Accrued Interest On Bonds: An Explanation Based On Brokers' Preference For "Clean" Price Data With A Critique Of Intermediate Accounting Textbook Explanations

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ABSTRACT

By convention U.S. bond markets announce the actual price of a bond as the sum the quoted price plus accrued interest. The economic meaning of accrued interest and its role in this price announcing convention is generally misunderstood by accounting textbook authors who mistakenly provide accrued interest with both an economic and administrative explanation. A cogent rationale for the broker price announcing convention is offered which places accrued interest in its proper context. Explanations of accrued interest found in a sample of intermediate accounting textbooks are also critiqued. The concept of negative accrued interest is also briefly discussed.

Keywords: Bond Valuation; Accrued Interest; Actual Price; Quoted Price; Dirty Price; Clean Price

INTRODUCTION

Although accrued interest on bonds is firmly imbedded in the broker convention of announcing bond prices as well as the bond accounting model taught in intermediate accounting textbooks, accrued interest on bonds is difficult to define theoretically. For example, the amount of accrued interest is not a present value based on the market rate of interest which underlies the exchange between buyer and seller. Rather, accrued interest is merely a fractional nominal interest payment. Indeed, van Deventer and Imai (1997:11) have gone so far as to describe accrued interest as an arbitrary amount with little or no basis in economic reality.

Despite its apparent theoretical flaws, intermediate accounting textbooks infuse their discussions of accrued interest with economic meaning. In addition, they may also suggest that the recognition of accrued interest confers various administrative benefits to issuers. Unfortunately, these textbook discussions are mistaken on both points. This paper seeks to clarify this confusion by placing accrued interest in its proper analytical context. First, the paper provides a context for the discussion and clarity of terms by briefly reviewing bond valuation with the broker price announcing convention in which accrued interest plays an important role. [The related concept of negative accrued interest is discussed briefly in an Appendix.] Secondly, the paper explains the rationale behind brokers' preference for clean price data and how quoted bond prices function as a clean price surrogate. Finally, the paper critiques the explanations of accrued interest found in four contemporary intermediate accounting textbooks. Throughout, mathematical expressions are rendered in Excel syntax and amortized cost accounting is assumed.

BOND VALUATION WITH BROKER PRICE ANNOUNCING CONVENTION

Theoretically, the actual price of a bond—the amount of cash exchanged between the buyer and seller—is equal to the present value of the bond's future cash flows discounted at the market rate of interest. Following Rusbarsky and Vicknair (1999), the actual price of a noncallable U.S. corporate bond with semiannual payments, can be expressed as a two-step calculation using Excel's PV and FV functions (expressed on a per $100 par value basis):[1]

V_α: =PV(Y/2,N,-100*R/2,-100) (1a)

AP: =FV(Y/2,α,0,-V_α) (1b)

where,

AP	=	actual price at the settlement date
FV	=	future value
N	=	number of interest payments remaining after the settlement date
PV	=	present value
R	=	annual stated rate
V_α	=	present value of the bonds cash flows on the previous interest date discounted at Y
Y	=	annual market rate at the settlement date
α	=	fraction of the current period elapsed prior to the settlement date

Rather than announcing the actual price directly, U.S. bond markets practice a circular price announcing convention wherein the amount of accrued interest (AI) is deducted from the actual price to obtain the quoted price (QP) and then the actual price is announced as the sum of the quoted price plus accrued interest:[2]

AI =100*R/2*α (2)

QP =AP-AI (3)

AP =QP+AI (4)

To illustrate, assume the following information about a U.S. corporate bond issue:

> ABC Company issued noncallable bonds with a total par value of $1,000,000. The bonds are dated April 1, 2009, mature on April 1, 2029, and pay interest semiannually on April 1 and October 1 at an annual stated rate of 8.000%. The bonds were issued on May 16, 2009 at a price which reflected a market rate of 8.125%. There were no issue costs.

According to equations (1a) and (1b) the actual price of the ABC bonds at the March 16 settlement date is $100.147344:

V_α = $99.155286 =PV(8.125%/2,20,-100*8%/2,-100)

AP = $100.147344 =FV(8.125%/2,45/180,0,-99.155286)

Using equations (2) and (3) the corresponding values of AI and QP are $1.00 and $99.147344, respectively:[3]

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[1] Equation (1a) can be embedded in equation (1b): =FV(Y/2,α,PV(Y/2,N,100*R/2,100)).

[2] This convention gives rise to widely misunderstood jargon such as that bonds sell for "price plus accrued interest", where "price" refers to the quoted price. For example, Rusbarsky and Vicknair (1999) identify bond valuation errors in intermediate accounting textbook illustrations valuing bonds sold between interest dates when Y=R using "price plus accrued interest" or similar jargon. Fundamentally, these textbooks misunderstand equation (3) and so mistakenly equate "price" with Vα rather than QP. It is interesting to note that the same or similar valuation errors appear in each of the four intermediate accounting textbooks cited in this paper.

[3] By convention 30/360 day counting is used and so each interest period is 180 days in length. May 16 falls 45 days [using Excel's DAYS360 function, =DAYS360("4/1/2009","5/16/2009") returns a value of 45] into the April 1-October 1 interest period and so α=45/180.

AI = $1.00 =100*8%/2*45/180

QP = $99.137344 =100.147344-1.00

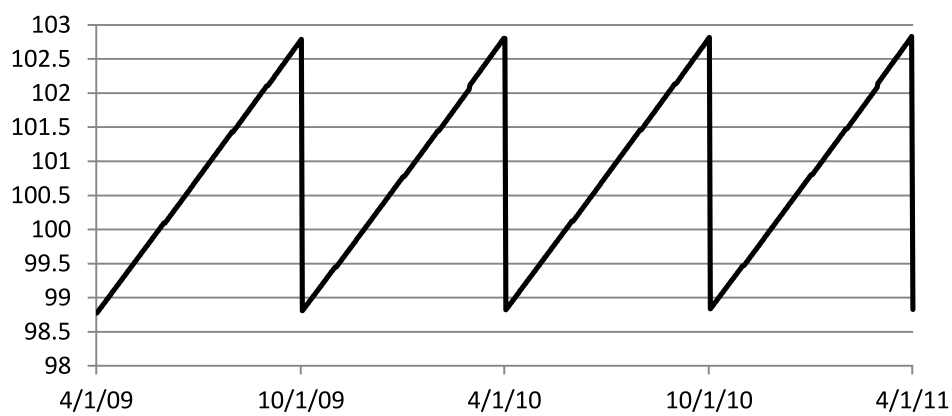
Finally, in accordance with the broker convention reflected in equation (4), the actual price of the ABC bonds is "reassembled" and announced to the market as "$99.147344 plus accrued interest of $1.000000":

AP = $100.147344 =99.147344+1.00

RATIONALE FOR THE BROKER PRICE ANNOUNCING CONVENTION

The actual price of a bond changes with the passage of time and as a result of changes in the market conditions reflected in the market rate Y. The passage of time effect occurs with or without a change in Y. To illustrate, Figure A plots the actual price of the ABC bonds over the first four interest periods holding Y= 8.125%.[4]

Figure A
Actual Price of ABC Bonds
First 4 Interest Periods, Y=8.125%



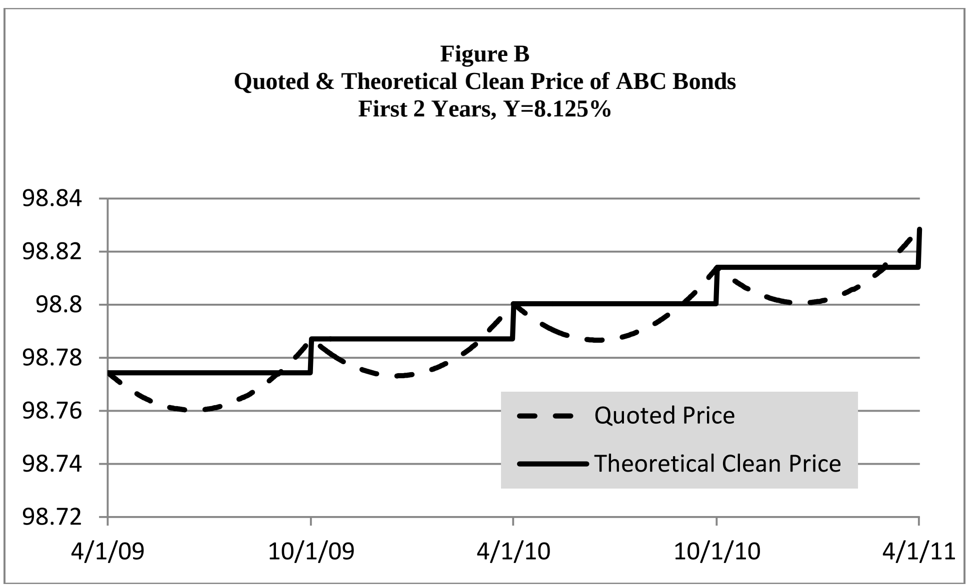
Observe in Figure A that actual price follows a characteristic "saw tooth" pattern as it rises along a compound interest curve[5] during each interest period, before peaking and dropping at each April 1 and October 1 interest payment date. Brokers are particularly interested in the impact of changing market conditions (the factors that change Y) on bond prices and so for analytical purposes only they prefer to work with bond price data without the "saw tooth" effect from the passage of time. The resulting "clean price" [6] is more stable over time than the actual price.[7] It is important to recognize that the broker preference for clean price data has no effect whatsoever on actual bond prices.

[4] In Figure A (as well Figure B) the paper assumes the market is open every day and ignores ex-coupon periods.
[5] Although the upward slope of the "saw tooth" appears to be a straight line, it actually follows a compound interest curve. For example, the linear midpoint between the actual price on April 1 and October 1, 2009 is $101.169378. However, the actual price at the midpoint is $101.149328.
[6] In contrast, in practice the actual price is commonly referred to as the "dirty price."
[7] See, for example, the discussion of dirty price, clean, price, and accrued interest, including diagrams illustrating the relationships at www.riskglossary.com/articles/bond_accrued_interest_htm.

ROLE OF ACCRUED INTEREST

The broker preference for clean price data and the broker price announcing convention converge in the convenience of accrued interest. A theoretical clean price could be easily calculated if values of Y were available on a timely basis.[8] However, in practice actual prices are given by the participants in the market and not by equations (1a) and (1b). Thus, trailing values for Y are not available until they are published by a data service generally much later than the trade date. For example, Moody's Bond Record is published but once a month.[9] This renders the calculation of a theoretical "clean" price impractical.

This is where accrued interest becomes useful. Because bond interest payments are contractually fixed (100*R/2) and α is easy to calculate, so then is accrued interest. By deducting accrued interest from the actual price, brokers have in the quoted price a theoretically flawed, but convenient clean price surrogate. For example, Figure B plots the quoted price versus the theoretical clean price for the ABC bonds over the same period as Figure A again assuming Y=8.125%.[10] Observe that the quoted price does not in fact plot as a straight line, but follows a "wavy" pattern which peaks at interest payment dates and ebbs in between.[11] In contrast, the theoretical clean price follows a step-wise path. Of practical significance to brokers is the fact that the paths of the quoted price and the theoretical clean price overlap somewhat. Thus quoted prices may be a relatively good surrogate for theoretical clean prices.



CRITIQUE OF TEXBOOK EXPLANATIONS

The author examined the discussions of bonds sold between interest dates in four contemporary intermediate accounting textbooks (Keiso et al, 2010; Nikolai et al, 2010; Spiceland et al, 2010; and, Stice et al, 2010) and found the following three explanations of accrued interest:

[8] Theoretically, the "clean" price for a particular AP is equal to the corresponding Vα.
[9] The lack of a timely market rate, may explain why the Wall Street Journal discloses the "current yield" (periodic dividend divided by quoted price) rather than the market rate for most bonds.
[10] With the market rate held constant, the theoretical clean price is the present value of the bonds at the beginning of the current interest period per equation (1a).
[11] This pattern is very subtle and so when AP is plotted against QP the magnitude of the "saw tooth" effect obscures the "wavy" pattern making QP appear to plot as a straight line. This apparent straight line may contribute to the misunderstanding noted earlier in footnote 2.

1. Accrued interest compensates the seller for holding the bonds for the fractional period prior to settlement.
2. Accrued interest simplifies the recording of interest expense in the period following the settlement date.
3. Accrued interest helps avoid the problem of splitting interest payments between two or more holders of the bonds.

The following sections examine and clarify each of the abovementioned explanations. It is instructive to note that none of the four textbooks describes accrued interest in the context of the broker price announcing convention or of a “clean” price metric.

Accrued Interest Does Not Compensate Sellers for Holding Bonds

All four textbooks assert that purchasers “pay” (Keiso et al 696; Spiceland et al 779; Stice et al 700) or sellers “collect” (Nickolai et al 656) accrued interest to compensate the seller for the interest earned during the fractional period prior to the settlement date. The following excerpt from Keiso et al is representative:

When companies issue bonds on other than the interest payment dates, buyers of the bonds will pay the seller the interest accrued from the last interest payment date to the date of issue. The purchasers of the bonds, in effect, pay the bond issuer in advance for that portion of the full six-months’ interest payment to which they are not entitled because they have not held the bonds for that period.

The above economic interpretation of accrued interest is flawed on several points. First, it confuses the nominal amount of accrued interest, $100*R/2*\alpha$, with the change in the bond’s present value during the fractional period prior to the settlement date, or $V_\alpha*((1+Y/2)^\wedge\alpha-1)$ (also $AP-V_\alpha$). To illustrate, assume Y=8.125% from April 1, 2009 through May 16, 2009. The present value (actual price) of the ABC bonds on April 1, 2009 would have been $99.155286 (N=20, R=8%, α=0). The actual price of the ABC bonds on the May 16, 2009 remains $100.147344 as computed earlier (N=20, R=8%, Y=8.125%, α=45/180). Thus, during the 45 days from April 1 to May 16 the economic value of the bonds increased by $0.992058 (100.147344-99.155286). Theoretically, it is this amount which measures the compensation received by ABC for holding the bonds. Importantly, per equation (1) this compensation is already included in the actual price (as well compensation for holding every other bond future cash flow for the 45 days).

In contrast, accrued interest for the same 45 day period is $1.00 as computed earlier. Although the difference of $0.007042 is not large ($0.77420 per $1,000 par bond or $77.42 for the entire bond issue) a rational purchaser theoretically would refuse to pay it because it would have the effect of slightly lowering the yield.[12]

More troubling to this author, however, is the invalid measurement principle implied in the explanation—that the growth in a bond’s actual price during any fractional period is a linear function of the annual stated (nominal) rate. Clearly this is contrary to the economic measurement principles reflected in equation (1a) and (1b) and may leave students confused concerning whether compound interest concepts are applicable only when bonds are valued on interest payment dates.

Accrued Interest Simplifies Subsequent Entries is Limited to Y=R and Interest is Prorated

Three textbooks (Nikolai et al, 656; Spiceland et al, 780, ftn 14; Stice et al, 701, ftn 7) assert either in the main narrative or in an accompanying footnote that recording accrued interest as a credit to Interest Expense simplifies the recognition of interest expense in the period following settlement. The following excerpt from Nikolai et al (656) is representative:

[12] Different assumptions about the value of Y on April 1, 2009 yield different measures of compensation (or loss) to the seller. For example, if the market rate on April 1 is 8.000%, the actual price on that day would be $100.00 and the compensation for holding the bonds from April 1 to May 16 would increase to $0.147344 (100.147344-100.00). Theoretically, this amount is equal to growth in the bond’s present value from April 1 through May 16 of $0.985341 [100*((1*8.125%/2)^(25/180))-1)] less the reduction in actual price of $0.837997 due to the increase in the market rate on May 16 from 8% to 8.125% [=FV(8%/2,45/180,0,PV(8%/2,20,-100*8%/2,-100))-FV(8.125%/2,45/180,0,PV(8.125%/2,20,-100*8%/2,-100))]. In contrast, the amount of accrued interest is inflexibly fixed at $1.00.

Recording accrued interest] reduces the record keeping for the first interest payment. This interest amount collected typically is credited to Interest Expense and is computed by multiplying the face value by the stated interest rate for the fraction of the year from the interest payment date prior to the sale date. On the next interest payment date, the company pays each bondholder six months of interest and records Interest Expense as usual.

Unfortunately, what these authors fail to mention or ignore is that this supposed recording keeping benefit is limited to the case where Y=R and interest expense is prorated between the fractional periods before and after an interest payment date. For example, assume the ABC bonds were sold on their April 1, 2009 dated date. Table A presents a partial amortization schedule including the first four interest periods assuming Y=8.000% and with interest prorated. Notice that the amount of interest expense for each whole interest period is equal to the $40,000 (1,000,000*8%/2) semiannual coupon payment and that the amount of interest expense attributable to each fractional period before and after December 31 is equal to the $20,000 (40,000*90/180) accrued interest attributable to each fractional period.

Table A
Partial Amortization Schedule for ABC Bonds
Y=8.000%, Interest Expense Prorated

Date	Cash	Interest Expense	Increase (Decrease)	Net Bonds Payable	Accrued Interest Attributable
4/1/2009				$ 1,000,000.00	
10/1/2009	$ 40,000.00	$ 40,000.00	$ -	1,000,000.00	
12/31/2009		20,000.00	20,000.00	1,019,803.90	$ 20,000.00
4/1/2010	40,000.00	20,000.00	-20,000.00	$ 1,000,000.00	20,000.00
10/1/2010	40,000.00	40,000.00	-	$ 1,000,000.00	
12/31/2010		20,000.00	20,000.00	1,019,803.90	20,000.00
4/1/2011	40,000.00	20,000.00	-20,000.00	1,000,000.00	20,000.00

Now contrast Table A with Table B which shows the amortization schedule for the same period with Y=8%, but assuming that interest expense is equal to the change in the bonds present value. Significantly, this approach is theoretically correct and consistent with the "saw tooth" effect illustrated in Figure A. Observe that while interest expense for each whole interest period is equal to $40,000, the amount of interest expense attributable to the fractional periods before and after December 31 is not equal to $20,000 confirming that the supposed record keeping benefit is generalizable only to cases where interest is prorated.

Table B
Partial Amortization Schedule for ABC Bonds
Y=8%, Theoretically Correct Interest Expense

Date	Cash	Interest Expense	Increase (Decrease)	Net Bonds Payable	Accrued Interest Attributable
4/1/2009				$ 1,000,000.00	
10/1/2009	$ 40,000.00	$ 40,000.00	$ -	1,000,000.00	
12/31/2009		19,803.90	19,803.90	1,019,803.90	$ 20,000.00
4/1/2010	40,000.00	20,196.10	-19,803.90	1,000,000.00	20,000.00
10/1/2010	40,000.00	40,000.00	-	1,000,000.00	
12/31/2010		19,803.90	19,803.90	1,019,803.90	20,000.00
4/1/2011	40,000.00	20,196.10	-19,803.90	1,000,000.00	20,000.00

Finally, consider Table C which shows a partial amortization schedule for the same period, with interest expense calculated according to the theoretically correct approach used in Table B, but assuming Y=8.125%. Observe that the amount of interest expense for each whole period is no longer equal to the $40,000 periodic interest payment, neither is the amount of interest expense for any fractional period equal to the $20,000 accrued interest attributable to that period confirming that the supposed record keeping benefit is not generalizable to cases where Y≠R.

Table C
Partial Amortization Schedule for ABC Bonds
Y=8.125%, Theoretically Correct Interest Expense

Date	Cash	Interest Expense	Increase (Decrease)	Net Bonds Payable	Accrued Interest Attributable
4/1/2009				$ 991,552.86	
10/1/2009	$ 40,000.00	$ 40,281.83	$ 281.83	991,834.70	
12/31/2009		19,946.08	19,946.08	1,011,780.78	$ 20,000.00
4/1/2010	40,000.00	20,053.92	-19,946.08	992,127.98	20,000.00
10/1/2010	40,000.00	40,305.20	305.20	992,433.18	
12/31/2010		19,803.90	19,803.90	1,012,391.30	20,000.00
4/1/2011	40,000.00	20,196.10	-19,803.90	992,750.78	20,000.00

The author appreciates that the prorating approach being implicitly taught here is a practical shortcut and that the difference between the prorating and theoretically correct approaches on the income statement and balance sheet is generally not material. However, as educators we would prefer that textbook illustrations focus first on sound measurement principles. If shortcuts which deviate from these principles, such as prorating interest between fractional periods, are pedagogically expedient, we would prefer that they be clearly explained in a note or appendix. Regardless, we question the pedagogical usefulness of a shortcut that is applicable only under very limited circumstances such as the one claimed by these authors.

The Nonexistent Problem of Splitting Interest Payments

One textbook (Stice et al, 2010:701) states that recording accrued interest is a practice which helps avoid the "problem an issuer of bonds would have in trying to split interest payments for a given period between two or more owners of the securities." But there is no splitting of interest payments between the buyer and seller. For example, assume the original ABC example and that on May 1, 2009 each $1,000 par ABC bond is sold to a different investor for its actual price of $1,000.47 ($100.047344*10 rounded to the nearest whole cent). Assume further that on October 1, 2009, each of the bonds is still held by the original investor and so ABC or it financial intermediary electronically transfers $40 ($40,000/1,000) to each of the 1,000 bondholders. The author fails to see how the recognition on May 1 of accrued interest in the amount of $10,000 (100*8%/28*45/180*1,000) facilitates the 1,000 separate interest payments of $40 (100*8%/2) on October 1 or how the failure to record accrued interest on May 1 creates a "splitting problem" for the issuer.

CONCLUDING COMMENTS

Accrued interest is best understood in the context of the broker price announcing convention and preference for "clean" price data. Notwithstanding accounting textbook descriptions, accrued interest contributes little to our understanding of the exchange between buyer and seller, neither does the recognition of accrued interest convey to issuers of bonds any practical administrative benefit. If indeed accrued interest is nothing more than an arbitrary amount with little or no basis in economic reality (van Deventer and Imai, 1997:11), then perhaps it is time to reconsider its continued use in the bond accounting model.

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APPENDIX

Negative Accrued Interest

The ex-coupon date is set at a fixed number of days before each coupon payment and is the last date settlement must occur in order for the purchaser to receive the next coupon. Thus, bonds sold ex-coupon do not include the upcoming interest payment. Purchasers and sellers are aware of this and an appropriate adjustment to the actual price is made. Theoretically, the actual ex-coupon actual price (AP_{ex}) can be calculated by deducting the present value of the coupon retained by the seller from the value of AP per equation (1), which assumes receipt of the coupon:

$$AP_{ex} = AP\text{-}PV(Y/2.1\text{-}\alpha,0,\text{-}100*R/2) \qquad (A1)$$

According to the broker pricing convention, the ex-coupon quoted price (QP_{ex}) is calculated by <u>adding negative accrued interest</u> (NAI) to the ex-coupon actual price. The ex-coupon actual price is then announced to the market as the ex-coupon quoted price <u>less negative accrued interest</u>:

$$NAI = 100*R/2*(1\text{-}\alpha) \qquad (A2)$$

$$QP_{ex} = AP_{ex}+NAI \qquad (A3)$$

$$AP_{ex} = QP_{ex}\text{-}NAI \qquad (A4)$$

To be consistent with their treatment of accrued interest, the author suspects that each of the four accounting textbooks cited in this paper would describe negative accrued interest as the amount sellers pay to purchasers (or purchasers receive from sellers) to compensate them for the fractional period that they hold the bonds, but don't receive the coupon. However, like accrued interest, the transfer of negative accrued interest to buyers is a fiction drawn from the broker price announcing convention. As suggested by equation (A1), theoretically the present value of the coupon retained by the seller is already incorporated by the market into the actual ex coupon price.

NOTES