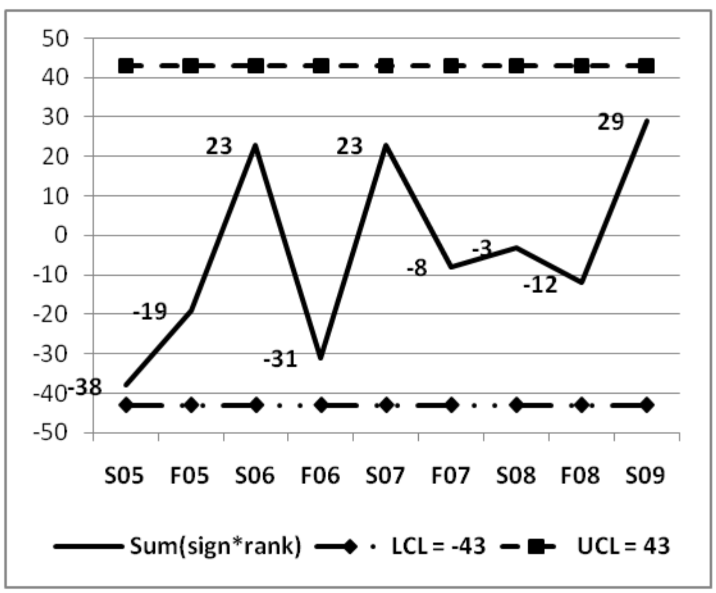

Figure 1: Control Chart for Spring 05 through Spring 09 with Target Median GPA of 2.6

Figure 2 indicates that in all semesters (except F06), the students' GPAs were on the positive control limit of 43, indicating a statistically significant positive shift from the target median GPA of 2.5.

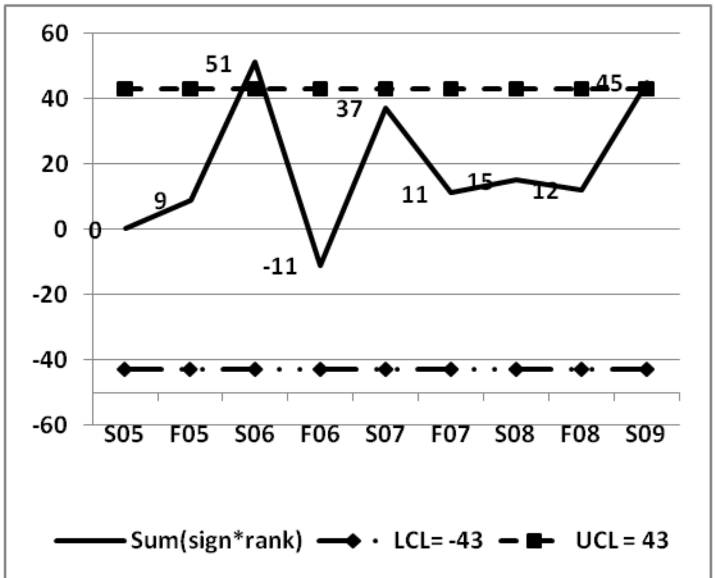

Figure 2: Control Chart for Spring 05 through Spring 09 with Target Median GPA of 2.5

Figure 3 indicates that in all semesters, the students GPAs were on the negative side of the median target of 2.8. In the Spring of 05, Fall 06, Fall 07, and Spring 08, the GPAs broke through the lower control limit of -43, indicating a statistically significant negative shift from the target median GPA of 2.8. Fall 08 and Spring 09 represented a slight upward shift when the GPAs climbed back above the lower control limit.

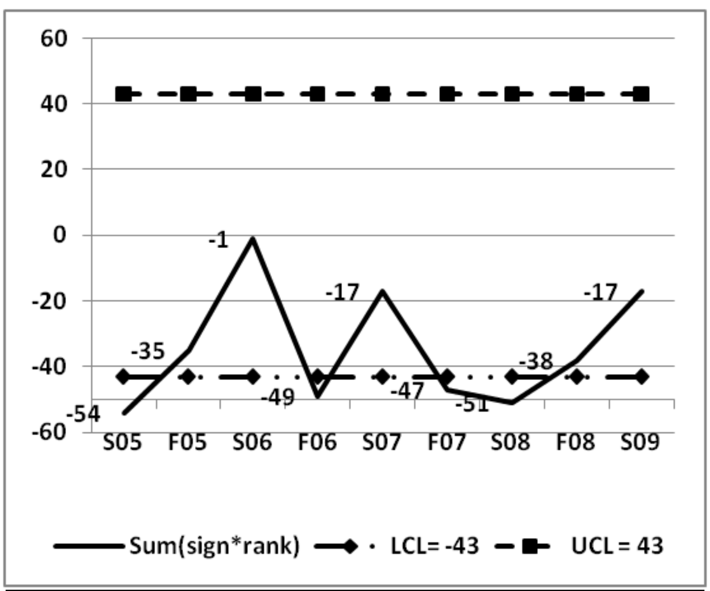

Figure 3: Control Chart for Spring 05 through Spring 09 with Target Median GPA of 2.8

WEAKNESSES AND IMPLICATIONS FOR FUTURE RESEARCH

This research has developed a means of monitoring student performance over time. However, the nonparametric procedure is intended to be applied to groups, such as a department within a college. The logical question that arises when the control charts indicate improved or declining performance is what procedure should be used for further analysis at the individual student level?

Procedures should be developed to determine whether positive or negative performance is due to a specific course(s) and how the courses are administered or whether it is due to individual student performance.

CONCLUSION

This study has illustrated how to use a nonparametric quality control chart based on signed-ranks to monitor the cumulative GPAs of students over a period of time and detect any statistically positive or negative shift from a desired target value. Unlike the traditional Shewhart X-bar control chart, the nonparametric control chart is distribution-free in the sense that its in-control average run length (or the false alarm rate) is the same for any underlying probability distribution that is continuous and symmetric. If the underlying distribution of the GPAs is markedly non-symmetric, a nonparametric control chart based on signs, such as that developed by Amin, Reynolds and Bakir (1995), should be used. Tables for the exact in-control average run lengths and false alarm rates, together with their corresponding control limits, are available in Bakir (2004).

A demonstration of the usefulness of this procedure was shown by an actual case study of the GPAs of management major students in the Department of Business Administration at Alabama State University for the period Spring 2005 through Spring 2009. The study showed when GPAs maintained a desired target GPA level of 2.600 and when the GPAs were significantly below the higher level GPA level of 2.800 (representing 70% of the maximum score of 4.000).

AUTHOR INFORMATION

Saad T. Bakir is a Professor of Economics at Alabama State University. He holds the following degrees: a Doctorate in Statistics, Virginia Tech, Blacksburg Va. (1977), M.S. (1968) and B.S. (1966) in Statistics, American University of Beirut, Lebanon. Dr. Bakir worked in Virginia Commonwealth University, University of Kuwait, and Virginia Tech. before joining Alabama State.

Bob McNeal, D.I.B.A., is an Assistant Professor of Management at Alabama State University. He holds the following degrees: a Doctorate in International Business, Nova Southeastern University, Ft Lauderdale Florida (2003), an MBA (Auburn University at Montgomery, Montgomery Alabama (1993), a BSBA in Hotel-Restaurant Management (1979), University of Denver, Denver Colorado, and a BS in Accounting (1969), Huntingdon College, Montgomery Alabama. Prior to entering higher education, Dr. McNeal worked in the private sector in government accounting and auditing in West Africa and the US and in the hospitality field.

REFERENCES

1. Amin, R, Reynolds, M. R., Jr. and Bakir, S. T. (1995). "Nonparametric Quality Control Charts Based on the Sign Statistic." *Commun. Statist.- Theory Meth*., 24(6), 1597-1623.
2. Bakir, S.T. (2004). "A Distribution-Free Shewhart Quality Control Chart Based on Signed Ranks." *Quality Engineering*, 16(4), 613-623.
3. Bakir, S. T. (2001). "Classification of Distribution-Frez Quality Control Charts." Proceedings of the Section on Quality and Productivity; American Statistical Association: Washington, DC.
4. Brassard, M., Finn, L., Ginn, D., and Ritter, D. (2002). The Six Sigma Memory Jogger II.
5. GOAL/QPC. Salem, N. H.
6. Chakraborti, S. and Graham, M. A. (2007). "Nonparametric Control Charts." *Encyclopedia of Statistics in Quality and Reliability*, John Wiley, New York.

7. Chakraborti, S., Van Der Laan, and Bakir, S. T. (2001). "Nonparametric Control Charts: An Overview and Some Results*." J. of Quality Technology*, 33(3), 304-315.
8. Deming, W. E. (1986). *Out of the Crisis*. Cambridge, Mass., MIT.
9. Gibbons, J. D. and Chakraborti, S. (2003). Nonparametric Statistical Inference. Marcel Dekker, Inc. New York.
10. Jenkins, Lee (1997*). Improving Student Learning: Applying Deming's Quality Principles in the Classroom.* ASQ Quality Press. Milwaukee WI
11. Hall, C., Smith, K. and, Chia, R. (2008). Cognitive and personality factors in relation to timely completion of a college degree. *College Student Journal*. (42.4) 1087-1099
12. Milton, O., Pollio, H. R., and Eison, J. A. (1986).*Making sense of college grades*. San Francisco: Jossey-Bass.
13. Montgomery, D. C. (2001) *Introduction to Statistical Quality Control*, 4th ed.; John Wiley & Sons., Inc., New York, NY.
14. Ridgell, S. D. and Lounsbury, J. W. (2004). Predicting academic success: general intelligence, "Big Five" personality traits, and work drive. *College Student Journal*, (38.4) 607-619.
15. Rode, J. C. Mooney,. Arthaud-Day, C. H, Near, M.L., Baldwin, J. P. Rubin, R. S., and Bommer, W.H. (2007). *Journal of Organizational Behavior, (28), 399-421*.
16. Ritter, Diane (1992). The Memory Jogger for Education. Goal / QPC, Methuen MA.
17. Ritter, Diane (1993)."Transforming TQM into the Classroom, "Competitive Times, Goal/QPC, Methuen MA.