Value At Risk, Minimum Capital Requirement And The Use Of Extreme Value Distributions: An Application To BRICS Markets

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ABSTRACT

This paper uses closing prices of the BRICS (Brazil, Russia, India, China, and South Africa) financial markets to implement a risk model that generates point estimates of both Value at Risk (VaR); and Expected Shortfall (ES). The risk model is thereafter backtested using three techniques namely the Basel II green zone, the unconditional test, and the conditional test.

We first filter the log-return data using an Autoregressive Regression model (AR) of order one for the conditional mean and an Exponential Generalised Autoregressive Conditional Heteroscedasticity of order one (EGARCH 1,1) for the conditional variance. We thereafter fit the filtered returns by using the Generalised Pareto Distribution (GPD) model before we compute both VaR and ES estimates. We find that the use of the GPD is well suited to financial markets that are highly exposed to global financial risks.

Our results show that both VaR and ES estimates for South Africa are very low when compared with those of other BRICS financial markets. We argue that South Africa's credit and loan regulations, pioneered by the National Credit Regulator (NCR), might have decreased its exposure to global financial risks. The resulting minimum capital requirement values are found to be significantly different depending on whether the Variance-Covariance or the GPD methodology is used. The backtesting methodologies show that the VaR model used in the paper is more robust and practically reliable.

Keywords: Extreme Value Theory; Generalised Pareto Distribution; Backtesting; Basel II Green Zone

1. INTRODUCTION

Market risk can best be described as the risk arising from an adverse movement of asset prices that are traded on a daily basis by any financial institution. Adverse price movements during a financial year have a significant impact on the performance of a financial institution. Longin (2000) points out that an adverse movement in asset prices, as opposed to everyday trading over the course of a financial year can play a greater role in the performance of a financial institution. It is therefore important that financial institutions meet the minimum capital requirements proposed by the Basel II committee and implement a sound and robust risk model that is able to forecast potential losses in the near future.

This paper implements such a robust market risk model and illustrates its applicability to the returns data of BRICS financial markets collected from INet-Bridge. This exercise consists of computing both VaR and ES future estimates, calculating their corresponding minimum capital requirement values as prescribed by the Basell II regulations.

The soundness and reliability of our risk model are both evaluated by making use of the covered and uncovered backtest techniques proposed by Kupiec (1995) and Christoffersen (1998), respectively. The paper focuses mainly on the impact of the Extreme Value Distributions (EVT) distributions on the minimum capital requirement.

Our findings highlight the importance of this type of distribution in modelling market risks. We show that financial markets that are highly exposed to global financial risks are better modelled using the Extreme Value Distribution method, in particular the Generalised Pareto Distribution model. This type of distribution is well suited to modelling market risks – especially those with special characteristics, such as excess kurtosis and skewness found in our sample data – since it contains both the sub-prime and the European debt crisis observed recently in the global financial markets.

Excess skewness and kurtosis often lead to a distribution that differs from normal distribution. As a result, VaR methodologies, such as the Variance-Covariance mothod, result in under-estimating future VaR estimates. Alexander (2001) confirms this, asserting that the assumption that asset returns are normally distributed does not yield the best VaR estimates. Alternatively, EVT-based distributions can be used in order to take into account excess skewness and kurtosis. There is extensive literature highlighting the importance of EVT-based distributions in risk management. For example, Longin (1996) uses EVT-based distributions to model the tails of equity profit and loss distribution. Longin (2000) argues that since the occurrence of extreme events is a concern for risk managers, the computation of an institution's minimum capital requirement should be considered to be an extreme value problem. Recent applications of EVT-based distributions include the work by Peiro (1999), who points out that a risk model that makes use of EVT-based distributions would obviously produce much better VaR estimates. Muteba Mwamba (2012) concentrates on a particular type of EVT distribution known as the Generalised Pareto Distribution (GPD) and shows that VaR estimates – which are computed based on the returns above, revealing a certain optimal threshold – are reliable, consistent and in line with the 99.9% significance level proposed by Basel II, since it generates less than three exceptions.[1]

Unlike most of the existent literature, this paper not only makes use of the EVT-based distributions to compute future VaR and ES estimates (as in Rootzen and Kuppelberg, 1999, or Yamai and Yoshiba, 2005), but also investigates the implications of such distributions on the minimum capital requirement. The rest of the paper proceeds with section 2 describing the methodology used in the paper while section 3 discusses the empirical results and is followed by a conclusion of the paper.

2. METHODOLOGY

In this paper, we assume that the return generating process has the following expression:

$$r_t = a_0 + a_1 r_{t-1} + z_t s_t \tag{1}$$

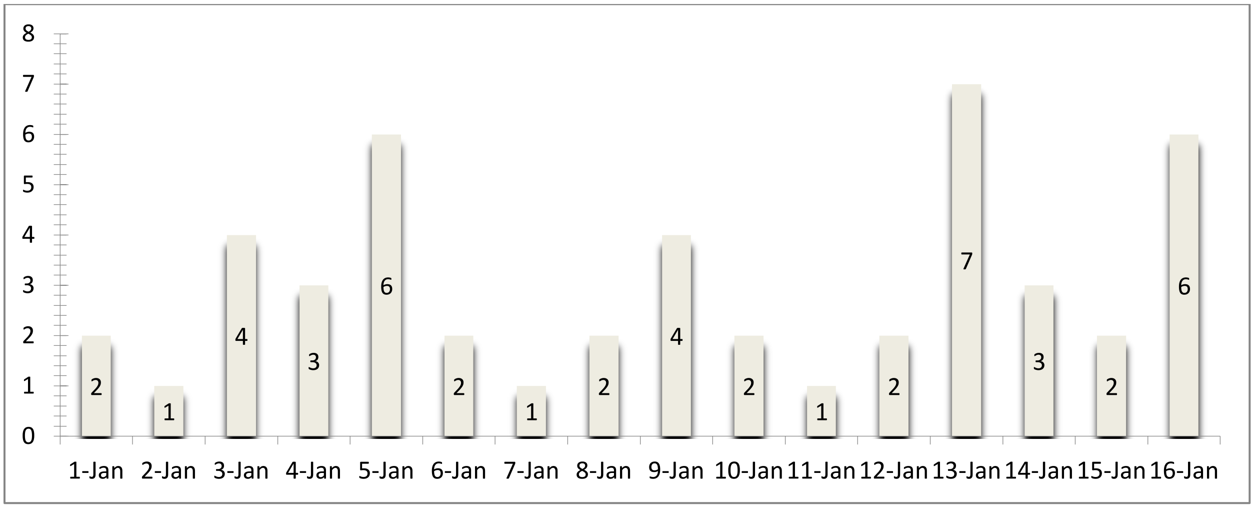
Here: $r_t, \mu_t(y), z_t\ \sigma_t$represent the return series; the constant term in the mean equation, the coefficient of the lagged returns, the standardised random variable, ; and the conditional volatility assumed to be a GARCH process.

To account for a leverage effect (Nelson, 1991) in the conditional volatility equation, we make use of an asymmetric EGARCH model given by:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma \frac{e_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|e_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \tag{2}$$

[1]An exception is observed when a realised loss is larger than the forecasted VaR. Therefore a good market risk model will have to produce few exceptions since it is able to forecast almost precisely the exact number of losses to be observed during the next trading period (that is, one trading year).

In the above, e_{t-1} and γ represent the previous period residuals from the mean equation and the leverage effect, respectively. w, b, and a are coefficients of the model to be estimated. Filtered returns (residuals obtained from Equation 1 above) are thereafter fitted to a GPD. Since we are interested in downside risk measure, we collect all negative filtered returns (losses) and multiply them by negative one. The process of fitting these losses to a GPD is described in more detail in Muteba Mwamba (2012). The following figure illustrates briefly how the GPD fits the losses of a portfolio over a given threshold, of let's say $u=3$.



Source: Taken from Muteba Mwamba (2012)

Figure 1: Loss Distribution With A Given Threshold Of $u=3$ During The Period Under Investigation.

This figure shows a hypothetical loss distribution along the x-axis, observed during the first half of January, marked as 1, 2, 3, 4, 5, 6, and 7, while the y-axis reports their magnitudes. Assume that the loss marked as 3 is our threshold. In this case, losses marked as 4, 5, 6 and 7 are considered to be extreme losses as they are larger than the threshold of u = 3. The limiting distribution of these extreme losses over the threshold $u = 3$ is known as GPD and is given by the following expression:

$$G_{\xi,\beta(u)} = \begin{cases} 1 - \left(1 + \xi \frac{x}{\beta(u)}\right)^{-\frac{1}{\xi}} : \xi \neq 0 \\ 1 - exp\left(\frac{x}{\beta(u)}\right) : \xi = 0 \end{cases}, \quad (3)$$

Here, x, ξ and $\beta(u)$ represent the extreme losses above a certain fixed optimum threshold, and illustrate the shape of the GPD and scale parameter, which is a function of the optimum threshold, u, respectively. Both the scale parameter and the threshold must be positive, that is, $\beta(u) > 0$; $x \geq 0$for$\xi \geq 0$and$0 \leq x \leq -\frac{\beta(u)}{\xi}$; for $\xi < 0$. The maximum likelihood method is used to estimate the scale and shape parameters of the GPD empirically. The method attempts to maximise the following log-likelihood function:

$$L(\xi,\beta) = -N_u Log(\beta) - \left(1 + \frac{1}{\xi}\right)\sum_{i=1}^{N_u} Log\left(1 + \frac{\xi x_i}{\beta}\right) \quad (4)$$

N_u is the number of extreme losses that exceeds the optimum threshold, u. Notice that the log-likelihood function depends only on the shape parameter, ξ, and the scale parameter, β;. Therefore the optimum threshold, u, must be determined beforehand. This operation involves the use of the mean excess function suggested by Davidson and Smith (1990). Following Muteba Mwamba (2012) the expression of the VaR based on the GPD is given by:

$$VaR\ (\alpha) = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{1-\alpha}{N_u/n}\right)^{-\xi} - 1\right) \quad (5)$$

Based on Equation 5, one can derive the expression of the ES, which is equal to the VaR plus the mean of all losses that are larger than the VaR itself:

$$ES(\alpha) = E\left(r_i \middle| r_i > VaR(\alpha)\right) = VaR(\alpha) + E\left(r_i - VaR(\alpha) \middle| r_i > VaR(\alpha)\right) \quad (6)$$

Therefore the expression of the ES based on the GPD is given by:

$$ES(\alpha) = \frac{VaR(\alpha)}{1-\hat{\xi}} + \frac{\hat{\beta}-\hat{\xi}u}{1-\hat{\xi}} \quad (7)$$

Backtesting

Backtesting is a set of statistical procedures designed to check if the realised losses are in line with VaR forecasts (Jorion, 2007). Practically, internally developed VaR models are supposed to produce a sequence of pseudo, out-of-sample VaR forecasts for the past 252 trading days, that is, one calendar year. Backtesting is based on comparing the realised losses and these VaR forecasts. In this case, the VaR backtesting procedure is based on the counting of exceptions. In this paper, the VaR forecasts are generated by the following expression:

$$VaR_{t+1} = \mu_{t+1}(y) + \sigma_{t+1}(y)F^{-1}(\alpha) \quad (8)$$

where $F^{-1}(\alpha)$is the α^{th} quantile of the cumulative function of the GPD. The three backtest procedures implemented include the Basel II green zone, the unconditional test (Kupiec, 1995) and the conditional test (Christoffersen, 1998).

Basel II Green Zone

The test is based on the number of exceptions produced over a period of one year (250 trading days) as prescribed by the Basel II. A model with more exceptions requires a higher capital requirement. The Basel II green zone[2] is illustrated below:

Table 1: Basel II Zones And Exceptions Based On A 250 Intra-Day Trading Sample

Zone	Confidence Level			Multiplication Factor				
	90%	**95%**	**99%**					
Green	0-32	0-17	0-4	3				
Yellow	33-43	18-26	5-9	3.4	3.5	3.65	3.75	or 3.85
Red	≥ 44	≥ 27	≥10	4				

Source: Basle Committee on Banking Supervision *(1996)*

A good model is expected to have few exceptions as shown in the green zone and thus receives a low penalty (multiplication factor) when determining the capital requirement.

The Unconditional Coverage Test (Kupiec, 1995)

Under the null hypothesis that the model is correct, the test determines whether the frequency of exceptions observed is consistent with the frequency of expected exceptions. We test the null hypothesis that 'the model is correct'. In this test, the number of exceptions is assumed to follow the binomial distribution:

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad (8)$$

where p and n represent the probability of an exception and the number of trials, respectively, given a certain confidence interval.

[2] Was introduced with the Market Risk amendment to the Basel I accord in 1996, and was unaltered with the introduction of Basel II in 2008.

The Conditional Coverage Test (Christoffersen, 1998)

This test determines the number of exceptions and the independence of the exceptions of the VaR model. Roy (2011) points out that testing the conditional accuracy of the VaR model is advisable as it becomes useful when volatility clustering is inherent. The test allows insight into the VaR model's accuracy and is given by:

$$TS = 2Log\left(\frac{L(I_{\propto})}{L(I_{\propto})}\right) \quad (9)$$

where $I_{\propto}$ is an indicator variable:

$$I_{\propto} = \begin{cases} 1 & exception\ occured\ during\ 10\ days \\ 0 & no\ exception\ occured\ during\ 10\ days \end{cases} \quad (10)$$

The null hypothesis of the conditional coverage test is that the number of exceptions is correct and that they are independent. In the logarithm of the ratio of the two likelihood functions, L forms the test statistic which is distributed, χ^2 (Christoffersen, 1998; Roy, 2011).

The resulting minimum capital requirement is computed as follows:

$$Min\ Cap\ Req = Factor \times Risk\ Measure \times Portfolio\ Value \quad (11)$$

where $Factor$, $Risk\ Measure$, and $Portfolio\ Value$ represent the factor assigned by the Basel II green zone test, the risk measure (VaR or ES) and the hypothetical portfolio value.

3. DATA AND EMPIRICAL RESULTS

Daily log returns of the Brazilian BOVESPA, the Russian Stock Exchange Index (RTS), the Chinese Shanghai Securities Exchange Composite Index (SSE), the Indian S&P BSE SENSEX and the South African All Share Index (ALSI), respectively representing the BRICS financial markets, were collected for the period of January 2009 to December 2012[3]. The year ending December 2012 was used for out-of-sample forecasting and backtesting of the VaR. The out-of-sample period constitutes 25% of the sample data which was sourced from INet-Bridge.

Point Estimate of the VaR

We first compute the point estimates of the VaR using the entire sample data. For comparative purposes, we use the Variance-Covariance method as the benchmark model. The VaR and ES percentage estimates are based on the Variance-Covariance model, with different levels of significance (α), which are exhibited in Figure 2 below:

[3] This sample period has been chosen because of data availability

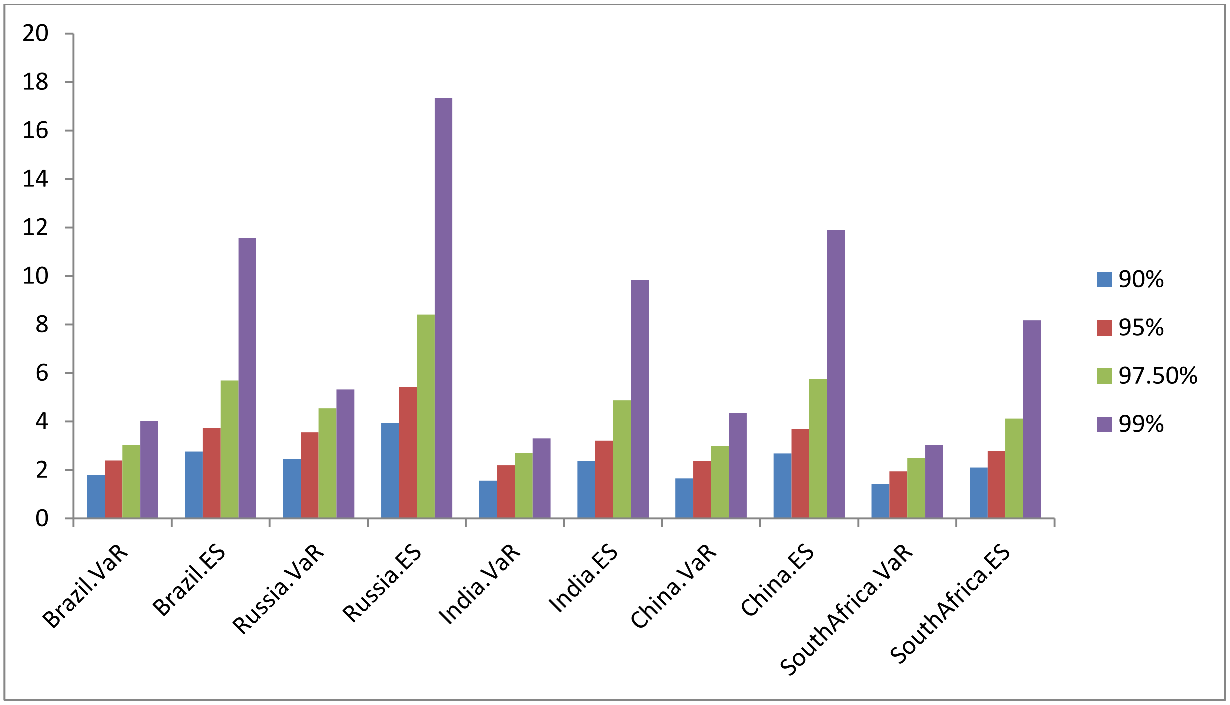


Figure 2: Variance-Covariance Estimates Of VaR And ES

Figure 2 reports the VaR and ES estimates of each BRICS market on the x-axis, while reporting the magnitudes in percentage values of each risk measure on the y-axis. The figure shows that the ES estimates are always larger than the VaR estimates. These results are consistent with the literature, according to which the ES estimate is equal to the sum of the VaR estimate and the mean of losses larger the VaR (Acerbi and Tasche, 2002). Further to that, Figure 2 also shows that, during our sample period, Russia has the highest market risk of around 17% as a result of its exposure to the European market, which was severely hit by the financial crisis of 2008 – 2009 and the ensuing debt crisis after the 200 financial crisis.

Similar to Figure 2, Figure 3 reports the EVT-based VaR and ES estimates of each BRICS market on the x-axis, while it reports the magnitudes in percentage values of each risk measure on the y-axis.

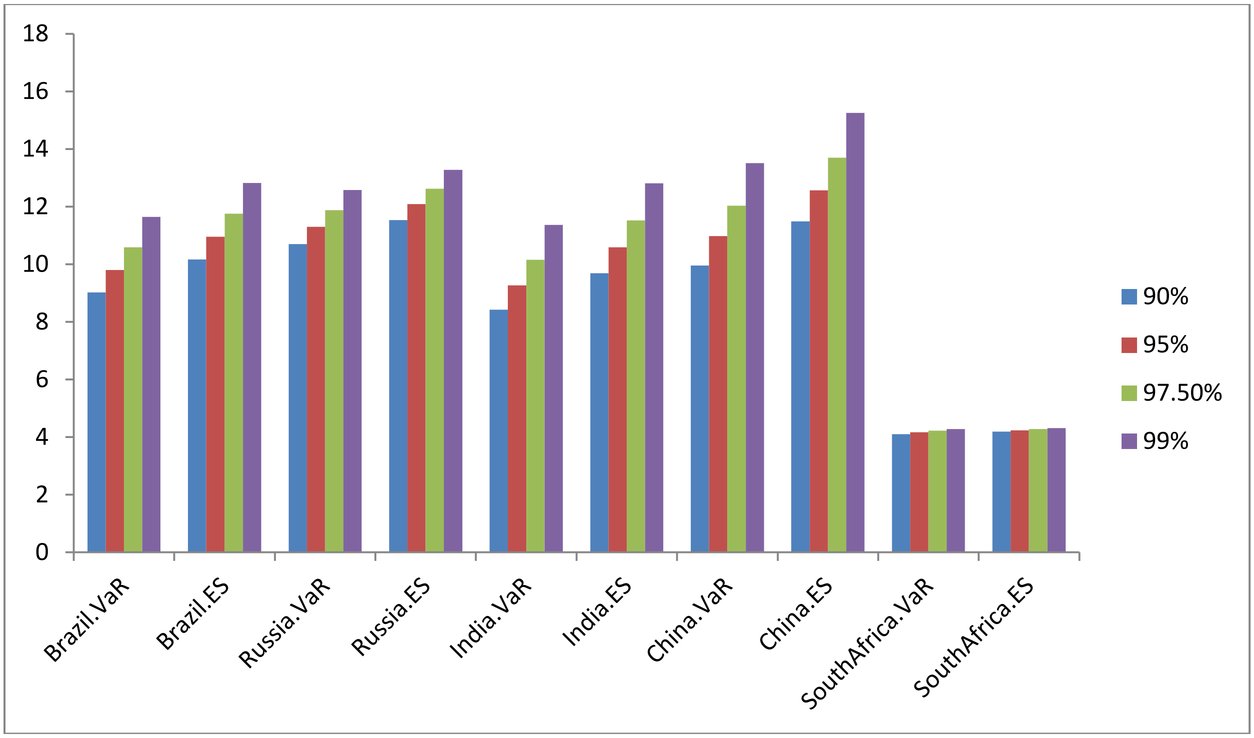


Figure 3: VaR & ES Estimates Using EVT

Figure 3 reports the EVT-based VaR and ES estimates in percentages for various levels of significance using Equations 6 and 8. At both lower and higher quantiles, we observe moderately significant lower VaR and ES estimates (around 4%) for the South African market in comparison with the other BRICS markets. This is attributed to the credit regulations in South Africa pioneered by the National Credit Regulator (NCR). The NCR, though regulations on credits and loans, reduced the exposure of South Africa to the global financial crisis and, therefore, to global market risk. For countries with high exposures to global market risk, both the VaR and ES estimates are found to be extremely larger, that is, between 9% and 13%, compared with potential losses observed in normal market conditions. This finding illustrates the importance of modelling market risk that is characterised by extreme market conditions.

Table 2: Minimum Capital Requirement of a Notional US $100 based on VaR and ES

	Brazil	***Russia***	***India***	***China***	***South Africa***
Variance-Covariance:					
VaR	$12.09	$15.96	$9.93	$13.09	$9.10
ES	$34.68	$52.00	$29.50	$35.68	$24.50
EVT-based (GPD):					
VaR	$34.94	$37.74	$34.10	$40.52	$12.84
ES	$38.47	$39.82	$38.41	$45.76	$12.95

Table 2 reports the minimum capital requirement on a notional amount of US $100. It highlights a significant difference between the capital requirement values that were computed with the Variance-Covariance method, and the ones computed with the EVT-based GPD method. The minimum capital requirement values between VaR and ES do not differ greatly in the EVT-based GDP method. However, there is a significant difference between the minimum capital requirement values based on the VaR and ES when the Variance-Covariance method is used. We argue that this difference is due to the assumption of normality underlying the Variance-Covariance method.

Backtesting The VaR Model

In this sub-section, we forecast the number of losses, as well as their magnitudes, during the last 250 trading days of our sample period. In fact, we consider the year ending in December 2012 as the out-of-sample space and use Equation 9 to backtest the VaR model.

Table 3: Conditional & Unconditional Backtests

	Brazil	*Russia*	*India*	*China*	*South Africa*
Expected exceptions	2.5	2.5	2.5	2.5	2.5
Actual exceptions	2	4	1	1	3
Actual %	0.01	0.02	0	0	0.01
Unconditional coverage test: null hypothesis: correct number of exceptions					
Test statistic	0.11	0.77	1.18	1.18	0.1
Critical Value	3.84	3.84	3.84	3.84	3.84
p-value	0.74	0.38	0.28	0.28	0.76
Reject null hypothesis?	No	No	No	No	No
Conditional coverage test: null hypothesis: correct number of exceptions and independence of failure					
Test statistic	0.14	0.9	1.19	1.19	0.17
Critical Value	5.99	5.99	5.99	5.99	5.99
p-value	0.93	0.64	0.55	0.55	0.92
Reject null hypothesis?	No	No	No	No	No

Table 3 reports the uncovered and covered backtests' results. These backtests are an important exercise in evaluating the validity of an internal risk model. We consider an out-of-sample space consisting of the last 250 trading days of the year ending in December 2012. At the 1% level of significance, we expect approximately three exceptions in 250 trading days. Table 3 also shows that our risk model produces at a 1% significance level less than four exceptions for all BRICS financial markets. Thus, the risk model falls within the green zone as shown in Table 1 above. We fail to reject, at 1% significance level, the null hypothesis that the model produces an acceptable number of exceptions for the uncovered test, and that these exceptions are independently distributed. These results suggest that our VaR and ES estimates are more robust and practically reliable.

4. CONCLUSION

This paper uses the closing prices of BRICS financial markets to evaluate both point estimate and the VaR forecast using the Variance-Covariance model and the Generalised Pareto Distribution method.

Our findings show that the application of the Generalised Pareto Distribution to the computation of VaR and ES is well suited to financial markets that are highly exposed to global financial market risks. We find that South Africa has low VaR and ES estimates due to its tough regulations on credits and loans, which limited its exposure to the global financial market and debt crises during the period under investigation.

Furthermore, we also find that the minimum capital requirement values computed with the VaR and ES were significantly different when one uses the Variance-Covariance or the EVT-based GPD method. We believe that this might have to do with the underlying normality distribution that the Variance-Covariance method assumes. We argue that the difference in results highlights the significance of the use of extreme value distribution, particularly during market distresses. The backtesting procedures show that our VaR methodology was robust and practically reliable.

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