A Seasoned Instructor's New Look At Fixed-Income Securities: An Exercise Using Discounted Bonds

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ABSTRACT

Having returned to teaching the basics of pricing fixed-income securities after several years, the author recalls the difficulty students have in understanding the total return provided by fixed-income securities that are purchased at either a discount or premium from face value. This teaching note attempts to clarify the concept by suggesting that separation of the return provided by coupon payments from the return provided by periodic changes in the fair market value may aid in student understanding. The integration of financial calculators in the classroom improves and simplifies this process for seasoned instructors.

Keywords: Bond Amortization; Yield to Maturity; Current Yield; Bond Pricing

INTRODUCTION

Students of finance eventually understand that a fixed-income security sells at a discount when the return provided by the coupon is insufficient to compensate the investor. This is because the investor seeks additional compensation by bidding less than the maturity value with the knowledge that additional return will be obtained when the security matures. A rational investor selling before the maturity date will seek the additional return by selling the security at a price higher than the purchase price. Therefore, the ability to demonstrate how an investor eventually earns his or her required rate of return is essential for the continuing finance student.

Instructors of finance often do not exploit the opportunity to show the process of change in the fair market value of a fixed-income security over time when interest rates remain constant. Several introductory texts also do not provide examples of this particular process before moving on to other factors that affect bond pricing and yields. Keown et al. [2011, 202] allude to the phenomenon during a discussion and example of the current yield on a coupon bond. Block and Hirt (2008, 291-292) provide a formula that includes a linear amortization of the bond discount or premium to approximate the yield to maturity, but do not mention that the amortization process of a discount or premium is actually non-linear. Ross et al. [2010, 191-200] raise the issue, and confirm the discount or premium with time value of money principles. However, it is presented in aggregate for the time to maturity rather than describing that the underlying bond price will change each coupon payment period if investor's required rate of return remains constant. The intent in these texts is to show the process of capital gains or losses and to explain why a previously issued bond may sell at a premium or discount.

Brealey et al. [2009, 164-169] come very close to an example of how the fair price of a bond changes over time when investors' required rate of return does not. The authors explicitly state:

When interest rates do not change, the bond price changes with time so that the total return on the bond is equal to the yield to maturity...

However, the issue is clouded by continuing the statement:

...The rate of return will be less than the yield to maturity if interest rates rise, and it will be greater than the yield to maturity if interest rates fall.

The resolution of the constant interest rate phenomenon, as stated by Brealey et al. [2009, 167], is then left to the student as an exercise.

The example(s) presented below fill a pedagogical gap that seems to exist in introductory finance texts. After initial exposure to basic fixed-income pricing, it is a common instructional strategy to vary the required rate of return while holding the number of coupon payments constant. This demonstrates mathematically why a security may sell at a premium or discount from face value. However, it is uncommon for an instructor (or a text) to then demonstrate the effect when the required rate of return is held constant and the number of coupon payments is varied. It is believed that students will have greater comprehension of the role capital gains and losses play in allowing an investor to earn his or her entire required rate of return when purchasing a fixed-income security if this instructional strategy is pursued. The discussion can then move on to the material many texts provide - changing both the required rate of return and the coupon payments remaining simultaneously.

FAIR MARKET PRICING OF A FIXED-INCOME SECURITY

The pricing of a fixed-income security follows a three step process: (1) find the present value of coupon payments that will be received as an annuity, (2) find the present value of the lump sum amount to be received at maturity, (3) combine the two results. When the discount rate (investor required rate of return) applied in this process is greater than the coupon rate, the fair market price of the security will be below the security's maturity value.

The process described in the previous paragraph is fundamental to the field of finance. Twenty years ago, pricing a fixed-income security could be an arduous process. The introduction and integration of financial calculators over the last two decades has made pricing fixed-income securities easily accessible to all students. Often, the most difficult detail for students to remember is to divide the required rate of return by two and multiply the number of years to maturity by two when determining the fair market price of a bond that pays interest semi-annually.

By focusing on mechanical functions, students and instructors often ignore the process by which investors receive their desired rate of return from fixed-income securities. The coupon payment provides only part of this return. In the case of discounted securities, the rest of the return must be obtained by either selling the security at a price higher than the purchase price, or holding it until maturity.

Example of the Fair Market Price of a Discounted Fixed-Income Security

Assume a corporate bond with a face value of $1,000, an annual coupon rate of 8%, four years to maturity, semi-annual coupon payments, and an investor who requires an annual return of 12%. Further, assume that the required rate of return on this security does not change over the four years until it matures.

The suggested process is to demonstrate how the fair market price of the security changes over time and more importantly, to explain how this change provides additional compensation to the investor. Calculator keystrokes that provide the current fair market price are:

Calculator Solution 1
The Fair Market Price of a Fixed Income Security

Calculator Key→	**PV**	**FV**	**PMT**	**N**	**I**
Input→	**Solve**	1,000	40	8	6
Solution→	**$ 875.80**				

It is suggested that the instructor use as an in-class exercise, homework, casework, or other example, a method similar to the following.

1. Students should first find the fair market price of the example security ($875.80).
2. The instructor should stress that if the security is purchased at that price, the investor must earn their desired 12% annual return (6% semi-annual) on the $875.80 invested.
3. The instructor can easily show that the first $40 semi-annual coupon payment returns only 4.57% on the $875.80 invested, which is below the semi-annual 6% desired by the investor.
4. How does the investor receive the entire 6% they demand?

The next steps reveal the remaining 1.43% return. The student is to find the fair market value that exists immediately after the security pays the first $40 coupon. The required rate of return (discount rate) is the same, the maturity value is the same, and the amount of the coupon payment is the same. The only variable that changes in the calculation is the number of coupon payments remaining. The same calculator keystrokes apply; changing only the number of coupon payments (N):

Calculator Solution 2
The Fair Market Price of the Security One Period Later

Calculator Key→	**PV**	**FV**	**PMT**	**N**	**I**
Input→	**Solve**	1,000	40	7	6
Solution→	**$ 888.35**				

The fair market price of the security has increased $12.55 from $875.80 to $888.35. This represents an increase of 1.43% in value which supplements the 4.57% return earned on the coupon payment. The investor's semi-annual required rate of return of 6% is preserved.

At this point, it is important that the instructor emphasize that if the investor is not able to sell the security for at least the fair market price of $888.35, the investor has chosen to reinvest. Therefore, the $888.35 must earn the investor's semi-annual required rate of 6% for the next six months.

The next coupon payment of $40 will provide a semi-annual return of 4.50% on the reinvestment. The difference between the semi-annual required rate of return and the return provided by the coupon must come from the increase in fair market value that occurs over the six-month period. The financial calculator solution, only changing the number of coupon payments remaining until maturity is:

Calculator Solution 3
The Fair Market Price of the Security Two Periods Later

Calculator Key→	**PV**	**FV**	**PMT**	**N**	**I**
Input→	**Solve**	1,000	40	6	6
Solution→	**$ 901.65**				

The $13.30 increase in fair market value, from $888.35 to $901.65, represents a 1.50% semi-annual rate of return on the reinvested $888.35. Again, the investor's semi-annual required rate of return is preserved by earning 4.50% from the coupon and 1.50% from a capital gain.

SUGGESTED MODEL EXERCISE

Given the importance of reconciling the return that an investor receives through the coupon payment and the return that can be realized through an increase in the fair market price of a discount fixed-income security, it is a worthwhile exercise for the students to complete at least the part of the following example.

Example Steps

1. The student is to find the fair market price of the security after *each* coupon payment. This can be accomplished in minutes, as only the value of variable N is changed.
2. For discount securities, the student should notice that the fair market price is increasing after each coupon payment.
3. The dollar difference in fair market price is found at each semi-annual juncture.
4. Determine the return from the difference in the fair market value based on the reinvestment of the fair market value after the prior coupon payment for each period.
5. Calculate the return provided by the coupon payment, also based on the reinvestment of the fair market value after the prior coupon payment.
6. Add the two rates of return to confirm the investor has (or could) indeed realize the desired rate of return.

Exercise Template

Period:	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Coupon		$40	$40	$40	$40	$40	40	40	$40
Maturity									$1,000
Variable N	8	7	6	5	4	3	2	1	0
Fair Value 0	$875.80								
Fair Value 1		$888.35							
Fair Value 2			$901.65						
Fair Value 3				$915.75					
Fair Value 4					$930.70				
Fair Value 5						$946.54			
Etc…							Etc…		
Following Section Is Optional **Completed In Class With The Instructor Present**									
Fair Value	$875.80	888.35	901.65	915.75	930.70	946.54			
$ Change		$ 12.55	$ 13.30	$ 14.10	$ 14.95	$ 15.84			
% Change		1.43 %	1.50 %	1.56 %	1.63 %	1.70 %			
Coupon Return		4.57 %	4.50 %	4.44 %	4.37 %	4.30 %			
Overall Return		6.00 %	6.00 %	6.00 %	6.00 %	6.00 %			

Instructors may wish to make note of the relative importance of the return from capital gains and the return from the coupon payment as the bond approaches maturity. As the bond's fair market price increases each period, the periodic return provided by the constant coupon diminishes. Therefore, the return provided by capital gains must increase in order to achieve the desired return. Framed in this perspective, it may be clearer to the student why the dollar change in the fair market price must necessarily increase each period.

CONCLUSION

Finance instructors often do not pursue a full explanation of how an investor earns his or her required rate of return from fixed-income securities. There is a tendency to define and provide examples of what it means when the security sells at a premium, a discount, or at par. It is suggested that a relatively simple homework assignment or in-class exercise can not only provide additional practice for students, but increase the opportunity to understand how an investor's required rate of return is fulfilled in the fixed-income market.

AUTHOR INFORMATION

Thomas W. Secrest earned a Ph.D. in finance from the University of South Carolina and began his career at Coastal Carolina University where he is the former director of the Center for Economic Development. A member of the SC Board of Regional Economic Advisors, he has served five governors in that capacity. He teaches in the areas of financial markets and institutions and real estate. Current research interests include fixed-income securities and agricultural risk management. Prior to his academic career, he was a senior financial analyst and head of cost accounting at First Federal Savings and Loan Association of Rochester, New York. E-mail: tom@coastal.edu

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