Applying The Learning Curve
To Operating Cash Flows
In A Capital-Budgeting Framework

Peyton Foster Roden, University of North Texas, USA
Pedro Lizola-Margolis, Universidad Autónoma del Estado de México, México

ABSTRACT

This paper discusses the way financial managers can and should include the learning curve and its impact on cash flows when evaluating proposed capital projects. After examining finance text books and survey results to show that the learning curve is an ignored part of financial decision making, we briefly develop the mathematics of the learning curve then illustrate its application to labor costs and operating cash flow. We calculate the difference between operating cash flows without and with learning to illustrate the systematic impact of the benefit of learning. We note that applying the learning curve to the estimation of expected operating cash flows offers financial managers the opportunity not only to enhance the evaluation process, but also to provide a useful way to look at investing and financing projects. (JEL: C60, M41)

Keywords: Operating cash flows, capital budgeting

INTRODUCTION

Each of us is aware at some level of the benefits associated with repeating a task. Consider hitting a wedge or a topspin forehand. At first, the process is an ordeal. Shanks, whiffs, and frustration abound. But the more we repeat the process, the better the result—perhaps not hitting the wedge within birdie range or the forehand for an outright winner, but clearly better than when we started. These activities reflect systematic learning by doing.

Learning by doing has a long history in economic theory to explain the increased productivity at a given capital stock. As Arrow notes, “. . . technical change in general can be ascribed to experience. . . .” (1962, p. 156) At the microeconomic level, the relationship between increased productivity and a given capital stock is the learning curve. Although a financial manager can apply the learning curve to many areas of financial decision making, we apply it in this paper to labor costs and the calculation of operating cash flow from a proposed capital project.

We concentrate on operating cash flow because its estimation is a process with which financial managers are familiar and so readers will readily see the impact of the learning curve. Moreover, ranking and selecting proposed capital projects—capital budgeting—is at the center of maximizing stock price and increasing shareholder wealth. Its theoretical and empirical relationship to firm valuation has been documented in the literature (Woods and Randall 1989, Vogt 1997) and included in the text-book literature of finance. So it is appropriate that we use this area of financial decision making to develop our discussion.

By extension, the learning curve has a benefit of disciplining managers and encouraging them to consider the systematic relationship between learning and doing. Indeed, including the learning curve in financial analysis should enhance discounted and non-discounted cash flow methods of ranking and selecting proposed capital projects, measuring the time and expense in negotiating costs of financing, and budgeting for departments within a company—as the benefits from learning are realized, fewer employees may be needed to perform tasks, permitting employees to be moved into other areas of the company. When managers submit a proposed capital project, they know they will be held responsible for assumptions and results. Many companies review a capital project one year or three years after acceptance to compare expected cash flows and expected return with their actual values. A post-completion audit
holds the manager accountable for assumptions and calculations in the original authorization request. Assumptions and calculations should be tangible, trackable, and actionable. Although a learning-curve percentage is not part of the language of finance and financial managers do not routinely use the term, they are intuitively aware of the benefit from learning and may include it unsystematically in cash-flow estimates. Including specific recognition of the learning curve, as this paper suggests, will enhance the post-completion audit by giving management assumptions and calculations that are tangible, trackable, and actionable for evaluation of a project. The post-completion audit can verify the source of the cost estimates (as developed with the learning curve), the relationship between budgeted and actual values, and take action to improve future budgeting.

The paper is organized as follows: Part One examines finance textbooks and survey results to indicate the absence of consideration of the benefits from learning. Part Two presents the mathematics of the learning curve with particular emphasis on the logarithmic equation for calculating the cumulative average time associated with each attempt. We show a short-cut method that financial managers can use without a calculator to find a first approximation. Part Three applies the marginal cost of labor (derived from estimates of the cumulative average cost) to the calculation of operating cash flows as part of the capital-budgeting process. Part Four is a summary and conclusion.

PART ONE. ABSENCE OF THE LEARNING CURVE IN THE FINANCIAL LITERATURE

Although the learning curve and its implications are part of the management accounting literature, they appear to be ignored in the finance literature. Consider the treatment of capital budgeting in finance textbooks. Table 1 presents results from examining the treatment of cash flows in several textbooks. Although not exhaustive, the sample is representative.

The conventional textbook presentation of capital budgeting ignores the benefits of learning when discussing the estimation of cash flows associated with a proposed project, the way the project is financed, and the various methods for measuring the expected impact on stock price and shareholder wealth. The emphasis in most presentations is consistent with the separation of the investment decision from the financing decision: Estimating incremental cash flows, determining the sources of financing and their costs, and finally measuring the expected impact on stock price and shareholder wealth (for example, with net present value and internal rate of return). Three text books in Table 1 (Beninga 2006; Brealey, Myers, and Allen 2006; Ross, Westerfield, and Jordan 2007) do not have a separate chapter discussing cash flows surrounding capital-budgeting analysis, so perhaps a discussion of the impact of learning on expected cash flows is justified on the basis of space consideration. However, even when cash flows are considered in separate chapters, the presentation ignores the impact of learning (Beasley and Brigham 2008; Brealey, Myers, and Marcus 2007; Emery, Finnerty, and Stowe 2006; Gitman 2006; Keown, Martin, and Petty 2008; Megginson and Smart 2006; Moyer, McGuigan, and Rao 2007).

In addition to textbooks, survey results suggest that the benefits of learning are ignored by financial managers, or perhaps by researchers developing the survey instrument. Surveys to determine the capital-budgeting methods used to evaluate projects reveal that DCF methods may be supplemented with non-DCF methods (Chen 1995; Bierman 1993; Gitman 2000; Graham and Harvey 2001; Block 2005; Danielson and Scott 2006). None of the survey results we examined, however, revealed evidence of the benefits of learning, perhaps because none of the questionnaires specifically addressed learning.

Failure to include the impact of learning on cash flows and project evaluation may be justified in replacement decisions because one could argue that the benefits of learning are already impounded in operations and cash flows. However, failure to include the impact in other types of projects leads to underestimating expected cash flows and expected profitability, thereby exposing the company and its shareholders to underinvestment and to failure to maximize shareholder wealth by increasing stock price.

PART TWO. DEVELOPING A LEARNING CURVE

Not all proposed capital projects are created equally. Some proposals are replacements of on-going projects while, at the other extreme, some are investments in completely new projects. Rigor applied to each type of project typically differs depending upon whether the project is a replacement or completely new and on the size of the firm. Replacement projects are typically subject to less analytical methods (Danielson and Scott 2006). Large firms are
associated with more sophisticated methods (Graham and Harvey 2001). If a project is a replacement or one in which
the company’s management has experience, then the benefits of learning may be already impounded in the process
and no incremental benefits from learning need be applied. However, if management has little or no experience in a
proposed project, then it makes sense to include in the analysis the benefit from learning.

**Model Building.** There is no single learning-curve model, as noted in Liao (1988). However expressed, the model
attempts to describe the systematic relationship between learning and doing. One expression relates repetitive activity
(experience) and the cumulative average time required to complete a task as follows:

\[ Y_n = aX^b \]  

(1)

Where \( Y_n \) = cumulative average time (or cost) through a given lot or attempt
\( a \) = constant; effort (or cost) to produce the first lot or batch of output
\( X \) = midpoint of a specific lot or batch of output; 1, 2, 3, … n
\( b \) = slope constant; negative because average effort (or cost) per lot (attempt) decreases with learning

Because time and cost are usually proportional, the dependent variable can be either cumulative average time
or cumulative average cost. The equation shows that the cumulative average time (or cost) \( Y \) required to produce a unit
of output decreases each time the quantity of output \( X \) increases. The equation shows the relationship to be non-linear
so that the benefit of learning decreases at a decreasing rate. For example, the first practice session with a wedge
provides more benefit than the second one, which in turn provides more benefit than the third one, and so on.

**Table 1. Text Book Consideration of the Benefit from Learning in Cash-Flow Estimation**

This sample of textbooks shows the extent to which each emphasizes cash-flow estimation as part of the capital-budgeting process
and ignores the learning curve. Absence of the learning curve was a universal characteristic in the textbooks surveyed by the
authors.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Title</th>
<th>Separate Chapter for Operating Cash Flows?</th>
<th>Discussion of Learning Curve?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beasley, Scott, and Eugene F. Brigham</td>
<td><em>Essentials of Managerial Finance, 14/e</em></td>
<td>Yes (Chapter 10)</td>
<td>No</td>
</tr>
<tr>
<td>Beninga, Simon</td>
<td><em>Principles of Finance with Excel</em></td>
<td>No</td>
<td>No</td>
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<tr>
<td>Brealey, Richard A., Stewart C. Myers, and Franklin Allen</td>
<td><em>Principles of Corporate Finance, 8/e</em></td>
<td>No</td>
<td>No</td>
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<tr>
<td>Brealey, Richard A., Stewart C. Myers, and Alan J. Marcus</td>
<td><em>Fundamentals of Corporate Finance, 5/e</em></td>
<td>Yes (Chapter 08)</td>
<td>No</td>
</tr>
<tr>
<td>Emery, Douglas R., John D. Finnerty, and John D. Stowe</td>
<td><em>Corporate Financial Management, 3/e</em></td>
<td>Yes (Chapter 10)</td>
<td>No</td>
</tr>
<tr>
<td>Gitman, Lawrence J.</td>
<td><em>Principles of Managerial Finance, 11/e</em></td>
<td>Yes (Chapter 08)</td>
<td>No</td>
</tr>
<tr>
<td>Keown, Arthur J., John D. Martin, and J. William Petty</td>
<td><em>Foundations of Finance, 6/e</em></td>
<td>Yes (Chapter 10)</td>
<td>No</td>
</tr>
<tr>
<td>Megginson, William L., and Scott B. Smart</td>
<td><em>Introduction to Corporate Finance</em></td>
<td>Yes (Chapter 09)</td>
<td>No</td>
</tr>
<tr>
<td>Moyer, R. Charles, James R. McGuigan, and Ramesh P. Rao</td>
<td><em>Fundamentals of Contemporary Financial Management, 2/e</em></td>
<td>Yes (Chapter 10)</td>
<td>No</td>
</tr>
<tr>
<td>Ross, Stephen A., Randolph W. Westerfield, and Bradford D. Jordan</td>
<td><em>Essentials of Corporate Finance, 5/e</em></td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
An effective way to implement Equation 1 is to transform the model into log-linear form by setting b (the exponent) as the ratio of the natural log of the learning-curve percentage \((lnLC)\) expressed as a decimal and less than 1.0, and the natural log of 2 \((ln2)\) for doubling. Such a model assumes that for each doubling of the units of output, the cumulative average cost or time decreases by a constant proportion called the learning-curve percentage:

\[
\text{Cumulative average time} = \text{Beginning time} \times \left( \frac{\ln LC}{\ln 2} \right)^{\text{Target lot}}
\]

\[
Y_n = a_0 \times \left( X_N^{ln LC \over ln 2} \right)
\]

(2)

The exponent is constant at each attempt measured as the natural log of the learning-curve percentage divided by the natural log of 2. A positive learning-curve percentage less than one has a negative log value (for example, \(ln0.80 = –0.22314\)). That makes the exponent negative and the learning curve downward sloping. Some business-financial calculators (such as the TI BAII Plus) have a natural log function \((ln)\) on the keypad, so finding the value of the exponent doesn’t require using a table of logarithmic values. We illustrate the calculation in the next section after showing a short-cut method for calculating the benefit of learning.

Financial managers don’t need a PhD in math or a brain the size of a watermelon to apply the learning curve when they don’t have a calculator with exponent and natural log keys. They can use a short-cut method by calculating the cumulative average time (CAT) for double the preceding lot (attempt) number beginning with the first lot. For example, you can calculate with the short-cut method the cumulative average time (or cost) for the first attempt, then for the second attempt, then for the fourth attempt, then for the eighth attempt, and so on, as follows:

\[
\text{CA } T_n = a_0 \times \left( \frac{\text{Percentage learning curve}_1 \times \text{Percentage learning curve}_2 \times \cdots \times \text{Percentage learning curve}_n}{\ln LC \over \ln 2} \right)
\]

\[
= a_0 \times (X^n)
\]

(3)

where \(n\) is the doubled \(n\)th attempt or lot and the learning-curve percentage is a decimal less than one.

A positive learning-curve such as 0.80 means that the second attempt (lot) has a cumulative average time or cost of 80 percent of the first attempt. The fourth attempt (lot) has a cumulative average time or cost of 80 percent of the second attempt, and so on systematically with each doubling.

Table 2 illustrates the results from applying an 80 percent learning curve to a process that requires an initial 10 hours to complete (Lot 1). Values in column 2 were calculated with Equation 2. Column 3 totals the cumulative average time through a given attempt (lot) to use in calculating the marginal time in column 4.

Three rows in Table 2 are highlighted: Lot 2, Lot 4, and Lot 8. We singled them out for special treatment because they show the result of the short-cut method in Equation 3. Each bold cumulative average time in column 2 is 80 percent of the previous bold value. Although results from the short-cut method are limited in usefulness because the method applies only to doubling each previous attempt, it serves as a first approximation of the more rigorous logarithmic form and furnishes a quick and easy check for logarithmic calculations.

**Learning-Curve Values.** Applying the learning curve is straightforward because business-financial calculators and spreadsheets do the heavy lifting of the calculations. After determining the learning-curve percentage and the initial time or cost, the financial manager calculates the exponent, stores it in the calculator, and then multiplies each attempt or lot by the stored coefficient. Alternatively, the financial manager may choose to use a spreadsheet to automate the process. It’s a piece of cake.
Before applying the learning curve in Equation 2, a financial manager must estimate two values, initial time (or cost) and the learning-curve percentage. Estimating initial time (or cost) is the domain of the cost accountant from applying experience or estimates to an activity. For example, the company that produces graphite shafts for golf clubs can use its experience in that process to estimate the starting value \( a_0 \) to the production of titanium shafts. Alternatively, using publicly available information from industry associations may help to determine a starting point.

We don’t want to trivialize the choice of the learning-curve percentage (80 percent in Table 2). Managers use two methods to determine the percentage. The first is an assumed rate of improvement, appropriate in some industries such as aerospace in which management has a great deal of experience with learning curves and production. The most common assumption is within the range of 90%–70%. The less the percentage, the greater the benefit from learning. An assumed rate is also appropriate for government contracts in which the percentage is dictated.

The second estimation method requires the use of data and curve fitting with a statistical package such as SPSS or the regression tool in Excel. Here, management would observe a process over some (brief) period of time, then use the curve-fitting tool to develop the exponent relating output with attempts (lots). Although a small sample increases estimation error and bias (for example, fitting the curve to three observations), curve estimation provides a useful starting point. Bailey (2000, p. 28) applies several models in an Excel add-in he makes available to identify statistically the shape of the curve.

Table 2. Log-Linear Cumulative Average Learning Curve

Column 2 reflects the direct application of Equation 2 to a process requiring an initial 10 hours to complete (Lot 1). The 80% learning-curve percentage in column 2 leads to a cumulative average total time in column 3 increasing at a decreasing rate as the benefits of learning unfold. The marginal time in column 4 is the change in the cumulative average total time associated with a subsequent attempt. Lot 2, Lot 4, and Lot 8 verify the application of the short-cut method in Equation 3: The cumulative average time in column 2 is 80 percent of the preceding value with each subsequent doubling.

<table>
<thead>
<tr>
<th>(1) Lot (Attempt)</th>
<th>(2) Cumulative Average Time</th>
<th>(3)=(1)×(2) Cumulative Average Total Time</th>
<th>(4)=Δ(3)/Δ(1) Marginal Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7.02</td>
<td>21.31</td>
<td>5.31</td>
</tr>
<tr>
<td>4</td>
<td>6.4</td>
<td>25.6</td>
<td>4.29</td>
</tr>
<tr>
<td>5</td>
<td>5.96</td>
<td>29.8</td>
<td>4.2</td>
</tr>
<tr>
<td>6</td>
<td>5.62</td>
<td>33.72</td>
<td>3.92</td>
</tr>
<tr>
<td>7</td>
<td>5.34</td>
<td>37.38</td>
<td>3.66</td>
</tr>
<tr>
<td>8</td>
<td>5.12</td>
<td>40.96</td>
<td>3.58</td>
</tr>
</tbody>
</table>

PART THREE. APPLYING THE LEARNING CURVE TO RANKING AND SELECTING PROJECTS

Capital budgeting provides a rich field for applying the learning curve. For example, learning by doing may lead to a systematic increase in operating income by increasing revenues more than operating expenses or by reducing operating expenses more than reducing revenues as the sales force becomes more proficient at selecting and contacting customers and the purchasing department identifies sources of material and becomes more adept at negotiating costs. The resulting increase in operating cash flow is a driver of positive net present value and increase in shareholder wealth. Alternatively, the marginal cost of capital may systematically decline as a result of the finance area of a company learning where to source finance or gaining experience in negotiating financing costs. The efficiency of financial markets suggests that benefits from learning by doing may be small in the finance function of a company. In a perfect capital market, the learning-curve percentage in Equation 2 and Equation 3 would be 100%, equivalent to no learning at all. The reader will note that a learning-curve percentage of 100% means that the numerator of the exponent in Equation 2 has a natural log of zero \((\ln 1 = 0)\) resulting in an exponent of zero. As a result, the cumulative average time at each attempt or lot is a constant measured as the beginning value multiplied by 1.
Benefits from learning are trackable in the estimation of operating cash flow from a proposed project because values are projected over time, thus allowing for learning by doing. Drtina and Largay (1985) and Thode, Drtina, and Largay (1986) discuss the calculation of operating cash flow, and we employ their method in our example calculations below. They use projected operating income statements for each period of a proposed project’s economic life to assure separation of the financing decision (the cost of capital) from the investment decision (operating cash flow). Separating the two decisions removes interest expense from operating cash flow and places it in the calculation of the cost of capital. The financial manager then uses the operating income statement directly or indirectly to calculate periodic operating cash flows.

Although the learning curve can be successfully applied to many areas of operating cash flow (sales revenue, material costs, and labor costs) we restrict our example to labor costs in a non-inflationary environment. We do so because labor is usually the cost most responsive to learning by doing and to keep the example straight-forward.

The following discussion first develops estimated operating cash flows from a proposed capital project in which labor cost reflects no benefit from learning by doing (equivalent to a 100% learning-curve percentage). We then apply an assumed 80% learning curve to labor costs and draw conclusions.

Table 3. Operating Cash Flow Without Learning

Financial managers calculate operating cash flows of a proposed capital project either directly by reading values from an operating income statement (top) or indirectly with an equation based on the statement (bottom). This Table shows a six-year project ignoring inflation and any benefits from learning by doing.

<table>
<thead>
<tr>
<th></th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>$600,000</td>
<td>$600,000</td>
<td>$600,000</td>
<td>$600,000</td>
<td>$600,000</td>
<td>$600,000</td>
</tr>
<tr>
<td>Less variable operating costs (30%)</td>
<td>180,000</td>
<td>180,000</td>
<td>180,000</td>
<td>180,000</td>
<td>180,000</td>
<td>180,000</td>
</tr>
<tr>
<td>Contribution margin (70%)</td>
<td>$420,000</td>
<td>$420,000</td>
<td>$420,000</td>
<td>$420,000</td>
<td>$420,000</td>
<td>$420,000</td>
</tr>
<tr>
<td>Less other operating costs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salaries and advertising</td>
<td>120,000</td>
<td>120,000</td>
<td>120,000</td>
<td>120,000</td>
<td>120,000</td>
<td>120,000</td>
</tr>
<tr>
<td>Labor (no learning)</td>
<td>96,000</td>
<td>96,000</td>
<td>96,000</td>
<td>96,000</td>
<td>96,000</td>
<td>96,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>24,000</td>
<td>38,400</td>
<td>23,040</td>
<td>13,824</td>
<td>13,824</td>
<td>6,912</td>
</tr>
<tr>
<td>Operating earnings before taxes</td>
<td>$180,000</td>
<td>$165,600</td>
<td>$180,960</td>
<td>$190,176</td>
<td>$190,176</td>
<td>$197,088</td>
</tr>
<tr>
<td>Less taxes (34%)</td>
<td>61,200</td>
<td>56,304</td>
<td>61,526</td>
<td>64,660</td>
<td>64,660</td>
<td>67,010</td>
</tr>
<tr>
<td>Operating earnings after taxes</td>
<td>$118,800</td>
<td>$109,296</td>
<td>$119,434</td>
<td>$125,516</td>
<td>$125,516</td>
<td>$130,078</td>
</tr>
<tr>
<td>Add back depreciation</td>
<td>24,000</td>
<td>38,400</td>
<td>23,040</td>
<td>13,824</td>
<td>13,824</td>
<td>6,912</td>
</tr>
<tr>
<td>Operating cash flow</td>
<td>$142,800</td>
<td>$147,696</td>
<td>$142,474</td>
<td>$139,340</td>
<td>$139,340</td>
<td>$136,990</td>
</tr>
</tbody>
</table>

Operating Cash Flow (Indirect Method)

\[
OCF = (EBIT + Depreciation)(1-T) + (Depreciation \times T)
\]

End of Year 1:

\[
OCF = (180,000 + 24,000)(1 - 0.34) + (24,000 \times 0.34) = 142,800
\]

End of Year 2:

\[
OCF = (165,600 + 38,400)(1 - 0.34) + (38,400 \times 0.34) = 147,696
\]

End of Year 3:

\[
OCF = (180,960 + 23,040)(1 - 0.34) + (23,040 \times 0.34) = 142,474
\]

End of Year 4:

\[
OCF = (190,176 + 13,824)(1 - 0.34) + (13,824 \times 0.34) = 139,340
\]

End of Year 5:

\[
OCF = (190,76 + 13,824)(1 - 0.34) + (13,824 \times 0.34) = 139,340
\]

End of Year 6:

\[
OCF = (197,088 + 6,912)(1 - 0.34) + (6,912 \times 0.34) = 136,990
\]
Ignoring Learning By Doing. Consider in Table 3 projected annual operating cash flow in the absence of learning for a six-year proposed project. The financial manager works with marketing managers, human resource managers, tax accountants, and engineers to forecast sales revenue, tax rates, and operating costs. Depreciation values in Table 3 (and in Table 4) use the modified accelerated cost recovery system with a five-year depreciable life (modified for the half-year convention) and assume a 34 percent marginal tax rate.

The top part of Table 3 calculates expected operating cash flow (OCF) at the end of each year using the direct method, so called because it uses values directly from the operating income statement without the need to apply an equation. Each year shows unchanged projected $96,000 labor costs as would be presented in the finance text books in Table 1—no learning at all.

For completeness, we show at the bottom of Table 3 the indirect method for calculating operating cash flow, so called because a manager first develop an operating income statement (as with the direct method), then apply an equation to find operating cash flow. Direct and indirect methods yield the same values, so the method is a matter of choice.

Considering Learning By Doing. Table 4 modifies values from Table 3 to reflect an 80 percent learning-curve percentage applied to labor costs. Labor cost in the first year is the same $96,000 as in Table 3. However, with an 80 percent learning-curve percentage, cumulative average labor costs decline systematically over the life of the proposed project.

Table 4. Operating Cash Flow With An 80 Percent Learning Curve

This table shows operating cash flows for the same proposed project as in Table 3 with the benefits of learning applied to labor. Labor is the marginal cost reflecting an 80 percent learning curve calculated with Equation 2. The systematic reduction in labor costs results in an increase in operating cash flow in periods 2–6 compared with results in Table 3.

<table>
<thead>
<tr>
<th>Year</th>
<th>Revenues</th>
<th>Less variable operating costs (30%)</th>
<th>Contribution margin (70%)</th>
<th>Less other operating costs</th>
<th>Operating earnings before taxes</th>
<th>Less taxes (34%)</th>
<th>Operating earnings after taxes</th>
<th>Add back depreciation</th>
<th>Operating cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$600,000</td>
<td>$180,000</td>
<td>$420,000</td>
<td>$24,000</td>
<td>$180,000</td>
<td>$120,000</td>
<td>$118,800</td>
<td>$24,000</td>
<td>$142,800</td>
</tr>
<tr>
<td>2</td>
<td>$600,000</td>
<td>$180,000</td>
<td>$420,000</td>
<td>$38,400</td>
<td>$184,799</td>
<td>$77,639</td>
<td>$121,967</td>
<td>$38,400</td>
<td>$160,367</td>
</tr>
<tr>
<td>3</td>
<td>$600,000</td>
<td>$180,000</td>
<td>$420,000</td>
<td>$23,040</td>
<td>$228,350</td>
<td>$82,491</td>
<td>$150,711</td>
<td>$23,040</td>
<td>$173,751</td>
</tr>
<tr>
<td>4</td>
<td>$600,000</td>
<td>$180,000</td>
<td>$420,000</td>
<td>$38,400</td>
<td>$242,624</td>
<td>$83,648</td>
<td>$160,129</td>
<td>$38,400</td>
<td>$180,000</td>
</tr>
<tr>
<td>5</td>
<td>$600,000</td>
<td>$180,000</td>
<td>$420,000</td>
<td>$13,824</td>
<td>$246,024</td>
<td>$86,858</td>
<td>$162,376</td>
<td>$13,824</td>
<td>$175,518</td>
</tr>
<tr>
<td>6</td>
<td>$600,000</td>
<td>$180,000</td>
<td>$420,000</td>
<td>$13,824</td>
<td>$255,464</td>
<td>$91,212</td>
<td>$168,606</td>
<td>$13,824</td>
<td>$175,518</td>
</tr>
</tbody>
</table>

Operating Cash Flow (Indirect Method)

\[
OCF = (EBIT + Depreciation)(1 - T) + (Depreciation \times T)
\]

End of Year 1:
\[
OCF = ([180,000 + 24,000](1 - 0.34) + (24,000 \times 0.34) = $142,800
\]

End of Year 2:
\[
OCF = ([184,797 + 38,400](1 - 0.34) + (38,400 \times 0.34) = $160,367
\]

End of Year 3:
\[
OCF = ([228,350 + 23,040](1 - 0.34) + (23,040 \times 0.34) = $173,751
\]

End of Year 4:
\[
OCF = ([242,620 + 13,824](1 - 0.34) + (13,824 \times 0.34) = $173,953
\]

End of Year 5:
\[
OCF = ([246,024 + 13,824](1 - 0.34) + (13,824 \times 0.34) = $176,200
\]

End of Year 6:
\[
OCF = ([255,464 + 6,912](1 - 0.34) + (6,912 \times 0.34) = $175,518
\]
Labor costs in Table 4 use marginal values and so cannot be taken directly from values calculated with Equation 2. Calculating marginal values begins with the cumulative average cost from Equation 2, then solves for the cumulative average total cost. Only then can we calculate marginal costs.

Consider Table 5 below, which shows the way we found the marginal values to use in Table 4. Column 2 begins with $96,000 labor cost in year one, then declines systematically to reflect the 80 percent learning-curve percentage used in Equation 2.

Table 5. Calculating Marginal Labor Costs

Marginal labor costs to use in Table 4 are calculated after developing the cumulative average total labor cost values in column 3 from cumulative average labor costs in column 2.

<table>
<thead>
<tr>
<th>Year</th>
<th>(2) Cumulative Average Labor Cost</th>
<th>(3) = (1)×(2) Cumulative Average Total Labor Cost</th>
<th>(4) = Δ(3) Marginal Labor Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$96,000</td>
<td>$96,000</td>
<td>$96,000</td>
</tr>
<tr>
<td>2</td>
<td>$76,801</td>
<td>$153,602</td>
<td>$76,801</td>
</tr>
<tr>
<td>3</td>
<td>$67,404</td>
<td>$202,212</td>
<td>$48,610</td>
</tr>
<tr>
<td>4</td>
<td>$61,442</td>
<td>$245,768</td>
<td>$43,556</td>
</tr>
<tr>
<td>5</td>
<td>$57,181</td>
<td>$285,920</td>
<td>$40,152</td>
</tr>
<tr>
<td>6</td>
<td>$53,924</td>
<td>$323,544</td>
<td>$37,624</td>
</tr>
</tbody>
</table>

Table 5 shows that the marginal labor cost calculated with the cumulative average time model requires the financial manager to use the change in the cumulative average total labor cost in column 3. Values in column 3 accumulate from values in column 2, the cumulative average labor cost. For example, we calculated the cumulative average labor cost in year 5 as follows:

\[
\text{Cumulative average cost} = \text{Beginning cost} \times \left( \frac{\ln L C}{\ln 2} \right)
\]

\[
= \frac{\ln 0.80}{\ln 2} \times 5
\]

\[
= 5 \times 0.3219
\]

\[
= 5 \times 0.5956
\]

\[
= 5 \times 0.5956
\]

\[
= 5 \times 0.5956
\]

Because each amount in column 2 of Table 5 is a cumulative average from the initial attempt or lot through the attempt considered, calculating marginal labor cost requires finding the cumulative average total cost in column 3, then calculating differences between subsequent total costs. Column 4 shows the results. The reader will note that with each doubling in column 1, cumulative average costs in column 2 behave as suggested by the short-cut method in Equation 3. Cumulative average labor cost for Year 2 is 80 percent of the value in Year 1 ($96,000×0.80). And the value for Year 4 is 80 percent of the value in Year 2 ($76,801×0.80). These results give us some assurance that calculations with the logarithmic equation are correct.

Figure 1 dramatizes the benefit from learning by illustrating the difference in labor costs in Table 3 and Table 4. Each estimate begins at $96,000 to reflect the labor cost during the initial year. The function labeled Without Learning (a 100% learning-curve percentage) shows labor cost to remain unchanged at $96,000 over the life of the
project. The function labeled With Learning reflects the 80% learning-curve percentage. The decline in labor costs arising from learning (reflecting the assumed decrease in time spent to produce output) contributes to increased operating cash flow over the expected life of the project.

**Figure 1. Undiscounted Labor Costs Without and With Learning**

Annual expected labor costs of a proposed capital-budgeting project is less when labor cost is adjusted for learning. Here, labor costs with learning (from Table 4) decline systematically to reflect an 80 percent learning percentage. The after-tax impact on operating cash flow is the difference between labor costs with and without learning multiplied by \((1-T)\), where \(T\) is the marginal tax rate applied to the proposed project.

We remind the reader that differences in labor cost and in resulting operating cash flows will vary depending upon such issues as the learning-curve percentage and the beginning amount of labor expense. For example, a 90 percent learning curve would show a less dramatic decline in the cumulative average cost, and a smaller percentage (for example 75%) would show a more dramatic decline. We are convinced, however, that financial managers should include the learning curve in financial analysis and then modify or dismiss it where conditions warrant. To ignore it altogether does a disservice to shareholders.

**PART FOUR. SUMMARY AND CONCLUSION**

This paper discussed the use of the learning curve in financial decision making. The text-book literature of capital budgeting ignores the influence of learning, and results of questionnaires surveying finance practitioners likewise suggest ignorance of the influence of learning. After noting that the learning curve is not part of the text book material and practitioner use in finance, we developed a log-linear model for estimating the cumulative average time or cost associated with a process. We then showed a simplified model for estimating the cumulative average time associated with the doubling of each subsequent attempt.

We showed a simple example of operating cash flow the way a financial manager can use the learning curve to forecast cumulative average labor costs and then use these values to find marginal labor costs. Although examples in this paper emphasized only labor cost, subsequent research can develop a more robust application to several of the variables. Moreover, surveys should ask explicitly about the use of the learning curve in financial decision making.

The learning curve is a useful tool for looking at the world and that financial managers (and students) should
know that the lower the learning-curve percentage (and steeper the learning curve), the more effective and efficient learning will be. Finance practitioners can develop learning-curve models for use in decisions, then discuss the relevance of the learning-curve percentage and beginning time (or cost). They can then refine the estimating process and evaluate results in a post-completion audit of a project by comparing actual performance with the expected performance developed with the learning-curve model.

To the finance instructor, introducing the learning curve in a course provides students with a richer and more realistic classroom experience than is the case without the learning curve. One of the authors of this paper introduces the short-cut learning curve in Equation 3 the first day of class to show students the way time devoted to mastering finance declines systematically as the student applies effort. The other author introduces the learning curve in discussing working-capital management. We contend that the issue is not whether you should introduce it or not. Clearly, you should. Students will be better trained in decision making and better prepared to discuss cost behavior with their business colleagues when comfortable with the learning curve.

AUTHOR INFORMATION

Peyton Foster Roden. BA in English literature, Baylor University. MA in Economics at the University of North Texas. PhD in Finance University of North Texas. Regents Professor of Finance at the University of North Texas. Certified Management Accountant (CMA) Certified in Financial Management (CFM). Roden@unt.edu.


Note

(1) The learning curve we use throughout the paper is the cumulative average-time model in which the average applied to all lots declines with learning. Here, marginal time is calculated after finding the cumulative average total time associated with an additional attempt or lot. Some managers use the incremental unit-time method in which the incremental time per lot declines by a constant percentage as output doubles. An explanation of the two methods is in Liao (1988, pp. 302-315) and in Horngren, Datar, and Foster (2003, pp. 340-342). For a readily available discussion of the two types of learning curves managers use, see the Management and Accounting Web page: http://www.maaw.info/IntroToMAAW.htm, Main Topics, Learning Curves

REFERENCES
