An Investigation Of The Effect Of Network Latency On Pedagogic Efficacy: A Comparison Of Disciplines

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ABSTRACT

E-learning has become a mainstream educational opportunity, as noted in U.S. News & World Report. Further, differences among college students have been documented in various disciplines. An experiment was conducted to determine the effects of network latency on pedagogical efficacy based on the students who were classified as in either humanities programs or engineering and science programs. The findings indicate that tolerances to screen update latencies are discipline-dependent and that students in engineering and science have a lower tolerance for screen update latency than students in the humanities.

Keywords: academic disciplines, e-learning, enjoyment, feedback delay, latency, learning styles, network delay, objective comprehension, pedagogical efficacy, self-reported comprehension

I. INTRODUCTION

The recent ranking of e-learning programs by *U.S. News & World Report* (2008) validates the arrival of this option for graduate study. At the classroom level, interactive web-based learning tools are increasingly integrated with other pedagogical activities. Flight simulators provide a staple to training programs. E-learning provides cost-effective, stimulating, convenient tools to convey the complex effect relationships and to provide "experienced-based" learning. For example, moving a slider can be used interactively to see how changing a resistor's value changes current flow through a current divider. A key component of e-learning is network delay between a student's action and the response from the program. Early research has indicated that an increase in the network delay impacts the learning experience (Squire, 2007).

Many studies have reported differences in learning styles, differences among academic disciplines and differences between subjects' perceived learning and actual learning. The application of e-learning technique crosses many academic disciplines. Perhaps the best known program, the University of Phoenix, offers in excess of 70 degrees at the associate through doctoral levels, in addition to certificate programs and individual courses. E-learning is not restricted to any particular group of individuals or disciplines. Consequently, developers of e-learning courses and software must understand how students in different academic disciplines react differently to e-learning courseware.

This paper will explore the effects of both the network delay and the subject's academic discipline on the learning experience of the subject. Using specifically developed interactive software application, which was embedded with different levels of network delay between the time a subject manipulates a control and an update appears on the computer screen, subjects were classified by the academic major. Finally, subjects assessed their

own perception of enjoyment and comprehension, while an objective measure of comprehension was calculated as well.

II.   LITERATURE REVIEW

The increasing demand for locally available higher education and the increasing technological capabilities requires the expanded use of e-learning on the web (Levin, 1998). Most cities have learning/testing centers offering on-line administration of most standardized tests, such as the Scholastic Aptitude Test (SAT) and Graduate Record Examination (GRE). Universities, such as the University of Phoenix (2008) have been offering distance-learning degrees for more than fifteen years. Newly admitted college students can take all of the pre-registration assessments online. The growth in the use of web-based learning tools and assessment will be continuing into the foreseeable future, which requires investigation into this new learning environment.

Learning requires the subject to complete a series of tasks in an appropriate order. Included in this sequence are "attention," "selective features of perception," and "semantic encoding." (Tuckman, 1992) The medium used to deliver the information to the learner affects the learner's ability to successfully complete each task. A key component of the learning process is feedback, which in a computer-based learning environment includes the time between the learner's input and the system's response via the computer screen, *latency*.

Campbell (1910) investigated the effects of delay on understanding the sender's speech when the budding telephone industry began to design echo suppression circuitry to improve transmission of speech. Brady (1971) reported that the relationship between the network delay, latency, and comprehension is not a simple linear relationship. He described the relationship as a two-piece linear function. At low levels of latency there was no apparent effect on comprehension, but at larger levels of latency he observed a rapid decline in understanding speech. Maddox et al. (2003) studied the effects of delaying feedback on different types of learning. Specifically, they compared the effects on rule-based skills, which require the learner to use an explicit reasoning process and on information integration skills, which require the learner to integrate existing knowledge. Their results suggested that the feedback delay had significantly negative impact information-integration learning, but not on rule-based learning. Pfordresher (2003) studied pianists' ability to perform short pieces with delayed auditory feedback. He reported that subjects receiving the delayed feedback had difficulty with the timing of the musical piece but did not make more errors than those subjects who received feedback on a traditional basis. Feedback delay definitely causes negative impact in the learning environment. However, based on Brady's study, the impact occurs after a threshold is reached in the delay.

Two additional aspects of learning have been reported in the literature. First, students in different disciplines demonstrate differences in learning. Common beliefs, such as, that engineers cannot write well and English majors cannot do mathematics, are based on stereotypes. However, actual differences among the students have been documented in previous studies. Vermunt (2005) analyzed differences among students in seven academic disciplines using the Inventory of Learning Styles. She reported that of all the variables, academic discipline revealed the strongest influence on learning style. Vermunt suggested that different academic disciplines "pose different demands on the way subject matter can best be studied." Second, self-assessment is not always consistent with actual performance. Bush (1989) demonstrated a mild inverse relationship between auditors' ability to predict future sales values and self-assessed confidence. Kruger and Dunning (1999) investigated the ability of students to assess their comedic abilities against the ratings of professionals. They reported that a negative relationship existed between the students' self-assessed abilities and those of the professionals except for the top quartile. Consequently, this paper also investigates the relationship between academic discipline and self-assessment of learning.

III.   METHODOLOGY

To assess the influence of latency on pedagogical efficacy, subjects were encouraged to participate in an experimental task to measure the effects of different levels of latency on three critical pedagogical aspects: subjects' self-reported enjoyment; subjects' comprehension, both as measured on a Likert scale; and objective comprehension as measured by multiple choice examinations.

An interactive software application was designed that embedded a hidden delay between the time a subject manipulates an interactive control and the time the feedback can be viewed on the screen. A screenshot of the application is shown in Figure 1, and the program and tutorial are available for download at http://academics.vmi.edu/ee_js/Research/Fourier_Synthesis/Fourier_Synthesis.htm. Different versions of the application were designed, each identical except for the delay (Squire, et al, 2008).

The experimental task (See Appendix.) appears to demonstrate how various functions can be created from sums of sinusoids. There were eight versions of the program. The only difference in each version was a hidden delay in multiples of 60 ms from zero to 420 ms. Subjects' enjoyment and learning are affected by the delay between user interaction with controls and screen update. Subjects were assigned randomly to each treatment. Subjects were required to report the treatment indicated on the title bar to ensure that subjects' responses were recorded for the correct treatment.

Based upon that data from a pilot study of 48 subjects, it was decided that 400 ms would be sufficient for the upper limit of latency. The choice of increment was based on a trade-off. There had to be sufficient number of bins to estimate the critical knee in the effects of latency and be few enough to include sufficient data in each bin. The study employed chose eight evenly-spaced latencies from zero to 420 ms which was consistent with the needs indicated by the pilot study.

A total of 251 students, from four universities, were randomly assigned to one of the eight applications. Authorization was obtained from the human subjects testing board to waive the requirement to inform students of the experimental nature of the task because: (1) The experimental task was completed anonymously. (2) The experimental task was administered as part of the academic curriculum. (3) The experimental task did not require an inordinate amount of time and had no adverse effects on the subjects. Subjects were not aware of the latency-testing aspect of the experimental task; they believed they were learning about applications of Fourier analysis. The testing was blinded to the subjects and scoring was blinded from the authors. The scoring was done by computer.

Each subject received a self-guided tutorial and the experimental task instrument including six demographic questions that included class year, age, gender, major, instructor, and university. The tutorial indicated which of the eight applications to download, including directions on how to download the correct version of the Fourier synthesis program. The remainder of the tutorial asked the subject to read a theory paragraph and to respond to a multiple-choice question that required the student to apply the information in the paragraph with the Fourier synthesis program. In total there were ten such paragraphs. The number of correct responses measured the subject's objective comprehension. Lastly, subjects self-reported their level of enjoyment and their level of comprehension.
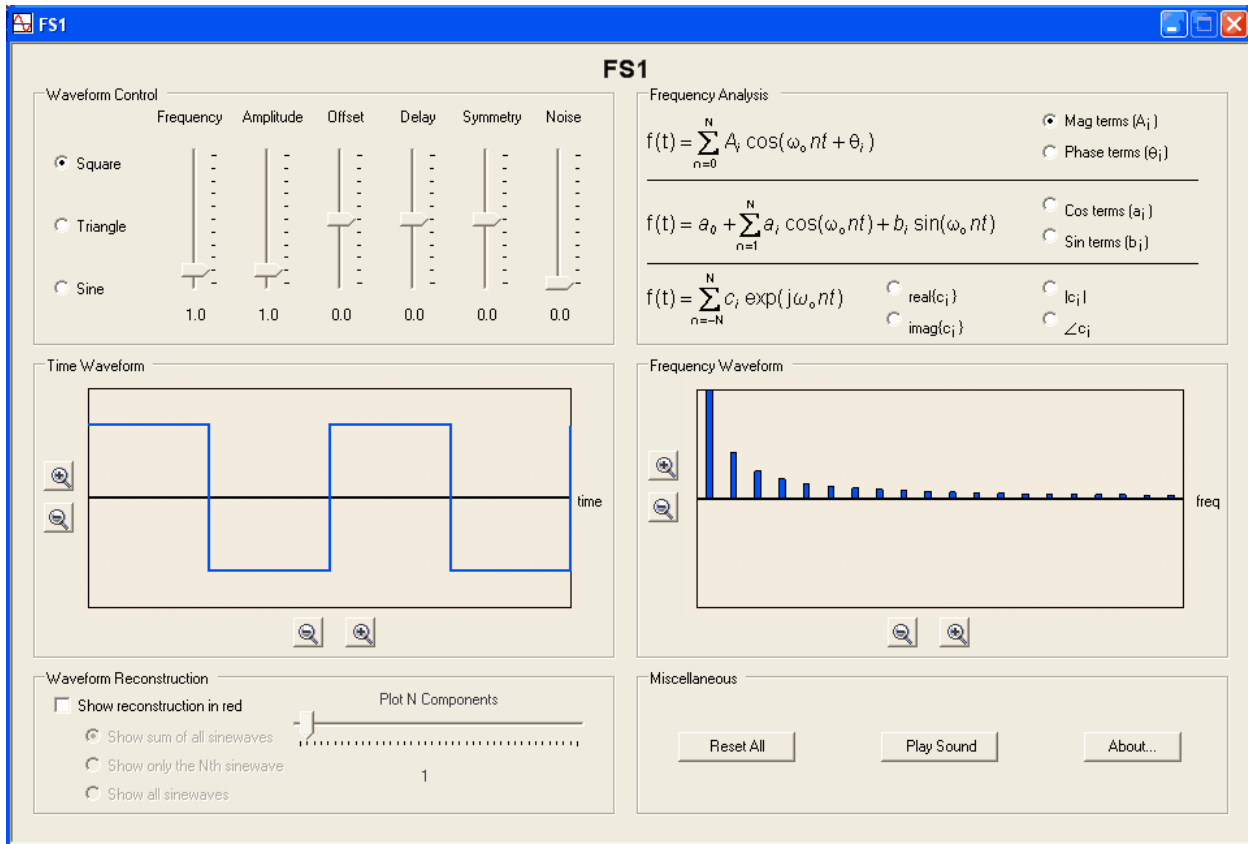
Figure 1: The Fourier Synthesis application program

A program, coded in Matlab, analyzed the data. The program scored the objective questions to provide a measure for objective comprehension. It also plotted the means of each variable at each level of latency with a bar for one standard deviation above the mean for each variable: enjoyment, objective comprehension, and self-reported comprehension. Further, the program provided a "best-fit" two piece continuous line or a single line to the data based on the calculated residuals. Finally, the standard deviation of the knee was estimated using Monte Carlo analysis techniques because the best-fit piecewise continuous lines are nonlinear.

III.   RESULTS

Comparative results of the relationship between latency and each of the variables – enjoyment, objective comprehension, or self-reported comprehension--are discussed below. Subjects were split between those which indicated a major classified as one in the "humanities" or as in "engineering or science." In total 96 subjects reported a major within the humanities and 155 subjects reported a major within the engineering or sciences. For each variable, we present the summary statistics and a best-fit bilinear (two piece continuous lines, one of which is horizontal) graph of the variable versus latency. The graph provides a bar graph for the mean response and an extension of one standard deviation above the mean.

Enjoyment vs. Latency

The mean responses on enjoyment for each group are plotted in Figure 2 and reported in Table 1. Both groups report a decline in mean response for enjoyment between the lower and higher level of latency. Although subjects in engineering and science express a slightly higher level of enjoyment when the latency is set at 0 ms, they

more quickly decline in enjoyment and appear to be strictly linear. The initial higher response of subjects from engineering and sciences may be an apparition caused by the familiarity of subjects with the experimental task and experiences created within the majors. The task involved manipulating various elements of the user interface, purportedly to teach Fourier analysis concepts.

Further, the difference in the responses indicate that there is a threshold before the level of enjoyment declines for subjects majoring in one of the humanities, where either there is no threshold or it is less than 60 ms for those majoring in engineering and science. The lack of a threshold effect for subjects majoring in an engineering and science field is also consistent with the notion of the familiarity with the experimental task. Subjects majoring in the humanities may initially experience additional frustration from the experiment, overriding any frustration caused by latency. Consequently, subjects experiencing a low level of latency would actually be expressing similar levels of frustration based on the experimental task.

Students of the Humanities   **Students of Engineering and Science**
Figure 2: The relationship between subjects' level of enjoyment and latency

Enjoyment									
	Latency (ms)	0	60	120	180	240	300	360	420
Humanities	Sample Size Mean Standard Deviation	12 4.583 0.996	14 4.286 0.726	14 4.357 0.842	15 4.600 0.633	10 4.500 0.527	13 3.462 0.877	9 4.222 0.441	9 4.000 0.8660
Engineering and Science	Sample Size Mean Standard Deviation	23 4.870 .344	23 4.174 .887	22 4.318 .9905	19 4.053 .848	15 3.467 1.126	15 3.533 .916	21 3.476 .8730	17 3.235 1.091

Table 1: Summary statistics of subjects' level of enjoyment and latency

Objective Comprehension vs. Latency

The results concerning the effect of latency on objective comprehension are similar to those related to enjoyment. Figure 3 provides the graphical representation of the mean responses on objective comprehension for each group. Table 2 provides the summary statistics. Both groups report a decline in mean response for objective comprehension between the lower and higher level of latency. Again, subjects in engineering and sciences score higher on objective comprehension and are more quickly affected by the increase in latency. Subjects from engineering and science begin to score lower when latency exceeds 180 ms, while subjects from the humanities begin to score lower when latency exceeds 360 ms.   Subjects from the humanities demonstrate a higher tolerance for increases in latency but are scoring lower on the objective comprehension. Further, the range of mean response is nearly approximately double for subjects majoring in the humanities. Although an increase in latency, in general, produces a reduction in objective comprehension, these differences are also consistent with the notion that the subjects from engineering and science disciplines are more familiar with the task. Hence, task familiarity may be a confounding variable.
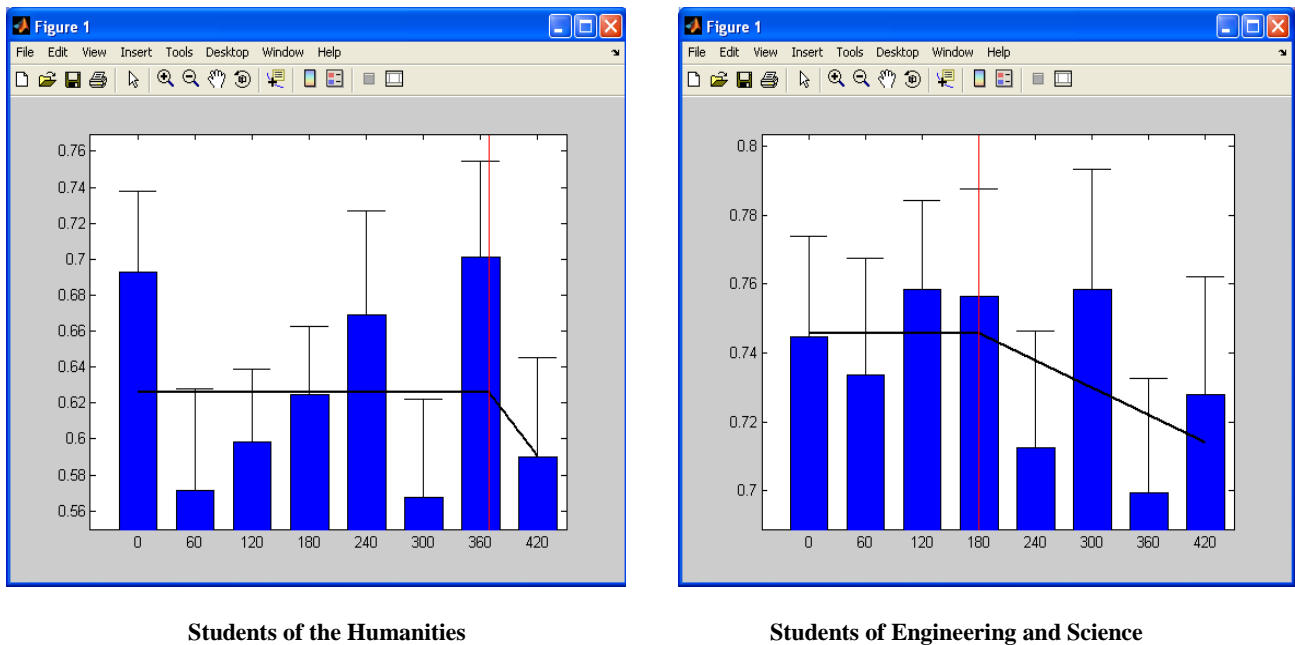

Students of the Humanities   **Students of Engineering and Science**

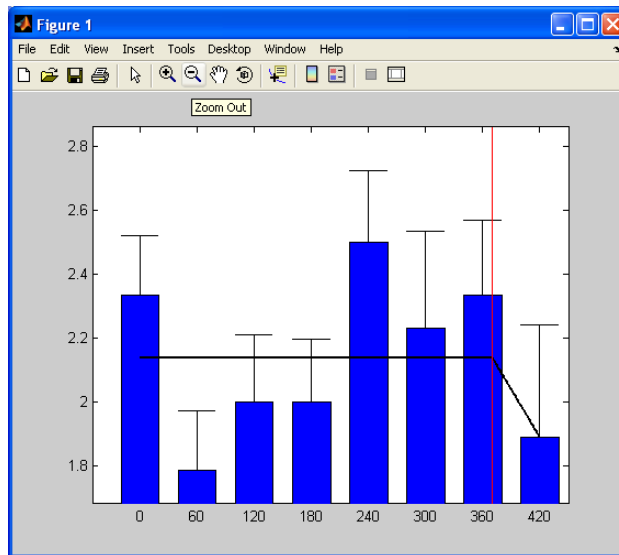**Figure 3: The relationship between subjects' level of objective comprehension and latency**

Self-Reported Comprehension vs. Latency

The results concerning the effect of latency on self-reported comprehension echo the previous findings. Figure 4 represents graphically the mean responses on self-reported comprehension for each group, while Table 3 presents the summary statistics. Higher levels of latency produced lowered mean responses for self-reported comprehension for both groups. Overall, subjects in engineering and sciences indicate higher levels of self-reported comprehension and are more quickly affected by the increase in latency. Self-reported comprehension has a notable difference, however. The thresholds, at which the effects of the higher level of latency begin to affect the mean responses, are closer. The difference is only 60 ms (360 less 300) for self-reported comprehension, while the differences were 180 ms (360 less 180 for objective comprehension and 180 and 0 for enjoyment) for the other variables. Unlike objective comprehension, self-reported comprehension has a greater range in the mean responses for subjects from the engineering and science disciplines. The inconsistency is similar to those found by Bush (1989). Overall, subjects experiencing a higher level of latency report lower levels of enjoyment and self-reported
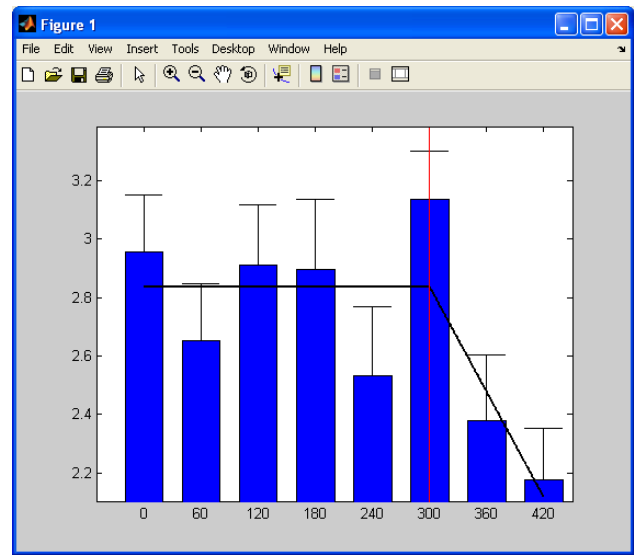
comprehension and score lower on objective comprehension. In additional, the subjects pursuing a degree in engineering were more comfortable with the experimental task.

Objective Comprehension									
	Latency (ms)	0	60	120	180	240	300	360	420
Humanities	Sample Size Mean Standard Deviation	12 0.693 0.156	14 0.571 0.211	14 0.598 0.153	15 0.625 0.146	10 0.669 0.184	13 0.567 0.198	9 0.701 0.1589	9 0.590 0.166
Engineering and Science	Sample Size Mean Standard Deviation	23 0.745 0.141	23 0.734 0.161	22 0.757 0.135	19 0.757 0.135	15 0.713 0.131	15 0.758 0.135	21 0.699 0.153	17 0.728 0.141

Table 2: Summary statistics of subjects' level of objective comprehension and latency

Students of the Humanities   **Students of Engineering and Science**

Figure 4: The relationship between subjects' level of self-reported comprehension and latency

IV.   CONCLUSIONS, LIMITATIONS, AND FUTURE CONSIDERATIONS

The responses indicate the two findings. First, an increase in user interface update latency of even 60 ms has a significant negative impact on objective learning. For each of the variables and for both groups of subjects, the mean responses were more negative at the higher levels of latencies. Although the exact relationships vary among the variables and for the groups, the impact was negative. With the exception of the enjoyment of the subjects pursuing degrees in engineering and science disciplines, there was a threshold below which there was not an impact from changes in latency, but after which the decline began. For those subjects pursuing engineering and science degrees, the decline in enjoyment began within 60 ms, the smallest interval in the study.

Self-Reported Comprehension									
	Latency (ms)	0	60	120	180	240	300	360	420
Humanities	Sample Size Mean Standard Deviation	12 2.333 0.651	14 1.786 0.699	14 2.000 0.785	15 2.000 0.756	10 2.500 .707	13 2.231 1.092	9 2.33 .707	9 1.889 1.054
Engineering and Science	Sample Size Mean Standard Deviation	23 2.957 0.928	23 2.652 0.935	22 2.909 0.972	19 2.895 1.049	15 2.533 0.916	15 3.133 0.640	21 2.381 1.0234	17 2.177 0.728

Table 3: Summary statistics of subjects' level of self-reported comprehension and latency

The second result is related to the experimental task. The experimental task required the subject to interact with a control panel associated with a visual response based on Fourier series. Those subjects who were classified as engineering or science students outscored the other group and were more sensitive to the changes in latency. Consequently, the experiment task may have intensified the effects for the subject from engineering and sciences, or hindered the effects for the subjects from the humanities. Further research will need to address the task to determine whether task or the personal bias of the subject groups is causing the differences.

The most significant limitation is the confounding of the experimental task and the definition of the two groups. The experiment clearly demonstrates the effects of the latency and the effects are different for the two groups. Future research will need to develop additional tasks which either are value neutral or favor subjects studying in the humanities. Another confounding variable which could be further studied is the effects created by pursuing a field of study. Subjects pursuing degrees that require the students to be more active with their computers may be acquiring a different attitude towards the task. Although latency and discipline affect the pedagogical efficacy, additional investigation will need to address the influence of prior learning experiences and the experimental task.

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Fourier Synthesis Tutorial
v.2.1

This tutorial is designed for students from many different academic levels and backgrounds. It occasionally uses terms from discipline-specific fields that you may not find familiar, but which you do not need to understand in order to complete the tutorial.

Background Information

The following information won't be used to identify you personally, but it will be used to identify ways to improve future software programs.

1. Please circle your college class year
   a. freshman
   b. sophomore
   c. junior
   d. senior
   e. grad student
   f. other (write in) _____

2. What is your age? _____

3. What is your gender? M / F

4. Name of your university _____

5. Name of your instructor_____

6. Please circle your major
   a. Computer Science
   b. Mathematics
   c. Engineering
   d. Humanities (history, English, music, etc.)
   e. Natural Science (physics, biology, chemistry, etc.)
   f. Social Science (economics, business, psychology, etc.)
   g. Undeclared or general studies
   h. Other (write in) _____

Download and Run the Program

7.   Point your browser to www.jimsquire.com. Go to *Research* and then to *Fourier Synthesis*. Download one application program from the table of 8 possible ones by right-clicking it, choosing "Save target as…" and then saving it to the desktop). As your instructor if you don't know which of the 8 programs you should download. Start by double-clicking the program (called FS with a digit, like "FS9.exe") on your desktop.

What is the name displayed on the title bar (the top of the program's window) that is running (e.g. "FS9")?
_____

Creating Waveforms

8.   Experiment with the Waveform Control upper-left box, and the "Square", "Triangle" and "Sine" waveform types. Results are shown in the Time Waveform plot. Briefly describe what each waveform type means:

    a.   Square_____

    b.   Triangle_____

    c.   Sine_____

9.   What does each of the following sliders do to the Time Waveform plot? You may find it useful to zoom in or out on the plot to observe the entire waveform; do this by using the magnifying glasses over the plot axes. Match the slider name on the left to its action on the right.

Slider Name	Action
___ Frequency	a. Adds a randomly-varying waveform
___ Amplitude	b. Alters the relative lengths of the rising segments and falling segments of each waveform
___ Offset	c. Shifts the waveform horizontally
___ Delay	d. Stretches the waveform horizontally
___ Symmetry	e. Shifts the waveform vertically
___ Noise	f. Stretches the waveform vertically

Finding the Sine Waves That Sum To Make An Arbitrary Waveform

A variety of operations are well-defined for symmetric, zero-offset sine waves, but not for other wave shapes. It is therefore useful to be able to approximate any periodic (repeating) waveform as a sum of sine waves, in which each of the summed sine waves has a unique frequency, amplitude, and delay. Remarkably, Jean Fourier published a method in 1822 that showed how to *exactly* recreate *any* wave shape as a (possibly infinite) sum of sine waves. (Mathematicians: yes, there are exceptions such as they can't have an infinite number of discontinuities; let's say any wave shape that you can draw can be decomposed into a sum of sine waves). If it's not obvious how a square wave with its discontinuities (vertical sides) can be fashioned using a summed set of smooth sine waves, it was not obvious to Fourier's contemporaries either, and the genius of his method ("Fourier series decomposition") was not widely recognized and put on firm mathematical ground until over 50 years after his death. (The secret, if you are interested, lies in the fact that an infinite number of sine waves have to be used in this case, and what "infinite" means in this particular sense).

The plot in the right of the program shows the Fourier series decomposition of the waveform plotted in the left window. Since both plots refer to the same waveform, avoid confusion by calling each by the left plot the "Time Waveform" and the right plot the "Frequency Analysis." Click the "reset all" button to show a symmetric square wave. The frequency analysis window shows a bar graph; the height of each bar represents the amount (the amplitude) of the sine wave at a frequency corresponding to the bar's location along the horizontal axis. The further the bar is to the right, the higher the frequency of the sine wave that it measures.

10. How does the frequency decomposition of the square, triangle, and sine waves change as you alter their amplitudes? (You may need to zoom out on the vertical axis of the frequency analysis plot to see everything by clicking on the icon of the vertical axis.) As the time waveform's amplitude increases, the height of the frequency decomposition bars (that correspond to the amplitude or height of sine wave at each frequency):

    a.   increase proportionally

    b.   decrease proportionally

    c.   most increase, but not all

    d.   do not change

11. Sine waves have 3 attributes: amplitude (their height), frequency, and phase (a shift to the left or right). You have so far used the program to determine the relative frequencies of the sine waves that sum to make an arbitrary waveform (the horizontal distances of each bar correspond to its frequency) and their amplitudes (the height of each bar corresponds to that sine wave's amplitude). To find each sine wave's phase, click the "phase terms" radio button in the upper-right panel. If you see nothing but a flat line it means that the phase of all the sine wave terms is zero. Try dragging the "delay" slider in the upper-left panel upwards to move the time waveform to the right. What happens to the phase of each sine wave?

    a.   each sine wave's phase increases by the same amount

    b.   each sine wave's phase decreases by the same amount

    c.   each sine wave's phase increases by an amount proportional to its frequency

    d.   d. each sine wave's phase decreases by an amount proportional to its frequency

The Fundamental Sine Wave

Press the "Reset All" button. Examine the square wave in the time plot window, and look at the frequencies and amplitudes of the sine waves that sum to create it in the frequency plot window. Move the "offset" slider in the upper left panel between 0 and 10 to translate the square wave up and down. That's like adding a constant value to the square wave, and a constant value has a frequency of zero. As you move the "offset" slider away from 0, a new half-width bar appears in the frequency plot window at 0 at the far left, corresponding to how much zero-frequency power is in the signal. Zero frequency is a constant because mathematically cos (0 t) = cos (0) = 1, a constant. The fundamental frequency of an arbitrary waveform refers to the frequency of the first non-zero-frequency sine wave…zero-frequency terms don't count.

Given the above, do the following sliders change the amount (the amplitude) or the frequency of the fundamental sine wave in an arbitrary waveform? (Try it!)

12. Frequency
    a.   changes only the amplitude of the fundamental sine wave
    b.   changes only the frequency of the fundamental sine wave
    c.   changes both the amplitude and frequency of the fundamental sine wave
    d.   changes neither the amplitude nor the frequency of the fundamental sine wave
13. Amplitude
    a.   changes only the amplitude of the fundamental sine wave
    b.   changes only the frequency of the fundamental sine wave
    c.   changes both the amplitude and frequency of the fundamental sine wave
    d.   changes neither the amplitude nor the frequency of the fundamental sine wave
14. Offset
    a.   changes only the amplitude of the fundamental sine wave
    b.   changes only the frequency of the fundamental sine wave
    c.   changes both the amplitude and frequency of the fundamental sine wave
    d.   changes neither the amplitude nor the frequency of the fundamental sine wave
15. Delay
    a.   changes only the amplitude of the fundamental sine wave
    b.   changes only the frequency of the fundamental sine wave
    c.   changes both the amplitude and frequency of the fundamental sine wave
    d.   changes neither the amplitude nor the frequency of the fundamental sine wave
16. Symmetry
    a.   changes only the amplitude of the fundamental sine wave
    b.   changes only the frequency of the fundamental sine wave
    c.   changes both the amplitude and frequency of the fundamental sine wave
    d.   changes neither the amplitude nor the frequency of the fundamental sine wave

Frequency Components Non-Zero Only At Some Harmonics

The Fourier decomposition of a periodic signal, such as the ones we are analyzing, does not have sine wave components at all frequencies; rather they only occur at integer multiples of the frequency of the periodic wave we are analyzing. That is, a 2 Hz square wave (that is, a square wave that makes 2 complete cycles each second) can be modeled as a sum of sine waves of various amplitudes and delays but with frequencies exactly equal to 2 Hz, 4 Hz, 6 Hz, 8 Hz, etc. The frequency of the periodic signal being analyzed is called the "fundamental frequency". The sine wave of the fundamental frequency is the left-most bar in the analysis window, and is usually the tallest, indicating that a fairly decent approximation to the original signal can be made using a single sine wave of that frequency. To see how good that approximation is, click the "Reset All" button, then the "Triangle" waveform type, and then the "show reconstruction in red" button in the lower-left panel. The red waveform shows the time waveform made by summing up a number of sine waves set by the slider titled "Plot N Harmonics". Mathematically, if you use the top equation selected in the upper right hand panel with N=1 you construct the red plot. Intuitively we see that just a single sine wave (called the "fundamental frequency" sine wave gives a fairly good approximation to a symmetric triangle wave.

17. Drag the "Plot N Component" slider to the right until the slider reads "4" to draw in red the waveform that is a sum of 4 sine waves of double, triple, and quadruple the frequency of the fundamental sine wave. Which of the following waveforms of different symmetries can be most accurately reconstructed as a sum of 4 sine waves? (all waveforms have a frequency, amplitude, and offset of 1 and a delay and noise of 0).

    a.   A square wave of symmetry = 0, frequency = 1, amplitude = 1, offset = 1, delay = 0, noise = 0
    b.   A triangle wave of symmetry = 0, frequency = 1, amplitude = 1, offset = 1, delay = 0, noise = 0
    c.   A square wave of symmetry = 9, frequency = 1, amplitude = 1, offset = 1, delay = 0, noise = 0
    d.   A triangle wave of symmetry = 9, frequency = 1, amplitude = 1, offset = 1, delay = 0, noise = 0

18. We have already noted that the Fourier decomposition of a periodic signal has sine waves appearing only at integer multiples of the periodic signal's fundamental frequency, called "harmonics". Some of these harmonics may have zero value. The simplest example is in the Fourier decomposition of a sine wave. Clearly, that can be perfectly modeled by a single sine wave of its fundamental frequency. Try it: click "reset all", then the "sine" wave type. There's only one blue bar, and by pressing the "show reconstruction in red" it's apparent that the waveform is perfectly reconstructed using a single sine wave. Next, drag the "symmetry" bar and note that immediately many harmonics appear. Now click the "reset all" again and choose the square wave. Drag it's "symmetry" slider. What is special about its harmonics when symmetry is set to zero? (If you can't make it exactly zero using the slider, use the "reset all" button again).

    a.   The amplitude of its even harmonics are zero
    b.   The amplitude of its odd harmonics are zero
    c.   The frequency of its even harmonics are zero
    d.   The frequency of its odd harmonics are zero

Student Evaluation

19. How irritating did you find the pause between the time you moved a slider and the time the screen updated?

    a. Not noticeable
    b. Slightly irritating
    c. Fairly irritating
    d. Very irritating
    e. Extremely irritating

20. How confident are you in your responses?

    a. Not sure at all
    b. Slightly confident
    c. Fairly confident
    d. Very confident
    e. Extremely confident

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