Generalized Hyperbolic Distributions And Value-At-Risk Estimation For The South African Mining Index

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ABSTRACT

South Africa is a cornucopia of mineral riches and the performance of its mining industry has significant impacts on the economy. Hence, an accurate distributional assumption of the underlying mining index returns is imperative for the forecasting and understanding of the financial market. In this paper, we propose three subclasses of the generalized hyperbolic distributions as appropriate models for the Johannesburg Stock Exchange (JSE) Mining Index returns. These models are shown to outperform the traditional assumption of normality and accommodate for a number of stylized features, such as excess kurtosis and volatility clustering, embedded within the financial data. The models are compared using the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC) and log-likelihoods. In addition, Value-at-Risk (VaR) estimation and backtesting were also performed to test the extreme tails. The various criteria utilized suggest the generalized hyperbolic (GH) skew Student's t-distribution as the most robust model for the South African Mining Index returns.

Keywords: FTSE/JSE Mining Index; Hyperbolic Distribution; Normal-Inverse Gaussian (NIG) Distribution; Generalized Hyperbolic Skew Student's *t*-Distribution; Value-at-Risk

INTRODUCTION

As a country in profusion of natural resources, South Africa's mining sector accounts for a considerable portion of the world's production and reserves in mineral riches such as gold, platinum, and coal. The mining sector is also crucial to South Africa's socio-economic development due to its historical importance and significant contribution to economic activities. Apart from this, when compared to other sectors of the South African market (e.g. the financials, industrials, or the tourism), the mining sector has its unique characteristics, such as its exposure to short-term extremes. For example, sudden and violent wildcat strikes as seen in 2012 resulted in a halt to mining activities. This resulted in short-term volatility spikes in share prices of mining companies and in the sector as a whole. Recently, the performance of the mining stocks has again come under further public interest due to strikes and gradual decline in output. The FTSE/JSE Mining Index (J177) series, which contains various major mining companies in South Africa, was developed with the intention to provide investors and analysts with reflections on the performance of the mining sector in the JSE. Hence, an accurate distributional assumption of the mining index may assist academics and practitioners alike with robust modeling and forecasting of the South African financial market and related applications. However, a gap exists in the current literature to identify the most appropriate distributional form of the mining index. In particular, one that may best describe or forecast the behavior of its returns under periods of market anomalies and extreme occurrences.

Although financial returns are classically assumed to be normally distributed, it is now widely documented that they really exhibit non-zero skewness and excess kurtosis. Mandelbrot (1963) has shown that returns data display heavier tails than the Pareto and Gaussian distributions and Fama (1965) found that extreme movements in financial returns occur more often than predicted by Gaussian models; i.e., returns are fat tailed. Ryderberg (1999) and Peiró (1999) also found that the actual distribution of financial returns illustrates asymmetric properties. Furthermore, Aas and Haff (2006) showed that returns of equity prices, exchange rates, and interest rates, measured over a short time interval, are often skewed with one heavy tail and one semi-heavy (or more Gaussian-like) tail. Various other stylized facts, such as volatility clustering and long range dependency, have also been studied in the literature (Tsay, 2010). A common model suggested for returns is the generalized autoregressive conditional heteroscedastic (GARCH) model of Bollerslev (1986). However, the GARCH model fails to fully explain long range dependencies.

Recently, the generalized hyperbolic distributions (GHDs) of Barndorff-Nielsen (1977) have been suggested to fit financial returns. Not only does the GHDs cater for skewness embedded in the data, but also accounts for extreme events that are not gradual in nature. Hence, it may be deemed to be particularly suitable in modeling the returns of the Mining index under focus. Since its formulation, GHDs has been successfully employed in diverse disciplines such as physics and biology (Blaesid & Sorensen, 1992). Eberlein and Keller (1995) were among the first to apply these distributions to finance and used the hyperbolic subclass to fit German financial data. This work was extended by Prause (1999) who applied GHDs to model financial data on German stocks and American indices. However, Vee, Gonpot, and Sookia (2012) has shown that the notion of different indices are depicted by the same distribution is false. Hence, whether the GHDs will provide similar levels of accuracy in fitting the South African Mining Index is not immediately obvious. Thus, robust statistical analysis and tests are required in order to obtain accurate results and make robust conclusions. In this paper, we pursue the above-mentioned and explore possible avenues to make further claims on the general applicability and practicality of our results.

In this paper, three subclasses of the GHDs, namely the hyperbolic, the normal-inverse Gaussian (NIG) and the generalized hyperbolic (GH) skew Student's *t*-distributions, are utilized for modeling the South African Mining Index returns. These distributions exhibit heavier tails and depict the excess kurtosis in financial data more accurately. The hyperbolic distribution has tails that behave exponentially, while the two tails of the NIG distribution are semi-heavy and non-identical. In addition, the NIG distribution is able to model both symmetric and asymmetric distributions. One would therefore expect the NIG distribution to model skewness well. However, it is only appropriate in situations where the tails are not too heavy. The GH skew Student's *t*-distribution stands out from the GHD subclasses with an important property in that its one tail has polynomial and the other exponential behavior. Such dissimilarity is suitable for modeling substantially skewed and heavy tailed data (Aas & Haff, 2006). Finally, we evaluate the performances of these models through several statistical tests and backtesting of their Value-at-Risk (VaR) estimates. VaR is widely used as a tool in the risk management arena[1]. Incorrect assumptions and inaccurate forecasts of VaR have certain implications, such as inadequate capitalization due to over- or underestimation of risk exposure. It is widely accepted that firm are more prone to failures due to the shortage of capital resulting from underestimation of VaR. However, a recent study by Beling, Overstreet, and Rajaratnam (2010) has also shown that there is a negative profit impact due to the misestimation of VaR in either direction under the Basel framework. Hence, this paper also adds to the current body of knowledge on accurate forecasts of VaR models, and the reduction of negative impact on profit due to the inaccuracies of VaR estimations.

The remainder of this paper is structured as follows. In Section 2, descriptive analyses of the South African Mining Index returns are provided. Section 3 introduces the generalized hyperbolic distributions and the three subclasses used for this research. Empirical estimation of the GHD parameters and goodness-of-fit test results are discussed in Section 4. Section 5 is devoted to VaR estimation and backtesting on the GHD subclasses. Finally in Section 6, we summarize and discuss our findings.

[1] Despite its criticisms in the recent economic meltdown, recent studies have shown that the majority of global banks have yet to abolish the use of VaR models (see Deloitte, 2013). Rather, the number of participants has increased since 2010.

DATA

The data used in this research is the daily closed JSE Mining Index supplied by McGregor BFA. It covers 3162 observations from 2 January 2001 to 22 August 2013. We take the first differences of the natural logarithm of the JSE daily mining indices; i.e., mining index returns. For an observed index level P_t, the corresponding one-day log-return on day *t* is defined as:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

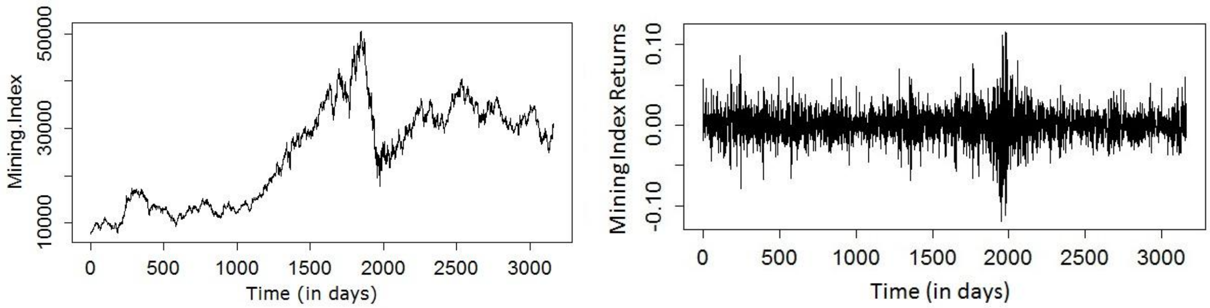

Figure 1: Time Series Plots of JSE Mining Index (left) and its Daily Log-Returns (right)

Figure 1 provides the time series plot of the daily JSE Mining Index and its one-day return series. The figures indicate heteroscedasticity and volatility clustering for the return series. A number of isolated extreme returns caused by unforeseen events or shocks to the mining industry are noticed, such as the 2009 financial crisis. The high frequency of extreme events is also evidenced on the Q-Q plot in Figure 2, showing that the tails of the data are significantly heavier than those of the normal distribution.

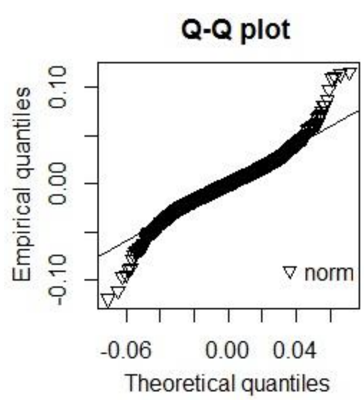

Figure 2: Normal Q-Q Plot of JSE Mining Index Returns

Stationarity of the return series are tested using the Augmented Dickey-Fuller (ADF) and the Philips-Perron (PP) unit root tests. The ADF test is set to lag 0 using the Schwartz Information Criterion (SIC) and the PP test is conducted using the Bartlett kernel spectral estimation method. Results reported in Table 1 indicate that the null hypothesis of unit root is rejected. Therefore, the return series of the JSE Mining Index can be considered to be stationary.

Table 1: Results for ADF and PP Unit Root Tests for Mining Index Returns

Unit Root Test	Test Statistic	*p*-value
ADF test	-53.50578	0.0000
PP test	-53.53378	0.0000

Table 2: Descriptive Summary Statistics of JSE Mining Index Returns

Minimum	Standard Deviation	Skewness	Kurtosis	Jarque-Bera Statistic (*p*-value)	Maximum	Mean	N
-0.119659	0.019720	0.045584	6.53579	1647.7 (0.00000)	0.116163	0.000434	3161

Table 2 provides descriptive statistics for the return series in consideration. We observe that the mean of the JSE Mining Index returns is positive, indicating that the overall mining index was slightly increasing. Furthermore, the large kurtosis of 6.53579 indicates the leptokurtic behavior of the returns. The series has a distribution with tails that are significantly fatter than those of the normal distribution. There is a small skewness of 0.045584, indicating that the two tails of the returns may behave slightly differently as commonly observed in financial returns (Rydberg, 1999; Aas & Haff, 2006). The Jarque-Bera test statistic also indicates that the data are non-normal. These factors motivate an analysis using both symmetric and asymmetric heavy tailed distributions.

THE GENERALIZED HYPERBOLIC DISTRIBUTIONS

The univariate GH distribution can be parameterized in many ways. We follow Prause (1999) and let:

$$f_X(x) = \frac{\left(\alpha^2-\beta^2\right)^{\lambda/2} K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)\exp\left(\beta(x-\mu)\right)}{\sqrt{2\pi}\alpha^{\lambda-1/2}\delta^{\lambda}K_{\lambda}\left(\delta\sqrt{\alpha^2-\beta^2}\right)\left(\sqrt{\delta^2+(x-\mu)^2}\right)^{1/2-\lambda}} \tag{1}$$

In the above expression, K_j is the modified Bessel function of the third kind of order j (Abramowitz & Stegun, 1972) and the parameters must fulfill the following conditions:

$\delta \geq 0, |\beta| < \alpha,$ if $\lambda > 0$
$\delta > 0, |\beta| < \alpha,$ if $\lambda = 0$
$\delta > 0, |\beta| \leq \alpha,$ if $\lambda < 0$

Various subclasses of the GHDs are obtained via different assumptions made on the parameters. In this paper, we concentrate on three of these subclasses.

The Hyperbolic Distribution

The hyperbolic distributions are characterized by having a hyperbolic log-density function whereas the log-density for the normal distribution is a parabola. Thus, one may expect the hyperbolic distributions to be coherent alternatives for heavy tailed data. The hyperbolic distribution is defined as a normal variance-mean mixture where the mixing distribution is the generalized inverse Gaussian (GIG) law with parameter $\lambda = 1$; i.e., it is conditionally Gaussian. More specifically, a random variable has the hyperbolic distribution if its pdf is given by:

$$f_{HYP}(x) = \frac{\sqrt{\alpha^2-\beta^2}}{2\alpha\delta K_1(\delta\sqrt{\alpha^2-\beta^2})} e^{-\alpha\sqrt{\delta^2+(x-\mu)^2}+\beta(x-\mu)} \tag{2}$$

where K_1 denotes the Bessel function of the third kind with index 1. The first two of the four parameters, namely α and β with $\alpha > 0$ and $0 \leq |\beta| < \alpha$, determine the shape of the distribution with α representing the gradient and β, the skewness. $\delta > 0$ is the scale parameter and $\mu \in \mathbb{R}$ is the location parameter. The calculation of the pdf is straightforward. However, the cdf has to be integrated numerically from Equation 2.

An interesting re-parameterization of the hyperbolic distribution is with $\xi = \left(1 + \delta\sqrt{\alpha^2 - \beta^2}\right)^{-\frac{1}{2}}$ and $\chi = \xi\beta/\alpha$. This has the advantage that ξ and χ are invariant under scale and location transformations. The new invariant shape parameters vary in the triangle $0 \leq |\chi| < \xi < 1$. For $\xi \to 0$, the normal distribution is obtained as a limiting case; for $\xi \to 1$, one gets the symmetric and asymmetric Laplace distribution; for $\chi \to \pm\xi$, it is a generalized inverse Gaussian distribution and finally, for $|\chi| \to 1$, we attain an exponential distribution.

The Normal-Inverse Gaussian (NIG) Distribution

The normal-inverse Gaussian (NIG) distribution was introduced by Barndorff-Nielsen (1997) as a subclass of the generalized hyperbolic laws obtained for $\lambda = -1/2$. The density of the normal-inverse Gaussian (NIG) distribution is given by:

$$f_{NIG}(x) = \frac{\alpha\delta}{\pi} e^{\delta\sqrt{\alpha^2-\beta^2}+\beta(x-\mu)} \frac{K_1(\alpha\sqrt{\delta^2+(x-\mu)^2})}{\sqrt{\delta^2+(x-\mu)^2}}, x \in \mathbb{R} \quad (3)$$

Like the hyperbolic distribution, the calculation of the pdf is straightforward but the cdf has to be integrated numerically from Equation 3.

The Generalized Hyperbolic (GH) Skew Student's *t*-Distribution

Letting $\alpha \to |\beta|$ in Equation 1, we obtain the GH skew Student's *t*-distribution:

$$f_X(x) = \frac{2^{1/2+\lambda}\delta^{-2\lambda}|\beta|^{1/2-\lambda}K_{1/2-\lambda}\left(\sqrt{\beta^2(\delta^2+(x-\mu)^2)}\right)\exp\left(\beta(x-\mu)\right)}{\Gamma(-\lambda)\sqrt{\pi}\left(\sqrt{\delta^2+(x-\mu)^2}\right)^{1/2-\lambda}} \quad (4)$$

where $\beta \neq 0$ and $\lambda < 0$. If $\beta = 0$, we get the non-central (scaled) Student's *t*-distribution. The most important property of this distribution is that it has one heavy tail and one semi-heavy tail. This makes it unique for modeling skew and heavy-tailed data, such as financial returns.

EMPIRICAL RESULTS AND DISCUSSION

The three subclasses of GHDs are now fitted to our JSE Mining Index returns. Figures 3, 4, and 5 show the histograms, log-densities, and Q-Q plots of the JSE Mining index returns fitted with the hyperbolic, the NIG, and the GH skew Student's *t*-distributions, respectively.

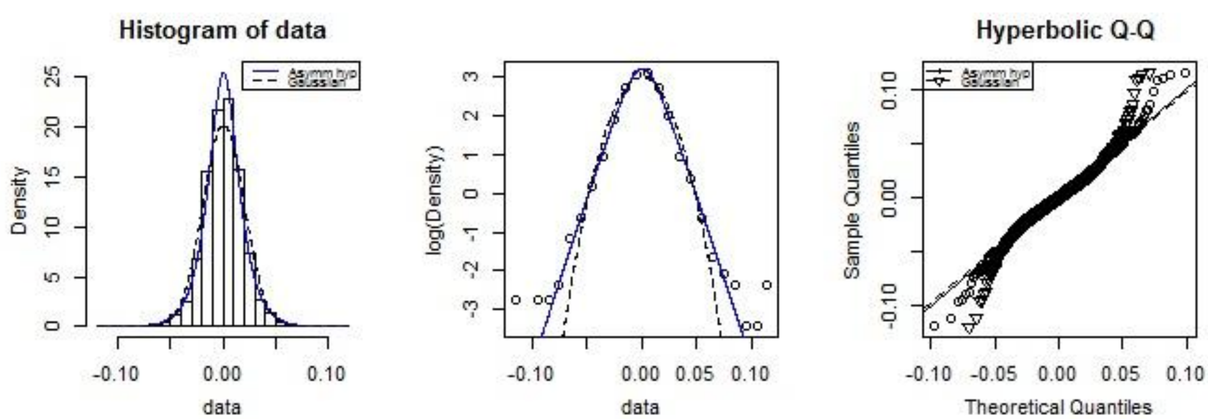

Figure 3: Histogram, Log-Density Plot, and Q-Q Plot Using the Hyperbolic Distribution

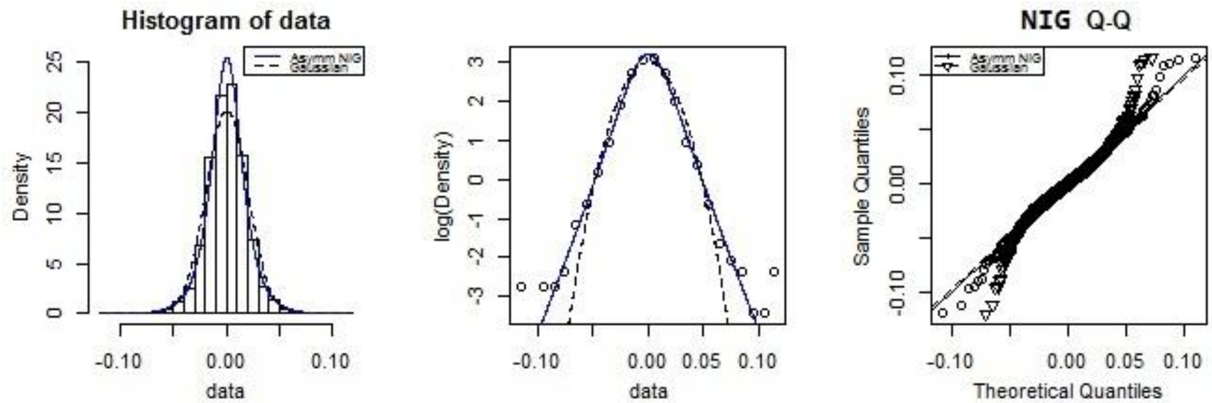

Figure 4: Histogram, Log-Density Plot, and Q-Q Plot Using the NIG Distribution

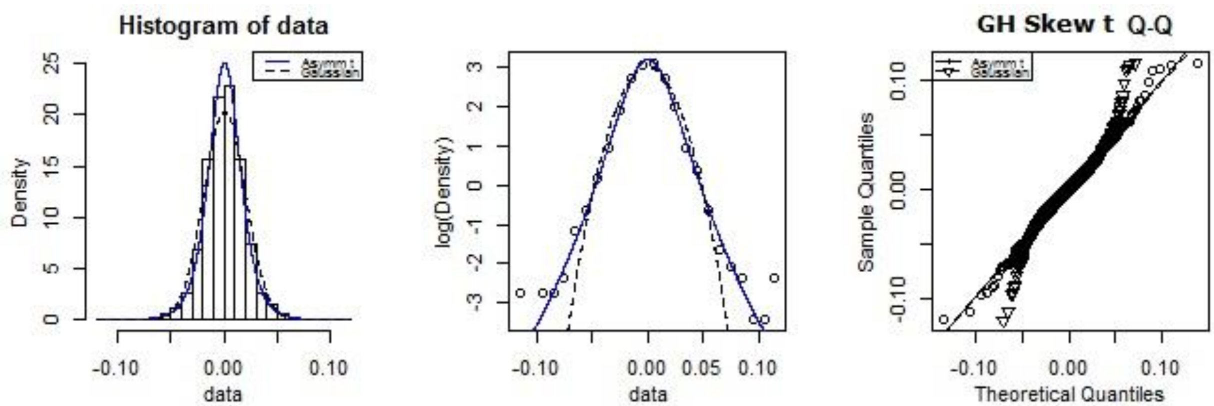

Figure 5: Histogram, Log-Density Plot and Q-Q Plot Using the GH Skew Student's *t*-Distribution

It is evidenced that heavy tailed distributions provide good fits to the data and, certainly, a much better fit than the normal distribution, in particular for the tails. All three GHD subclasses seem to provide a better depiction for the leptokurtic behavior of the JSE Mining Index returns. The log-density plots and the Q-Q plots also indicate that the three distributions gave superior tail fits than the normal distribution. A combined Q-Q plot is also given in Figure 6, which shows that the GH skew Student's *t*-distribution as the best model for the returns.

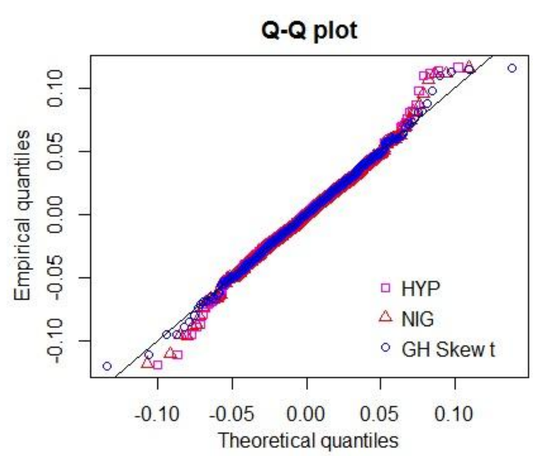

Figure 6: A Combined Q-Q Plot for all Three GHDs

Table 3 shows the parameter estimates from the JSE Mining index returns with the hyperbolic distribution, the normal-inverse Gaussian (NIG), and the GH skew Student's *t*-distribution.

Table 3: Parameter Estimates for GHDs

Parameters	α	δ	β	μ	λ
Hyperbolic fit	85.04677	0.01328366	0.5499572	0.0002288457	1
NIG fit	58.40504	0.02223585	0.4481596	0.0002651808	-0.5
GH skew Student t	0.3595371	0.03356477	0.3595371	0.0002954415	-2.44783

Table 4 presents the results from the Kolmogorov-Smirnov and the Anderson-Darling test. We observe that all three models produce high *p*-values when fitted to the data. These are strong evidence that we cannot reject the null hypothesis that the returns data follow these GHD subclasses. Both tests also indicate that GH skew Student's *t*-distribution as the best model (with the highest *p*-value and lowest distance in both tests). The Anderson-Darling test, due to its greater emphasis on the tail fits, shows a greater difference between the GHD subclasses than the results from Kolmogorov-Smirnov test.

Table 4: Results from Kolmogorov-Smirnov Test and Anderson-Darling Test

	Kolmogorov-Smirnov		Anderson-Darling	
Model	**Statistic**	***p*-value**	**Statistic**	***p*-value**
Hyperbolic	0.0152	0.462	1.2527	0.2483
NIG	0.0142	0.5469	0.7241	0.5391
GH skew *t*	0.0125	0.7028	0.4158	0.8329

Table 5: Akaike Information Criterion, Bayesian Information Criterion, and Log-Likelihood Values

Model	Hyperbolic	NIG	GH skew *t*
AIC	-16129.59	-16142.84	-16153.82
BIC	-16128.19	-16142.78	-16145.71
Log-likelihood	8068.793	8075.418	8080.911

The values of the AIC, BIC, and log-likelihood are presented in Table 5. With the lowest AIC, lowest BIC, and highest log-likelihood values, the GH skew Student's *t*-distribution may be deemed the best fit for the JSE Mining Index returns.

VALUE-AT-RISK ESTIMATES AND BACKTESTING

Apart from the robust statistical tests of model fit based mainly on the centre of the distribution, an additional analysis at the extreme tails is required. Value-at-Risk (VaR) is a frequently used measure of potential risk for losses in financial markets (Duffie & Pan, 1997). It is used, by financial institutes, to calculate the maximum loss over a given time horizon. Hence, its calculations concentrate on the tails of a distribution. For an accurate VaR estimation, the underlying model for financial returns must present a good depiction of the data at the extreme points. In this section, we use the estimated models from Section 4 to determine the risk for long and short trading positions of the JSE Mining Index under the context of VaR. Model robustness is based on the ability to forecast accurate VaR estimates for adequate capitalization. Finally, we test VaR model specifications and effectiveness by utilizing the widely accepted Kupiec likelihood ratio (LR) test (Kupiec, 1995).

For the three GHD subclasses and the Gaussian distribution, we predict the 1-day VaR at levels 0.1%, 1%, 5%, 95%, 99%, and 99.9%. The overall VaR estimates over the whole period of our data are compared to the empirical VaR values in Table 6. It can be seen, as well-known in the literature, that the Gaussian distribution tends to underestimate the VaR values. Whereas, the GHDs have both semi-heavy and heavy tails, thus proves to be better candidates for risk management.

Table 6: VaR Values of JSE Mining Index Returns Based on Empirical and Theoretical Distributions

Distribution	0.1%	1%	5%	95%	99%	99.9%
Empirical	-0.09410734	-0.05024998	-0.02985183	0.03044534	0.05135094	0.1077018
Normal	-0.06049589	-0.04543452	-0.03199761	0.03286497	0.04630188	0.06136326
Hyperbolic	-0.07832758	-0.05100169	-0.0315451	0.03262381	0.05231199	0.07997555
NIG	-0.08166794	-0.05100841	-0.030773	0.03181721	0.05230215	0.08336401
GH skew t	-0.08971199	-0.05087033	-0.03023131	0.0311885	0.05214742	0.09201655

The Kupiec LR test utilizes the fact that a good model should have its proportion of violations of VaR estimates close to the corresponding tail probability. The method consists of calculating x^{α} the number of times the observed returns fall below (for long positions) or above (for short positions) the VaR estimate at level α; i.e., $r_t < VaR^{\alpha}$ or $r_t > VaR^{\alpha}$, and compare the corresponding failure rates to α. The null hypothesis is that the expected proportion of violations is equal to α. Under this null hypothesis, the Kupiec LR statistic, given by:

$$2\ln\left(\left(\frac{x^{\alpha}}{N}\right)^{x^{\alpha}}\left(1-\frac{x^{\alpha}}{N}\right)^{N-x^{\alpha}}\right)-2\ln\left(\alpha^{x^{\alpha}}(1-\alpha)^{N-x^{\alpha}}\right) \tag{5}$$

is asymptotically distributed according to a Chi-square distribution with one degree of freedom.

Table 7: Number of Violations of VaR for Each Distribution at Different Levels

Distribution	0.1%	1%	5%	95%	99%	99.9%
Normal	19	53	135	128	48	16
Hyperbolic	7	29	139	129	31	8
NIG	6	29	151	138	31	6
GH skew t	4	29	154	148	32	5

Table 8: *P*-values for the Kupiec Test for Each Distribution at Different Levels

Distribution	0.1%	1%	5%	95%	99%	99.9%
Normal	**< 0.0001**	**0.0005**	0.0539	**0.0113**	**0.0065**	**< 0.0001**
Hyperbolic	0.0630	0.6361	0.1128	**0.0145**	0.9129	**0.0228**
NIG	0.1558	0.6361	0.5623	0.0947	0.9129	0.1558
GH skew t	0.6503	0.6361	0.7400	0.4073	0.9445	0.3405

Table 7 presents the number of violations of VaR for the different models, at different levels, and Table 8 gives the corresponding *p*-values of the Kupiec LR test. At 5% level of significance of the Kupiec LR test, the Gaussian distribution is rejected on all levels of VaR, except the 5% long trading position. In addition, the hyperbolic distribution is rejected at 95% and 99.9% VaR levels. The NIG and GH skew Student's *t*-distributions were both effective and well specified on levels of VaR. However, under the 10% level of significant, NIG is rejected at 95% VaR. It is clear that the GH skew Student's *t*-distribution has the highest *p*-value at all VaR levels and is the most robust model on all levels of significance.

CONCLUSION

In this paper, we evaluated the performance of generalized hyperbolic distributions in characterizing the South African Mining Index returns. In particular, we utilized the hyperbolic, the normal-inverse Gaussian as well as the GH skew Student's *t*-distributions. These models are able to capture certain stylized facts, such as skewness and both symmetric and asymmetric heavy tails, which provide a higher degree of accuracy when fitted to financial returns data. Our statistical analyses conducted include histogram fitting, log-densities and Q-Q plots. In all cases, we conclude that the GHD outperforms the classical normality assumption of financial returns. In addition, the goodness-of-fit tests failed to reject the null hypotheses at all levels of significance, suggesting minimal error bias. A comparison of the Akaike Information Criterion, Bayesian Information Criterion and the log-likelihood values indicates that the GH skew Student's *t*-distribution was the most robust. This result was finally confirmed by the models' performances in VaR estimation and results from the Kupiec likelihood test, where the GH skew Student's *t*-distribution is best for estimating VaR. Given that GHDs provide a better fit to the data, further work may also be to incorporate such distributions and compare VaR model performances under the framework of the well-studied unconditional variance GARCH-based VaR models and its metamorphoses, which accounts for properties such as long-memory and asymmetry.

R and *EViews* were used in this paper to produce figures and results of various tests.

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REFERENCES

1. Aas, K., & Haff, D. H. (2006). The generalized hyperbolic skew Student's *t*-distribution. *Journal of Financial Econometrics, 4*(2), 275-309.
2. Abramowitz, M., & Stegun, A. (1972). *Handbook of mathematical functions with formulas, graphs, and mathematical tables*. National Bureau of Standards Applied Mathematics Series 55. Tenth Printing.
3. Barndorff-Nielsen, O. E. (1977). Exponential decreasing distributions of the logarithm of particle size. *Proceedings of the Royal Society London, A, 353*, 401-419.
4. Barndorff-Nielsen, O. E. (1997). Normal inverse Gaussian distributions and stochastic volatility modeling. *Scandinavian Journal of Statistics, 24*(1), 1-13.
5. Beling, P., Overstreet, G., & Rajaratnam, K. (2010). Estimation error in regulatory capital requirements: Theoretical implications for consumer bank profitability. *Journal of the Operational Research Society, 61*, 381-392.
6. Blaesid, P., & Sorensen, M. K. (1992). *Research Report No. 24. Department of Theoretical Statistics*. University of Aarhus.
7. Bollerslev T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics, 31*, 307-327.
8. Deloitte (2013). *Global risk management survey eighth edition: Setting a higher bar.* [Online] Retrieved from http://www.deloitte.com/assets/Dcom-UnitedStates/Local%20Assets/Documents/us_fsi_aers_ global_risk_management_survey_8thed_072913.pdf
9. Duffie, D., & Pan, J. (1997). An overview of value at risk. *The Journal of Derivatives, 4*(3), 7-49
10. Eberlein, E., & Keller, U. (1995). Hyperbolic distributions in finance. *Bernoulli, 1*(3), 281-299.
11. Fama, E. (1965). The behavior of stock market prices. *Journal of Business, 38*, 34-105.
12. Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives, 2*, 173-184.
13. Mandelbrot, B. (1963). The variation of certain speculative prices. *Journal of Business, 36*, 394-419.
14. Peiró, A. (1999) Skewness in financial returns. *Journal of Banking & Finance, 23*, 847-862.
15. Prause, K. (1999). *The Generalized hyperbolic model: Estimation, financial derivatives and risk measures.* (Doctoral Thesis). University of Freiburg.

16. Rydberg, T. H. (1999). Generalized hyperbolic diffusions with applications towards finance. *Mathematical Finance*, *9*, 183-201.
17. Tsay, R. S. (2010). *Analysis of financial time series* (3rd ed.). New Jersey: Wiley & Sons.
18. Vee, D. N. C., Gonpot, P. N., and Sookia, N. (2012). Assessing the performance of generalized autoregressive conditional heteroskedasticity-based value-at-risk models: A case of frontier markets. *Journal of Risk Model Validation, 6*(4), 95-111.