IRR: A Blind Guide

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ABSTRACT

Over the past 60 years the internal rate of return (IRR) has become a major tool in investment evaluation. Many executives prefer it to net present value (NPV), presumably because they can more easily comprehend a percentage measure. This article demonstrates that, except in the rare case of an investment that is followed by a single cash return, IRR suffers from a definitional quandary. Is it an intrinsic measure, defined only in terms of itself, or is it defined by the efforts of active investors? Additionally, the article explains significant problems with the measure - reinvestment issues, multiple IRRs, timing problems, problems of choice among unequal investment opportunities, and practical difficulties with multiple discount rates. IRR is a blind guide because its definition is in doubt and because of its many practical problems.

Keywords: IRR; PV; NPV; Internal Rate of Return; Return on Investment; Discount Rate

INTRODUCTION

The internal rates of return (IRR) and net present value (NPV) have become the primary tools of investment evaluation in the last 60 years. Ryan and Ryan (2002) found that 76% of the Fortune 1000 companies use IRR 75-100% of the time. Earlier studies (Burns & Walker, 1987; Gitman & Forrester, 1977) indicate a preference for IRR over NPV, and a propensity to use both methods over others such as payback and return on funds employed. The researchers hypothesize that executives are more comfortable with a percentage (IRR) than a number (NPV).

A review of 14 respected finance texts (see Appendix), and numerous articles, as cited in this paper, suggest that there is confusion over the meaning of IRR and that it does not answer the question, "What is the return on *this* investment?" IRR has a number of drawbacks inherent in its use—problems of reinvestment of cash flows, multiple IRRs, and investment ranking and timing. Finally, most investment opportunities exhibit different discount rates for its initial investment, cash flows, and terminal values. Varying discount rates make IRR analysis extremely complex. These drawbacks invalidate IRR as a tool for most decision-making.

WHAT DOES IRR REALLY MEAN?

Beaves (1988, 280) notes:

Net present value and rate of return are essentially single-period indices designed for projects that have no intermediate cash flows. These indices are uniquely determined only when the investor's initial wealth commitment, Wo, and his terminal wealth, Wn, are uniquely determined. Without a reinvestment assumption, Wo and Wn are uniquely determined only for projects that have no intermediate cash flows.

Beaves argument suggests that if \$100,000 were invested at t = 0 and \$150,000 cash was received at the end of three years, the rate of return that discounts the \$100,000 to the \$150,000 would be 14.47%. Thus, \$150,000 / $(1 + .1447)^3$ = \$100,000. The 14.87% is the IRR or the return on the \$100,000 investment.

The implications of IRR, however, have been the subject of considerable confusion and debate when intermediate cash flows are introduced. Difficulties surrounding the concept occur when the reinvestment assumption is considered.

The Passive Investors' Definition

Beaves (1988, 280) continues, "These single-period indices can be generalized to projects that have intermediate cash flows, but such generalization requires a reinvestment assumption whether implicit or explicit." Several authors take issue with Beaves' point, arguing that IRR refers only to the percentage derived *because* of the calculation (Bierman & Smidt, 1957; Lohmann, 1988; Karathanassis, 2004; Crean, 2007). In other words, the cash flow determines the IRR, not the reinvestment assumption. There is no explicit or implicit reinvestment as Beaves suggests.

Additionally, Crean (2007) points out the tendency of textbook authors to define IRR as that discount rate which equates NPV to zero. A review of 14 respected finance textbooks published since 2000 bears this out (see Appendix.). For example, Brealey, Myers, & Allen (2011, 108) state, "The internal rate of return is defined as the rate of discount that makes NPV = 0." Gitman (2009, 431-432) comments, "The **internal rate of return (IRR)** is the discount rate that equates the NPV of an investment opportunity with $0 (because the present value of cash inflows equals the initial investment)."

Crean (2007) and others maintain that IRR must be defined in terms of itself as the discount rate that equates the estimated outlay to the actual or estimated cash flows earned by the investment. When the above discount rate is used, NPV does equal zero, but this fact is extraneous to the formal definition of IRR. The return calculated by IRR is internal to itself, calculated on the basis of its own cash flow only, hence "internal" rate of return. If cash flow is the only factor determining IRR, then investors are passive when it comes to reinvestment of the proceeds. Interestingly, Brealey, et.al. (2011, 109) agrees with this as well: "The internal rate of return is a profitability measure that depends solely on the amount and timing of the project cash flows.")

To understand Crean's (2007) point, he recommends profiling the IRR. The profile makes a distinction between "return *on* investment" and "return *of* investment" as shown in Profile A of Table 1.

Table 1: Calculating IRR

Time Period	**Year 0**	**Year 1**	**Year 2**	**Year 3**
Profile A: Investor Receives 24.3% Return on Unpaid Principal Balance; Does Not Reinvest				
Return On Investment: 23.4% Interest		23,375	17,152	9,473
Return Of Investment		26,625	32,848	40,527
Remaining Principal		73,375	40,527	0
Free Cash Flow to Investor	(100,000)	50,000	50,000	50,000
Average / Annual ROI	23.4%	23.4%	23.4%	23.4%
IRR	23.4%			
Profile B: Free Cash Flow Remains in Investment: Investor Cannot Reinvest				
Return On Investment		0	0	50,000
Return Of Investment		0	0	100,000
Free Cash Flow to Investor	(100,000)	0	0	150,000
IRR	14.5%			

Profile A assumes an annual cash inflow that is amortized over time. The *return on investment* row is the return on the remaining unamortized investment. Multiplying the remaining investment by the expected IRR return of 23.4% derives the values in A. Thus, $100,000 X .234 = $23,375 (accounting for rounding). The difference between the return and the $50,000 is the *return of investment*, amounting to $26,625 in the first year. The remaining principal at the end of year 1 is $100,000 - $26,625 or $73,375. The principal at the end of year one is multiplied by 23.4% to obtain the return on investment for year two. This process is continued until the remaining principal balance is paid off.

There is no reinvesting going on by the investors in Crean's profile, nor is there any comparison of the IRR to a required return. What is calculated is the return on the investment of $100,000. The annual return on investment is the average return on investment and equals the IRR.

His profile shows that investors may as well be absent, or storing the annual return under mattresses or in safety deposit boxes, after the investment is made. Either they cannot reinvest, should not reinvest, or choose not to reinvest. However, if the $50,000 per year is merely accumulating, the investor will have $150,000 and no more at the end of year 3. In that case, the investor's IRR will be 14.5% as shown in Profile B.

Both the 23.4% and the 14.5% fit the definition of IRR - that rate which equates the cash outlay to the cash inflows, yet both cannot be right. IRR is truly a return on an investment in the relatively rare case when an investment is made and a single return occurs at some future time. In that light, the return on the above investment will be 14.5%. The IRR, as an internal measure, is self-contradictory.

The Active Investors' Definition

When multiple cash flows occur after the investment is made, active investors will take action to reinvest the proceeds. As Solomon (Beaves, 1988, 276) points out, "In order to make a fair comparison [between two investment alternatives], an explicit and common assumption must be made regarding the rate rate (sic) at which funds released by either project can be reinvested up to the terminal date..." When that planned reinvestment takes place or is considered, the return from the intervention is no longer internal to the original investment. The very existence of exogenous factors—the active investors and an expected return on the cash flows—transform it into an external rate of return, more commonly referred to as the modified rate of return (MIRR).

MIRR differs from IRR in two important ways. First, MIRR assumes that the return to be calculated is on the cash flows, not the original investment. Second, the return on these cash flows is compounded over time. Compounding the positive cash flows forward to the terminal period at the expected reinvestment rate, and then finding that discount rate that equates the terminal value to the initial investment, is how MIRR is calculated.

Table 2 demonstrates the transformation. Profile C shows that if the active investors reinvest expecting no return, the MIRR will be the same as if the passive investors would not or could not reinvest. Profile D demonstrates that the return of investment under the Crean assumption cannot be $100,000. In that case, the return is only 11.3%. Profile E shows that the cash flow cannot be reinvested at a rate that does not compound. In that case, the return is 22.8%. To obtain an IRR comparable to that of the passive investors, the active investors must collect the sum of the cash flows as well as the compounded return on those cash flows. The passive investors' IRR is the return of the original investment plus a non-compounded return on that investment.

Table 2: Calculating MIRR

Time Period	**Year 0**	**Year 1**	**Year 2**	**Year 3**
Profile C: Active investor Reinvests Free Cash Flow at 0%				
Free Cash Flow	($100,000)	$50,000	$50,000	$50,000
MIRR	14.5%			
Profile D: Active investor Reinvests Free Cash Flow at a Compound Rate of 23.4% and Recovers Original Investment				
Free Cash Flow	(100,000)	50,000	50,000	50,000
Year 1 Compound Return On Investment				26,107
Year 2 Compound Return On Investment				11,688
Year 3 Return On Investment				0
Total Return On Investment				37,795
Return Of Investment				100,000
Free Cash Flow to Active investor	(100,000)	0	0	137,795
MIRR	11.3%			

Table 2: Continued

Profile E: Investor Reinvests Free Cash Flow at 23.4% Rate (Not Compounded) and Recovers Entire Free Cash Flow				
Year 1 Interest @ 23.4%				23,375
Year 2 Interest @ 23.4%				11,688
Year 3 Return On Investment				0
Total Return on Investment				35,063
Total Cash Flow Over Three Years				150,000
Free Cash Flow to Active investor	(100,000)	0	0	185,063
MIRR	22.8%			
Profile F: Investor Reinvests the Free Cash Flow at a Compound Rate of 23.4% and Recovers Entire Free Cash Flow				
Free Cash Flow	(100,000)	50,000	50,000	50,000
Year 1 Compound Return On Investment				26,107
Year 2 Compound Return On Investment				11,688
Year 3 Return On Investment				0
Total Return On Investment				37,795
Return Of Investment				150,000
Free Cash Flow to Active investor	(100,000)	0	0	187,795
MIRR	23.4%			

For active investors, IRR becomes a special case of MIRR where the reinvestment rate, as determined by the active investor, is exactly the same as the IRR as shown in Profile F. Recall that the passive investors' only determinate of the IRR is the cash flow itself. If the IRR determines the active investors' reinvestment rate, then the reinvestment rate is forecast by the investment's own cash flow. Given the active investors' definition, the result is a non sequitur - it cannot follow that a cash flow can predict its own reinvestment rate. For the active investor, IRR is a blind guide because its reinvestment rate cannot be determined by its cash flows.

Passive investors will argue that cash flow does not forecast its own reinvestment rate. The IRR is endogenous—it merely is. This poses a curious quandary - the passive investors simply invest and stand aside while their investment earns (in this example) 23.4%. The active investors must seek out alternative uses for their cash flows - ones that return exactly 23.4%.

PROBLEMS WITH IRR

There are four major technical problems with IRR. Three of these are examined in the academic literature of the 1950's (Alchian, 1955; Solomon, 1956; Lorie and Savage, 1955; Bierman & Smidt, 1957). The author summarized these in an earlier paper (Kierulff, 2008). A fourth problem has yet to be satisfactorily resolved.

Multiple IRRs

Rene Descartes (1596-1650) made the following discovery in mathematics: a series of numbers may have as many IRRs as the number of its sign changes. In the normal IRR problem, there is only one sign change—negative outflow to positive inflows. If there are more than one sign changes, however, multiple IRRs can occur. Plath and Kennedy (1994, 82) provide a simple example - a negative 16 in year zero, followed by 100 in year one and another negative 100 in year two. In this case, there will be two IRRs to correspond with the two sign changes - negative to positive and positive to negative. Discounting the 100 and the year-following negative 100 by either 25% or 400% will cause them to equal the initial negative 16.

There are business cases where several sign changes can occur. Brealey et. al. (2011, 110-111) discuss the case of a strip mine where the initial outflow to start up the mine is followed by positive cash flows and then a negative cash flow when the land is returned to a condition equivalent to its original state.

Unequal Investments

One intractable IRR problem is that of comparing unequal investments and cash flows. The problem also affects NPV as illustrated in Table 3.

Table 3: Comparing Unequal Investments

Time Periods	t = 0	t = 1	t = 2	t = 3
Alternative 1	($100,000)	$50,000	$50,000	$50,000
IRR	23.4%			
NPV	24,343			
Alternative 2	(200,000)	100,000	100,000	100,000
IRR	23.4%			
NPV	48,685			

Alternative 2 is twice Alternative 1 with twice the NPV, but both have the same IRR - 23.4%. Which to choose? There is no issue if the alternatives are not mutually exclusive and the company has enough ready liquidity to invest. If the company's hurdle rate for both is under 23.4% percent, it should invest in both; if the rate is above 23.4%, neither qualifies.

However, if the alternatives are mutually exclusive or funds are limited - say $200,000 in this case - the firm has no fair choice. The problem is incomplete. To make a valid cost-benefit comparison, either the costs (investment) or the benefits (resulting cash flows) must be equal. If the firm makes an investment of another $100,000 (Alternative 3) in conjunction with Alternative 1 and the former investment provides a NPV of $30,000, Alternatives 1 and 3 together would be preferred.

The Timing of Cash Flows

The timing of cash flows will affect the ranking of projects by NPV v. IRR - a critical issue when there are more good opportunities than funds to invest. Projects with later cash flows display lower IRRs while NPVs are relatively high. When projects return cash quickly, IRRs are high and NPVs tend to be relatively lower. Thus, rankings by size of IRR or by NPV may be different.

Table 4: Three Investment Opportunities

Time Period	Year 0	Year 1	Year 2	Year 3
Free Cash Flow: Alternative 1	($100,000)	$50,000	$50,000	$50,000
IRR	23.4%			
NPV	24,343			
Free Cash Flow: Alternative 2	(100,000)	20,000	50,000	80,000
IRR	18.8%			
NPV	19,609			
Free Cash Flow: Alternative 3	(100,000)	80,000	50,000	10,000
IRR	26.0%			
NPV	21,563			

Table 4 demonstrates how differences in ranking may occur. Alternative 1's cash flow is constant, while Alternative 2's cash flow is small at the beginning and large later on. Alternative 3's cash flow is the reverse of

Alternative 2. Ranking by NPV and IRR favors Alternative 1 over Alternative 2. However, ranking by IRR favors Alternative 3 over the other two, even though the former has a smaller total cash flow and its NPV is lower than Alternative 1's. The reason is that Alternative 3's early cash flows amortize the investment faster, thereby increasing the return on the remaining investment in later years. Table 5 compares Alternative 1 to Alternative 3. When ranking is necessary because of budgetary constraints, textbook authors recommend using NPV.

Table 5: Alternatives 1 And 3 From Table 4

Time Period	**Year 0**	**Year 1**	**Year 2**	**Year 3**
Alternative 1: Investor Receives 24.3% Return on Unpaid Principal Balance; Does Not Reinvest				
Return On Investment: 23.4% Interest		23,375	17,152	9,473
Return Of Investment		26,625	32,848	40,527
Remaining Principal		73,375	40,527	0
Free Cash Flow to Investor	(100,000)	50,000	50,000	50,000
Average / Annual ROI	23.4%	23.4%	23.4%	23.4%
IRR	23.4%			
NPV	24,343			
Alternative 3: Investor Receives 26% Return on Unpaid Principal Balance; Does Not Reinvest				
Return On Investment: 26% Interest		25,987	11,951	2,063
Return Of Investment		54,013	38,049	7,937
Remaining Principal		45,987	7,937	0
Free Cash Flow to Investor	(100,000)	80,000	50,000	10,000
Average / Annual ROI	26.0%	26.0%	26.0%	26.0%
IRR	26.0%			
NPV	21,563			

Multiple Discount Rates

To have meaning, the active investor must compare the discount rate - IRR (MIRR) - with the investment's hurdle rate or the company cost of capital. However, the usual free cash flow set will have many discount rates.

For example, it is often unrealistic to assume that the terminal value of an investment will bear the same risk or inflation rate as its cash flow. Does the salvage value of a piece of equipment or plant five or ten years from now have the same systematic risk as the cash flows they generate? Is estimating the value of a company or product line in five or ten years by their intrinsic or market values as certain as estimating their cash flows during the period? Others argue that the discount rates for operating cash flows vary because the inputs - risk, inflation, and liquidity preference - differ over time periods (Esty, 1999; Ang & Liu, 2004).

Although the issue is not entirely settled and is beyond the scope of this article, fixed investments are fixed costs in a determinate system, such as IRR. Investments (fixed costs) are unsystematic—riskless for a well-diversified investor; all of the variations (risks) in investment costs offset one another and are therefore negated by sufficient diversification. In these cases, the discount rate would be the riskless rate (inflation and liquidity preference). If the active investor were insufficiently diversified, the fixed investment discount rate could be negative in some cases because of unsystematic risk (Everett & Schwab, 1979; Berry & Dyson, 1980; Booth, 1983).

While varying rates can be accounted for in NPV, there is no convenient way to compare a hurdle rate or cost of capital with any one or a combination of those rates. It is possible to perform a specially modified IRR analysis on the discounted cash flows, including the discounted terminal value. However, the fixed and variable cash flows and terminal value must be discounted at their appropriate rates first. This IRR then becomes a weighted average measure. If it is greater than zero, NPV will be positive, and if it is zero, NPV will be zero. By itself the specially modified IRR conveys little new information because it is compared with zero and there is no single hurdle rate.

SUMMARY AND CONCLUSIONS

IRR is that discount rate which equates an investment to its resulting cash flow. It is not a measure of investment attractiveness until a cost of capital or hurdle rate becomes available as a comparator. It is not dependent upon anything but the size and timing of the free cash flow involved. Since the IRR is determined by its cash flow and active investors typically reinvest cash flows, IRR will be misleading if active investors cannot find opportunities that result in the same IRR.

As a practical matter, it is unreasonable to expect future investment opportunities to have the same IRR as the project under consideration. This is especially the case with high-risk investments demonstrating high potential returns. Nonetheless, active investors are locked into reinvesting cash flows at the project's IRR for that IRR to have meaning. The active investors must assume that they not only *can*, but also *will* find such investments and *will* invest in them. With IRR, it is not enough to be concerned with the potential of the investment under consideration; one must also reflect on the future use of cash flows.

As part of a measure of financial attractiveness, IRR has several other deficiencies. Multiple sign changes result in multiple IRRs. Active investors must either equalize investment costs or their cash flow benefits if they wish to compare opportunities properly. NPV and IRR do not always rank projects equally. Finally, a free cash flow may have different discount rates to account for cash flows, terminal value, and initial investment; IRR is a single measure and is not compatible.

The above factors must lead to the conclusion that IRR is a limited decision tool—financial analysts are well advised to use it with caution.

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APPENDIX

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