Risk Sharing

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ABSTRACT

The amount of risk sharing among countries is theoretically affected by trade policy, market openness, and monetary policy. To what extent each of these characteristics affect risk sharing is the object of this study. I use a risk sharing index akin to a correlation coefficient and compare it with the correlation between the returns on several market indexes, and with the correlation between changes in consumption. I find that the financial integration of markets does not necessarily imply risk sharing or co movements in consumption. The results also suggest that two countries being aligned in monetary policy do not ensure risk sharing.

Keywords: International Finance, Risk Sharing

INTRODUCTION

The financial crises of the last decade have shown a remarkable capacity to rapidly spread across borders, affecting not only financial markets, but also the level of consumption of several regions of the world. Integration in the world markets can be financial, economics, or both. The question is whether these different types of integration imply each other. It is not clear, a priori, that increased financial integration implies increased correlation in consumption, nor it is clear that it implies increased risk sharing.

I use a risk sharing index akin to a correlation coefficient and compare it with the correlation between the returns on several market indexes, and with the correlation between temporal changes in consumption. I find that the financial integration of markets does not necessarily imply risk sharing or co movements in consumption. The results also suggest that two countries being aligned in monetary policy, as measured by the risk free rate, do not ensure risk sharing.

Integration of markets can be defined in several ways; markets can be considered to be integrated if assets with similar risk traded in different markets yield the same return (Bekaert and Harvey, 1995), a definition that is mostly concerned with financial assets. Integration can also be economic, in which case we expect their population consumption changes to be similarly affected by world economic conditions. Economic and financial integration implies some influence of the return on market indexes over output (see Dumas, Harvey and Ruiz, 2003). If individuals smooth consumption through time by buying a set of securities today that will pay off in the future (Cochrane, 2005; Duffie, 1988), and world financial markets are integrated (Bekaert and Harvey, 1995; Carrieri, Errunza and Hogan, 2003), changes in the value of foreign securities may affect home consumption via a change in the ratio of present to future consumption by individuals. In perfect and completely integrated markets, the changes in consumption should be such that the intertemporal rate of substitution between countries is perfectly correlated (Cochrane, 2005; Gollier, 2004), implying perfectly correlated stochastic discount factors. What is more, under the assumption of equal utility functions, changes in consumption between countries should be perfectly correlated and depend only on the state of the world economy. In small agrarian villages in India individual consumption changes are correlated, and depend mostly on issues that affect the whole village and not on issues that affect individuals, suggesting a level of efficient risk sharing (Towsend, 1994). At the international level, several of the homogeneities existing in small villages, or in a particular country likely cease to exist, suggesting that risk sharing across countries may be low. Backus, Foresi, and Telmer (2001) show that the stochastic discount factors for two countries follow the relationship $m_h e_{t+1}/e_t = m_f$, where m_h and m_f refer to the stochastic discount factor in the home and foreign country respectively, and e refers to the exchange rate.

The integration in world capital markets that has occurred since the 1980's (see Bekaert and Harvey, 2000), due mainly to liberalizations in emerging countries, should bring increased risk sharing (Bekaert, Harvey, and Lundblad, 2005), which should ultimately be reflected in increased consumption correlations across countries.

Suppose two countries whose flows of capital are liberalized but have no commercial exchange (no flow of goods). If we were to measure integration by means of the correlation between the stock market indexes, we would likely conclude that these two countries are highly integrated. If, to the contrary, integration is measured by the correlation between the stochastic discount factors, according to Brandt, Cochrane, and Santa Clara (2006) we would likely conclude that there is no integration. The flows of goods, services, and capital vary widely across countries, which suggest the exercise of comparing the results of the possible measures used for integration. An index based on the stochastic discount factor is used to assess the degree of risk sharing existing between countries.

Financial market integration is measured by an index similar to the one used to measure risk sharing using the stochastic discount factors. This index is akin to a correlation coefficient; in fact, the calculations show that their value is practically the same. High values of the index suggest that the countries have integrated financial markets. A caveat with the interpretation of correlated indexes is that if two indexes are highly correlated but the assets in said indexes cannot be bought by individuals from another country, the integration is nonexistent despite the high correlation. Despite this caveat, I include the index between markets as one of the measures to be used for comparison because the markets used have been long opened to the world.

As stated before, when measuring integration between countries, consumption should be taken into consideration. The integration of the countries based on consumption is measured by an index similar to the one used to measure risk sharing. This index is compared with the one obtained based on market returns and the risk sharing index obtained by using the stochastic discount factors.

LITERATURE REVIEW

The extent to which markets are integrated, both financially and economically, is an issue that seems particularly important in light of the events of the 2007 financial crisis, which began in the financial markets but spilled later to the real economy. Bekaert and Harvey (1995) posit that the world financial markets have become more integrated through time, especially after the 1980s, where barriers to flow of capitals were lowered, mostly in emerging markets. Carrieri, Errunza, and Hogan (2007) study the integration of eight emerging markets to the world market, they find that country risk is still relevant; their study also supports increased market integration through time. Brandt, Cochrane, and Santa Clara (2006) compare the volatility of changes in exchange rates with the volatility of changes in the stochastic discount factors for several countries; they conclude that marginal utility growth should be highly correlated across countries; implying high risk sharing. Dumas, Campbell and Ruiz (2003) conclude that under the integrated markets hypothesis, national output is a good predictor of stock returns, a result that links the financial markets with the real economy. One of the benefits of integrated markets is increased risk sharing, which lowers the cost of capital and increases growth (Bekaert, Harvey, and Lundblad, 2005). I measure risk sharing as the correlation between the countries' intertemporal rate of substitution, hence, two countries have high level of risk sharing if changes in their marginal utilities of consumption are highly correlated. Risk sharing, as measured by intertemporal rate of substitution, require in principle flow of goods between countries, without it the exchange rate would adjust to exactly offset the relative differences in capital market returns and will have no effect in the relative marginal utility of consumption (Brandt, Cochrane, and Santa Clara, 2006). One of the objectives of this paper is to assess whether flow of goods between countries and risk sharing are related.

Investors allocate their resources between investment (deferred consumption) and present consumption, where the proportion allocated to each depends on their time preference. A corollary to the two fund theorem states that an investor will hold a portfolio formed by the risk free asset and the risky asset, or the market, as it is considered in CAPM. Harvey (1988) states that the yield curve can predict future consumption, the idea of this assertion is that if investors expect difficult economic times ahead, they will buy the risk free asset to smooth consumption, pushing up its price and lowering its return. Also, if investors decide to buy the risk free asset, they will consume less today. Thus, if the risk free asset return effectively reflects the proportion of wealth that investors dedicate today to consumption and investment, the relative changes of the risk free asset return between two

countries could be indicative of the relative consumption changes between them. Thus, relative changes between the countries risk free asset returns can be linked to relative changes in the stochastic discount factor. By comparing risk sharing based on stochastic discount factors and in the risk free rates, the question asked is whether a similar monetary policy, reflected in similar risk free rates, is reflected in the risk sharing index.

METHODOLOGY

To find a relation between the stochastic discount factor and the risk free asset return, I begin with the relation between the stochastic discount factors for two countries presented in Backus, Foresi and Telmer (2001). This relation is presented in equation 1, where m^i, i in [h,f] represent the stochastic discount factor and e^s, s in [t,t+1], represent the exchange rate in direct quote.

$$\ln(m^h) - \ln(m^f) = \ln(e^{t+1}/e^t) \quad (1)$$

To find a relationship between the stochastic discount factor and the risk free asset return, the uncovered interest rate parity relation is used. Writing the uncovered interest rate parity between t and t + 1 as

$$e^{t+1}/e^t = \exp(r^h - r^f) \quad (2)$$

and replacing it in equation (1) we find that.

$$\ln(m^h) - \ln(m^f) = r^h - r^f \quad (3)$$

Taking variance at both sides of equation (3) suggests that the variation in the difference of the stochastic discount factors between countries ought to be equal to the variation in the difference of their risk free rates. Equation (3) suggest a relation between monetary policy and the intertemporal rate of substitution, it suggest that two countries that maintain a constant difference between their risk free interest rates will have perfect risk sharing. Perfect risk sharing is characterized by the equality of the marginal utility of consumption in the countries of interest, or equal stochastic discount factors (Cochrane (2005), Gollier (2004)). If perfect risk sharing between countries occur, we have $m^h = m^f$, from where, taking logarithms at both sides, subtracting and taking variance we get

$$\text{var}(\ln m^h - \ln m^f) = 0 \quad (4)$$

Equations (3) and (4) suggest that if the difference in the risk free rates is constant for two countries, they will have perfect risk sharing. To test whether risk sharing obtained by using stochastic discount factors differs from what is obtained by using risk free rates, I calculate the variance for both sides of equation (3). In order to do so, the risk free security is defined following Black (1972), meaning, orthogonal to all other securities in the market but possessing variance. This definition contrasts with the definition where the risk free rate has no variance. The definition of a risk free asset as an asset that has no variance has the implication, under interest rate parity, that risk sharing is perfect across countries. The definition of a risk free asset as one that has variance but is orthogonal to the rest of the assets does not have such implication. The equation used to calculate the stochastic discount factor is

$$E\left[\frac{d\Lambda}{\Lambda} I_N\right] + E[dr] + E\left[\frac{d\Lambda}{\Lambda} dr\right] = \mathbf{0} \quad (5)$$

Where Λ represents the stochastic discount factor, dr is the [Nx1] vector of stochastic returns and I_N is the [Nx1] vector of ones. Since the risk free asset is assumed to be orthogonal to all other assets and with variance different from zero, the return on the risk free asset is stochastic and given by

$$dr_f = r_f\, dt + dz_f \quad (6)$$

where it is assumed that $E[dz_f dz_i] = 0$ for all $i \neq f$.

For all assets different than the risk free asset, the return is given by

$$dr_i = \mu_i\, dt + dz_i \tag{7}$$

in particular, for the foreign currency, the return dr = de/e, where e correspond to the direct quote exchange rate. Also, the stochastic discount factor satisfies

$$\frac{d\boldsymbol{\Lambda}}{\boldsymbol{\Lambda}} = \alpha dt + \beta dz \tag{8}$$

Where α and β are constants to be determined for each country.

Consider two countries, A and B each endowed with a risk free asset and a stock market; also, consider that these countries allow unrestricted trading in securities between them. With these conditions, solving for the constants α and β for countries A and B and replacing in equation (8) gives the following expression for the stochastic discount factors:

$$\frac{d\boldsymbol{\Lambda}_A}{\boldsymbol{\Lambda}_A} = -\left[r_{fA} + \sigma_{fA}\right]dt + r_A'\Sigma_A^{-1}\mathbf{dz_A} \tag{9}$$

And

$$\frac{d\boldsymbol{\Lambda}_B}{\boldsymbol{\Lambda}_B} = -\left[r_{fB} + \sigma_{fB}\right]dt + r_B'\Sigma_B^{-1}\mathbf{dz_B} \tag{10}$$

Where r_{fA} corresponds to the return on the risk free asset for country A and σ_{fA} corresponds to the standard deviation of the return on the risk free asset for country A. The [Nx1] vector r_A corresponds to the vector of returns for all securities different than the risk free and the [NxN] matrix Σ_A corresponds to the covariance matrix of the securities in country A, including currency and excluding risk free rate. The same notation applies for the securities in country B.

To evaluate equation (3), the natural logarithm of the stochastic discount factor has to be calculated; applying Ito's lemma to d(lnΛ) yields the following expressions for $d(\ln\Lambda_A)$ and $d(\ln\Lambda_B)$.

$$d\left[ln\Lambda_A\right] = -\left[r_{fA} + \sigma_{fA} - 0.5 r_A'\Sigma_A^{-1}r_A\right]dt + r_A'\Sigma_A^{-1}\mathbf{dz_A} \tag{11}$$

And

$$d\left[ln\Lambda_B\right] = -\left[r_{fB} + \sigma_{fB} - 0.5 r_B'\Sigma_B^{-1}r_B\right]dt + r_B'\Sigma_B^{-1}\mathbf{dz_B} \tag{12}$$

Equations (11) and (12) imply that the variances of $d[\ln\Lambda_A]$ and $d[\ln\Lambda_B]$ and the covariance between them is given by:

$$var\left[d[ln\Lambda_A]\right] = r_A'\Sigma_A^{-1}r_A\,dt \tag{13}$$

$$var\left[d[ln\Lambda_B]\right] = r_B'\Sigma_B^{-1}r_B\,dt \tag{14}$$

$$cov[dln\Lambda_A\, dln\Lambda_B] = r_A'\Sigma_A^{-1}\Sigma_{AB}\Sigma_B^{-1}r_B\,dt \tag{15}$$

Where $\Sigma_{AB}\, dt = E[dz_A dz_B]$.

From equation (4), perfect risk sharing between countries requires that

$$\frac{2cov[dln\Lambda_A\, dln\Lambda_B]}{var[dln\Lambda_A] + var[dln\Lambda_B]} = 1 \qquad (16)$$

Notice that the expression in the left in equation (16) is in the region [-1,1] and takes the value zero if there is no risk sharing and the value 1 in the case of perfect risk sharing, making it akin to a correlation coefficient. Hence, risk sharing will be measured by the following expression:

$$\frac{2cov[dln\Lambda_A\, dln\Lambda_B]}{var[dln\Lambda_A] + var[dln\Lambda_B]} \qquad (17)$$

To make measures comparable, the integration of financial markets and the co-movements in consumption will be measured by using an index similar to equation (17), with the change in the stochastic discount factor being replaced by return on the market and change in consumption respectively.

EMPIRICAL RESULTS

The equations presented are calculated for the US, Canada, France and England. For all the countries, the risk free rate is approximated by the yield on the three month zero coupon bond issued by the treasury of the respective country; the stock market indices utilized are FTSE100, S&P 500, TSX composite, and CAC40. The data is monthly from January 1999 to December 2008.

The characteristics of the zero coupon bonds, indexes, and currencies are presented in table 1.

Table 1: Variable characteristics

variable	N	average	std
return on the TSX index	120	0.0389	0.1612
return on CAD	120	0.0261	0.0840
three month t-bill, Canada	121	0.0345	0.0034
return on the S&P 500	120	-0.0247	0.1503
three month t-bill, US	121	0.0312	0.0050
return on the CAC-40	120	-0.0076	0.1866
three month t-bill, France	123	0.0308	0.0028
return on the EUR	120	0.0254	0.1007
return on the FTSE-100	120	-0.0388	0.1489
three month t-bill, UK	121	0.0461	0.0027
return on the GBP	120	-0.0091	0.0844

Characteristics of the variables used to calculate the stochastic discount factors. The average is the annualized monthly average and the standard deviation is the annualized monthly standard deviation.

The variance of the stochastic discount factors and the covariance of the countries stochastic discount factors is:

For Canada and the US
$var[dln\Lambda_{US}] = 0.070$, $var[dln\Lambda_{CAN}] = 1.996$, $cov[dln\Lambda_{US} dln\Lambda_{CAN}] = 0.112$

For the UK and the US
$var[dln\Lambda_{US}] = 0.031$, $var[dln\Lambda_{UK}] = 1.173$, $cov[dln\Lambda_{US} dln\Lambda_{UK}] = 0.088$

For France and the US
$var[dln\Lambda_{US}] = 0.035$, $var[dln\Lambda_{FR}] = 0.884$, $cov[dln\Lambda_{US} dln\Lambda_{FR}] = -0.0507$

Based on the results for the variances and covariances of the stochastic discount factors, the risk sharing index as presented in Equation (17) is for US/Can = 0.108, US/UK = 0.147, and US/FR = -0.110. The results from calculating the risk sharing index suggest that there is more risk sharing between the US and the UK than between the US and Canada, which may be surprising given the commercial ties and geographic closeness of the US and Canada. Also, the results suggest that marginal rate of substitution in France moves in opposite direction than in the U.S., implying risk shifting rather than risk sharing.

If risk sharing is effectively related with the flow of goods between countries, the biggest commercial partner with the US should have the biggest risk sharing coefficient. The average, maximum and minimum percentage of US imports and exports from (and to) Canada, the UK, and France are presented in table 2.

Table 2: Percentage of exports / imports of goods from the US perspective

Country	Average (percent)	Max (percent)	Min (percent)
Canada	22.6 / 17.9	23.9 / 19.4	20.1 / 16.0
UK	4.9 / 3.4	5.7 / 3.8	4.1 / 2.8
France	2.5 / 2.3	2.7 / 2.7	2.2 / 2.0

Percentage of the total exports and imports from, and to, the US that go to Canada, the UK, and France. The time series is from 1997 to 2008. The data is presented as exports / imports.

Table 2 shows that, out of the three countries considered, Canada is the biggest commercial partner with the US for both exports and imports, with the UK being the second and France the third. Considering the magnitude, US exports to Canada are on average 4.6 times those to the UK; which in turn are approximately double the exports to France. The relation between imports from the US is very similar to the exports. To the contrary, the risk sharing index between the US and UK is 1.35 times the risk sharing index between the US and Canada. The comparison of the risk sharing index with the flow of goods between the countries suggests that bigger flow of goods does not necessarily imply higher risk sharing.

The possibility of trading securities in international markets supposedly increases risk sharing and lowers the cost of capital (Bekaert, Harvey, and Lundblad, 2005; Arrow, 1964; Diamond, 1967). As a comparison to the risk sharing coefficient obtained by using the stochastic discount factor, I calculate the index presented in equation (17) between the stock markets returns of the US, France, Canada and the UK. The index between the stock markets of US and Canada is 0.742, between the US and France it is 0.835 and between the US and UK it is 0.836. The risk sharing coefficients obtained by using the stochastic discount factors are not totally in line with the index obtained between stock markets. The highest risk sharing coefficient, between the US and UK, corresponds to the highest index between stocks markets. The second highest index between the markets corresponds to the U.S. and France, while the risk sharing index between these countries is the lowest. Canada has the second highest risk sharing index after the UK, but the index between markets in the third, after the UK and France. A caveat with the previous calculation is that for some countries only a part of the index is within reach of foreign investors (Carrier, Errunza, and Hogan, 2007; Bekaert, Harvey, and Lundblad, 2005) which could distort the comparison between risk sharing index and index returns. I expect the fact of a proportion of the index not being available to foreign investors not to be a problem in this case because the markets used in this study are all developed markets long opened to the world. The results of the comparison between the market returns index and the risk sharing index suggest that the degree of integration of capital markets is not necessarily related with risk sharing. Also, the comparison of the risk sharing indexes between the countries considered in this study suggests that distance is not a decisive factor for integration and risk sharing.

When central banks set the reference rate, the objective is to influence the economy through incentives for spending and investing (lower interest rates), or the opposite. From this perspective, the marginal utility of consumption can be modified by changes in the interest rate, which will be reflected in changes in the stochastic discount factor. Equation (3) proposes a relation between the differences in the risk free rates and the stochastic discount factors.

I calculate the variance for both sides of equation 3, the results are presented in Table 3.

For the three pairs of countries considered, the term var $[r^h - r^f]$ is practically zero while the term var$[d\ln\Lambda_A - d\ln\Lambda_B]$ is on average 1.296; these numbers show the volatility of the stochastic discount factor to be much higher than the volatility of the difference between the risk free rates.

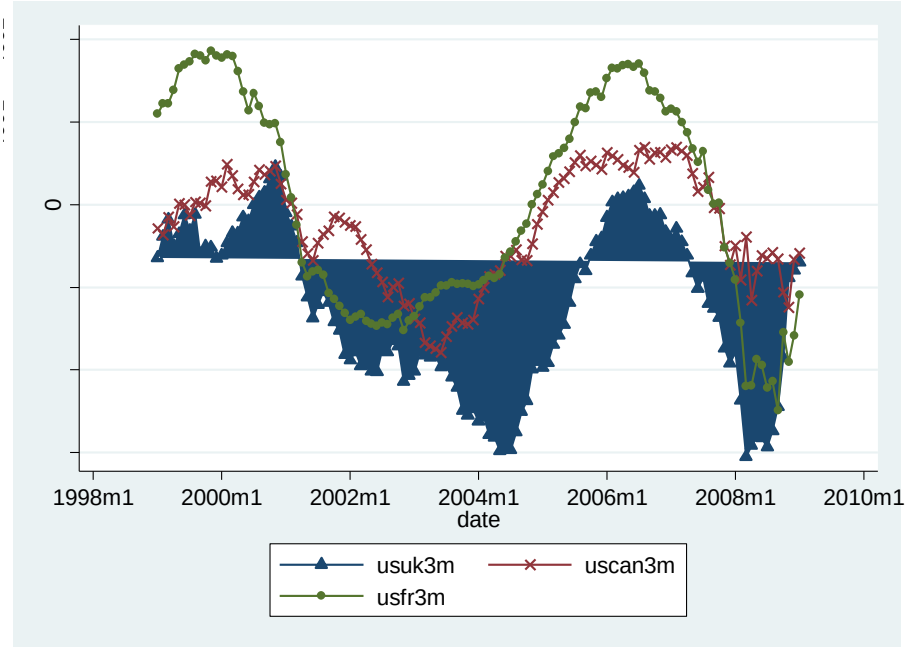
The volatility of the difference in the risk free rates suggests perfect risk sharing across the countries in the study, which is not supported by the stochastic discount factors. Figure 1 presents the difference in the risk free rates for the time considered.

Table 3: Characteristics of both sides of equation 3

	N	var $[r^h-r^f]$	var$[d\ln\Lambda_A - d\ln\Lambda_B]$
U.S./Canada	121	4.63e-7	1.843
U.S./U.K.	121	9.59e-7	1.026
U.S./France	121	1.64e-6	1.020

Characteristics of the difference in the yield on the risk free asset, the yields are annualized.

Figure 1: difference between the risk free rates of the US, Canada, UK, and France



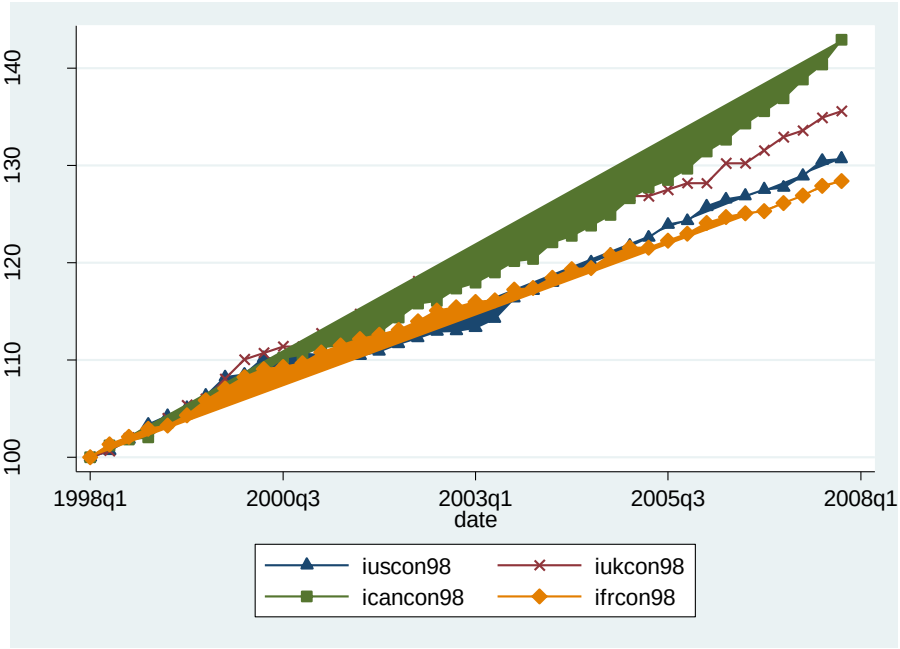
The figure presents the difference between the 3 month t-bill rate of the US and the UK (usuk3m), the US and Canada (uscan3m), and the US and France (usfr3m). The rates are not annualized.

The risk sharing implied by the difference in the risk free rates compared with the risk sharing implied by the difference in the stochastic discount factor suggest that consumption may not be readily influenced by changes in the risk free rate. A situation where consumption was not influenced by changes in the risk free rate happened in Japan, in the mid 1990's.

Ultimately risk sharing as used in this paper refers to consumption, hence, the next step is to study the index between the changes in consumption for the countries used for this study. Consumption for the countries used in this study is presented in figure 2 (year 1980 = 100).

I expect the changes in consumption to follow the stochastic discount factor, low risk sharing should imply low correlation between changes in consumption, the opposite should be true of high risk sharing. To calculate the differences in consumption and their correlation, the quarterly series of individual consumption for the US, Canada, France, and the United Kingdom is utilized. The index between the differences in consumption is obtained by calculating the difference between the logarithm of consumption at time t and t-1 for each of the countries, and then calculating the index according to equation (17). Individual consumption is quarterly data covering from 1980 to2007 and is obtained from the national accounts presented by the corresponding central banks. The index for changes in consumption for the countries used in this study is 0.293 between the US and the UK; between the US and Canada is 0.427, and between the US and France it is 0.040.

Figure 2: Indexed consumption for the US, UK, Canada, and France



The figure presents the indexed consumption, with the first quarter 1998 equal to 100, for the US (iuscon98), UK (iukcon98), Canada (icancon98), and France (ifrcon98).

The index for changes in consumption does not agree with the risk sharing index, the highest index for co-movements in consumption correspond to the U.S. with Canada, while the risk sharing between them is less than the risk sharing between the U.S. and the UK. The results from the index for co-movements in consumption compared with the results from the index for market returns suggest that the latter does not determine the former.

CONCLUSION

Integration in the world markets can be financial, economics, or both. The question is whether these different types of integration imply each other. It is not clear, a priori, that increased financial integration implies increased correlation in consumption, nor it is clear that it implies increased risk sharing. I find that high correlation in the financial markets is not related with high risk sharing or with increased correlation in consumption. I also find that high flow of goods between countries does not imply high levels of risk sharing, suggesting that flow of goods may not be as necessary to achieve risk sharing. As it is to be expected, flow of goods (imports/exports) is associated with the correlation in consumption.

AUTHOR INFORMATION

Paolo Miranda has a Bachelor in Mechanical Engineering, Master in Business Administration, and a Ph.D. in Finance. He has taught at the University of New Mexico and Purdue University Calumet and worked for years in the Engineering industry. His research interests are International Finance, Asset Pricing, and Corporate Governance.

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