Monte Carlo Simulation Of The Portfolio-Balance Model Of Exchange Rates: Finite Sample Properties Of The GMM Estimator

Hong-Ghi Min, (Email: hmin@icu.ac.kr), Information and Communications University, Republic of Korea

ABSTRACT

Using Monte Carlo simulation of the Portfolio-balance model of the exchange rates, we report finite sample properties of the GMM estimator for testing over-identifying restrictions in the simultaneous equations model. F-form of Sargan's statistic performs better than its chi-squared form while Hansen's GMM statistic has the smallest bias.

INTRODUCTION

Consistent estimation of linear simultaneous equations model requires minimal set of identifying restrictions in the system and validity of those restrictions can be tested if a model is over-identified. While some statistics are developed and those statistics are widely applied to test the specification of the estimated equations, test the validity of the instrumental variables, and to test if the estimated equations encompass the reduced form little is known the small sample properties of those statistics. For this reason, We investigate the small sample properties of four different over-identifying test statistics for simultaneous equations model including Hausmann(1990)'s Likelihood-ratio test, Sargan(1958, 1960)'s minimum characteristic root statistic, Sargan(1980)'s corresponding F-statistic (Sargan, 1980), and Hansen(1982)'s GMM statistic.

RESPONSE SURFACE METHODOLOGY, TEST STATISTICS, AND THE DGP

Response Surface Methodology

To overcome the specificity of Monte Carlo studies to the particular parameter and sample sizes employed, we utilize response surface methodology of Hendrey (1984). Let π_T be the finite sample probability of the test statistic lying in the critical region. In an experiment the DGP generates N sets of replications and the statistics lie in the critical region S out of N times. An explicit relationship can be defined as equation (7) in section 2.3.

$$E(s) = \pi_T = f(\theta, T) \tag{1}$$

Monte Carlo estimator s gives:

$$E(s) = \pi_T = f(\theta, T) + \varepsilon,\ \varepsilon_i \sim D(0, \pi_T[1- \pi_T]/N) \tag{2}$$

Let π_a is the analytically calculable asymptotic power of the test and $p^+(\theta,T)$ is the discrepancy between the finite sample and asymptotic power, i.e., $T^{-1/2}\{p(\theta,T^{-1/2}) - q(\theta,T^{-1/2})\}$.

Then,

$$p^+(\theta,T) = T^{-1/2}p(\theta,T^{-1/2}) \tag{3}$$

holds.

$p^{+}(\theta, T^{-1/2})$ is $O(T^{0})$ and is a polynomial in powers of $T^{-1/2}$ and the elements of θ.
Using (1)-(3), we can set up a stochastic relationship between a feasible and unbiased estimator of π_T and the unknown quantity π_a, θ, T as equation (4).

$$s - \pi_a = T^{-1/2} q(\theta, T^{-1/2}) + e \tag{4}$$

where $q(\theta, T^{-1/2})$ is an approximation to $p(\theta, T^{-1/2})$ and the error e is the combination of ε and $T^{-1/2}\{p(\theta, T^{-1/2}) - q(\theta, T^{-1/2})\}$. The choice of $q(\theta, T^{-1/2})$ is arbitrary but a finite polynomial in θ and $T^{-1/2}$ is common and coefficients on the terms of $q(\theta, T^{-1/2})$ may be estimated by least squares.

The Test Statistics And Their Asymptotic Properties

We consider four statistics whose statistical descriptions are given in Table 1.

Table 1: Statistics Of Over-Identifying Restrictions

Statistics			Asymptotic Distributions (Degrees of freedom)
Name	**Type**	**Sources**	
C_0	IV	Sargan (1958)	Chi-squared (n)
F_0	IV	Sargan (1980)	Chi-squared with (m-k_0)
G_0	GMM	Hansen (1983)	F(m- k_0,T-m)
L_0	Likelihood- ratio test	Harvey (1990)	Chi-squared(n)

(1). T is the econometric sample size, m is the number of instrumental variables, and k_i is the number of coefficients estimated under H_i . (2). The statistic F_0 is [C_0/ (m-k_0)]*[(T-m)/(T- k_0)]/[1- C_0/(T- k_0)] which is C_0/ (m-k_0) plus finite sample adjustment arising from the finite sample boundedness of C_0. (3). n is the number of over-identifying restrictions in the system.

The Data Generating Process: Portfolio-Balance Model Of Exchange Rates

True reduced form [equations (7)-(8)] is derived from the true structural form [equation (5)-(6)]. For alternative structural form, $x_{2,t-1}$ and $x_{7,t-1}$ are falsely included into the system [equations (9)-(10)].

$$y_t = \alpha_1 Y_t + \beta_1 (y_{2,t} - x_{7,t}) + \gamma_1 X_t + \varepsilon_{1,t}, \ (\varepsilon_{1,t} : \varepsilon_{2,t})' \sim NID\ (O, \Sigma) \tag{5}$$

$$Y_t = \alpha_2 + \delta y_{1,t} + \gamma_2 X_t + \varepsilon_{2,t} \tag{6}$$

where (y_t, Y_t)' and X_t are 2×1 and 7×1 vectors of endogenous and exogenous variables at time t (t=1,….,T); $X_t' = (x_{1t}, x_{2t}, x_{3t}, x_{4t}, x_{5t}, x_{6t}, x_{7t})$; β and δ are simultaneity parameters.

$$y_t = \alpha_3 + \gamma_3 X_t + \mu_{1t}, \ (\mu_{1t} : \mu_{2t})' = \Omega \tag{7}$$

$$Y_t = \alpha_4 + \gamma_4 X_t + \mu_{2t} \tag{8}$$

Alternative structural model is given as equations (9) and (10).

$$y_t = \alpha_5 + \beta_1 (y_{2,t} - x_{7,t-1}) + \beta_2 (x_{2,t} - x_{2,t-1}) + \gamma_5 X_t + \varepsilon_{1,t}, \ (\varepsilon_{1,t} : \varepsilon_{2,t})' \sim NID\ (0, \Sigma) \tag{9}$$

$$Y_t = \alpha_6 + \delta y_{1,t} + \gamma_6 X_t + \varepsilon_{2,t} \tag{10}$$

Independent samples of endogenous variables are generated by the population defined in equation (5)-(10) $\dot{a}$ la Basmann (1960). Error terms are generated from linear transformation of ε_i -N(0,1) and coefficients of transformation are from the Choleski decomposition of the variance-covariance matrices. The fraction of rejections s (=S/1,000) is an unbiased Monte Carlo estimator of the unknown finite sample rejection frequency π_T .

EXPERIMENTAL DESIGN AND COMPUTATIONAL ASPECTS

The Monte Carlo design variables for the econometric model (7)-(9) are θ and T , where,

$$\theta = (\beta, \delta, W, T) \in \Theta = \{\theta \,|\, |\Omega| > 0; |\Sigma| > 0\} \tag{11}$$

and $W = |\Sigma| / |\Omega|$, Σ is the variance-covariance matrix of the reduced form equations and Ω is the variance-covariance matrix of the structural form equations.

$$T \in \Im = [T_a, T_b] \tag{12}$$

where T_a and T_b are the smallest and largest econometric sample size considered. For a given statistic ϕ (say) and critical value δ , the objective of Monte Carlo study is to find statistics' finite sample rejection frequency $\pi_T \equiv prob(|\phi| \geq \delta)$ for the DGP and those relationships of interest over $\Theta \times \Im$.

$$\pi_T \equiv prob(|\phi| \geq \delta \,|\, \theta, T) = g(\theta, T) \tag{13}$$

The value of key parameters β_1, δ, W, T cover a range typical of econometric models estimated with actual data: β_1 =(0.3,0.8), β_2 =2.244, δ =(0.4,0.7), T=(20,40,60,80), W =(0.4,0.6,0.8,0.9). All other parameters are from reported values of Branson et al (1979). The number of replications is 1,000. Given the choice of parameters, a full factorial design is adopted, resulting in 64 experiments in all. Structural model is estimated by two-stage least squares and Ordinary least squares are used for the reduced form. The asymptotic powers of four statistics were calculated with critical values corresponding to the 5 percent.

POST-SIMULATION ANALYSIS

This section approximates the finite sample properties of the test statistics by various analytical and numerical-analytical formulae, and examines how well these formulae perform. Response surface regressions are reported in Table 2.

Nominal Size Of Four Statistics

Response surface of nominal size is reported in the first panel of the Table 2. Most of the estimated coefficients are significant but that of S_0. Size is well approximated for G_0 while S_0 is poorly approximated by the sample size and the ratio of the determinants.

Using a conservative estimate of 0.016 for the standard deviation of sample proportions, we find that most estimators are significantly larger than the nominal size of 0.05. Most strikingly, L_0 mostly reject true null hypothesis implying that we need to a small sample adjustment for L_0. Sargan (1980)'s Chi-squared test (S_0) is biased and over-reject in most of the cases in finite sample except for the case when simultaneity parameters are β =0.8, δ =0.4, and T=80. F-form of Sargan's statistic (F_0) is less biased than S_0 but properties of F_0 resemble those of S_0 as sample size increases. Hansen's GMM statistic (G_0) is also biased but nominal size of G_0 approaches to the value of 0.05 while S_0 departs significantly from 0.05. G_0 has the least bias for the nominal size.

Asymptotic Power Of Four Statistics

Figure 1 shows that, for most of the cases, estimated nominal power decreases as the econometric sample size increases and it increases with the increase of w. Asymptotic power is best approximated for G_0 and then F_0 while S_0 is poorly approximated.

Table 2: Response Surface Regressions

PANEL A			
	Nominal Size of		
	G_0	**S_0**	**F_0**
Constant	2.5151(0.2439)**	1.9174(0.3184)**	1.7900(0.3278)**
T	-0.0324(0.0032)**	-0.0203(0.0043)**	-0.0186(0.0044)**
W	0.9434(0.1802)**	0.6384(0.2444)**	0.8187(0.2516)**
τ	-57.991(6.085)**	-36.458(7.942)**	-46.874(8.177)**
Adj R^2	0.607	0.261	0.405

PANEL B			
	Asymptotic Power of		
	G_0	**S_0**	**F_0**
Constant	2.1533(0.2025)**	1.5877(0.3051)**	1.8591(0.3032)**
T	-0.0270(0.0027)**	-0.1363(0.0041)**	-0.0173(0.0041)**
W	0.7237(0.1496)**	0.4901(0.2341)*	0.8497(0.2327)**
τ	-45.537(5.052)**	-28.064(7.61)**	-48.597(7.563)**
Adj R^2	0.6082	0.1627	0.5082

PANEL C				
	Finite Sample Power of			
	L_0	**G_0**	**S_0**	**F_0**
Constant	0.0603(0.3088)	-0.4286(0.1573)*	-0.1383(0.207)	0.1934(0.1711)
T	0.0040(0.0100)	0.0085(0.0021)**	0.0027(0.0029)	0.0193(0.0023)**
W	0.0100(0.2281)	-0.1859(0.1162)	0.0663(0.1596)	-0.0444(0.1330)
τ	-1.5383(7.704)	12.401(3.924)**	1.6375(5.2384)	1.9975(4.1438)
Adj R^2	0.0856	0.2041	0.0101	-0.0623

Note: Double asterisks denote significance at 1 % critical level and single asterisk for 5%.

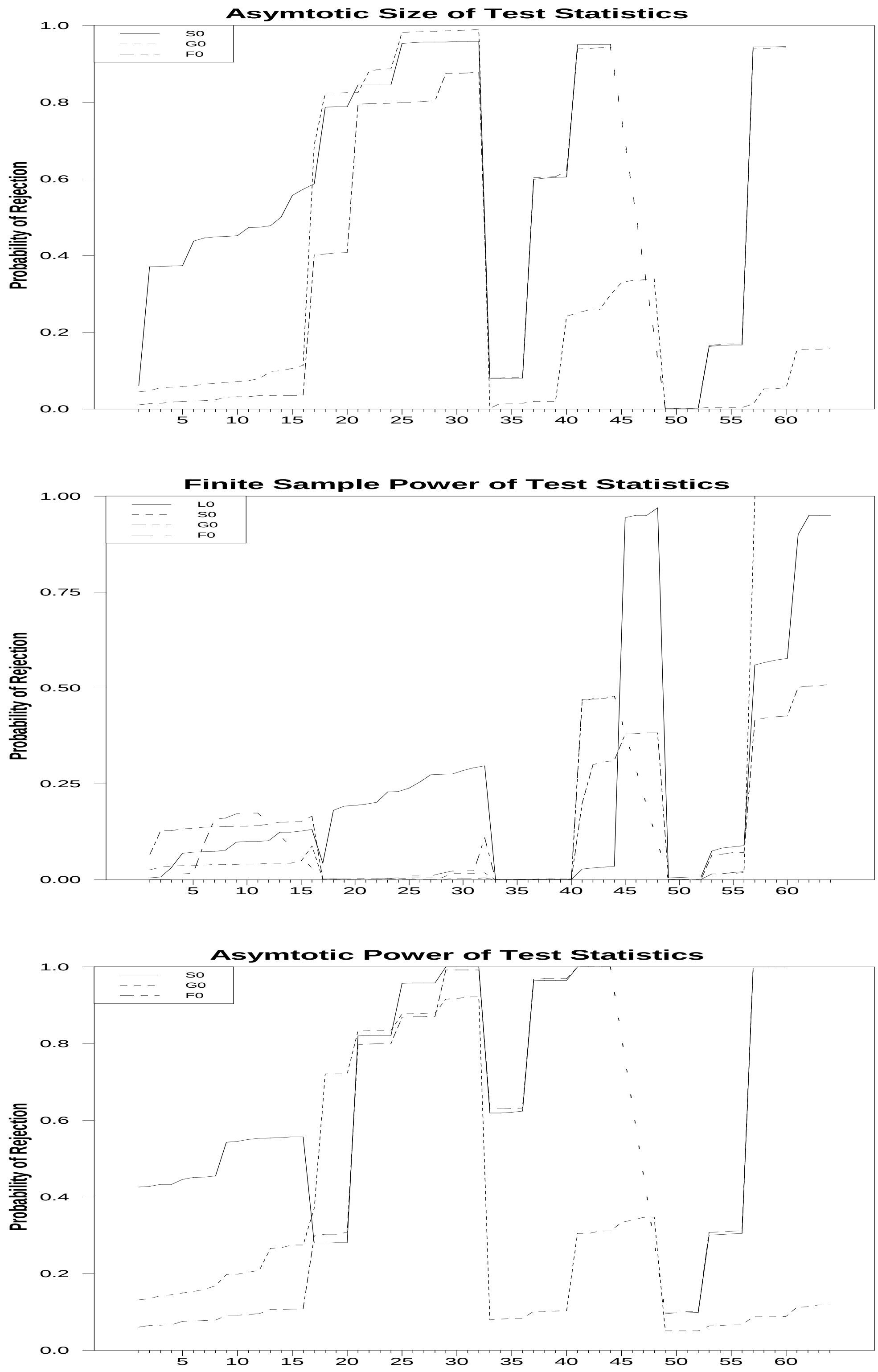
L_0 has rejection probability of one in most of the cases and S_0 appears to be more powerful than F_0 and G_0 unless simultaneity parameter is large. Another interesting feature is that S_0 outperform F_0 even in small samples but G_0 has the smallest rejection probability.

Finite Sample Power Of Four Statistics

In order to compare tests in terms of their power of a given size, the critical value for each test is set with reference to the empirical distribution of the statistic corresponding to the empirical size of 0.05. In each replication, the false null is rejected if the test statistic exceeded the empirical critical value. Most of the estimates for the response surface regressions reported in Table 2 are insignificant and they poorly approximate the finite sample power of these statistics. However, for G_0, the estimated coefficient of T is significant and finite sample power of G_0 increases with the increase of T. For all statistics, empirical power increase with the increase of sample size and empirical power decrease with the increase of w for most of the cases.

Out of four statistics, L_0 is the most powerful test statistic in most of the cases and its finite sample power increase with the increase of the econometric sample size. G_0 comes next and then F_0 and $S_{0.}$ However, performance of F_0 and S_0 are quite similar. Estimated finite sample power is much smaller than its asymptotic power substantially. Most significant departure between asymptotic power and finite sample power comes for S_0 while G_0 approximates its asymptotic power well. Finite sample power of L_0 is larger than other statistics and this increases with the increase of econometric sample size.

Figure 1: Finite Sample Properties Of L_0, G_0, S_0, and F_0



CONCLUSIONS

We conducted a Monte Carlo experiment to investigate the small sample properties of the four statistics which test the over-identifying restrictions in the simultaneous equations model. The likelihood-ratio test tends to reject the null hypothesis even when the errors in the model are consistent with the statistic's embodied hypothesis in the two equations model in finite sample. Small sample adjustment of Godfrey and Pesaran (1983) can be a solution for this bias. The problem seems less severe with test statistics of S_0, F_0, and G_0. The use of G_0 helps a little but it is also biased in finite sample. F-form of Sargan's statistic (F_0) performs better than its Chi-squared form (S_0). Hansen's GMM statistic (G_0) has the smallest bias. Different methodology and software may allow more extensive design and more efficient simulation and control variate might help estimate π_T more efficiently.

REFERENCES

1. Basmann, R. L., 1960, On Finite Sample Distributions of Generalized Classical linear identifiability Test Statistics, *Journal of American Statistical Association*, 55, 650-659.
2. Branson, W. H., Halttunen, H., and Masson, P., 1979, Exchange Rates in the Short-run: Some Further Results, *European Economic Review*, 12, 395-402.
3. Byron, R. P., 1974, Testing Structural Specifications Using the Unrestricted Reduced Form, *Econometrica*, 42, 869-883.
4. Ericsson, N. R., 1991, Monte Carlo Methodology and the Finite Sample Properties of the Instrumental variables Statistic for Testing Nested and Non-nested Hypothesis, *Econometrica*, 42, 869-883.
5. Godfrey, L. G. and Pesaran, M. H., 1983, Tests of Non-nested Regression Model: Small Sample Adjustment and Monte Carlo Evidence, *Journal of Econometrics*, 21, 133-154.
6. Harvey, A., 1990, *The Econometric Analyses of Time Series*, London:Phillip Allan.
7. Hansen, L. P., 1982, Large Sample Properties of Generalized Method of Moments Estimator, *Econometrica*, 50, 1029-1054.
8. Hendrey, D. F., 1984, Monte Carlo Experimentation in Econometrics, Chapter 16 in *Handbook of Econometrics*, volume 2, ed. By Grilliches and Intrilligatot, M. D., Amsterdam: North-Holland. 937-976.
9. Sargan, J. D., 1958, The Estimation of Economic Relationships Using Instrumental Variables, *Econometrica*, 26, 393-415.
10. Sargan, J. D., 1980, Some Approximations to the Distribution of Econometric Criteria Which are Asymptotically Distributed as Chi-squared, *Econometrica*, 48, 1107-1138.