

# Cost Swaps And Risk: An Analysis Of The Effect Of Cost Swaps On Degree Of Operating Leverage And Break-Even Points

Bruce Howard, (E-mail: [bruce.howard@wheaton.edu](mailto:bruce.howard@wheaton.edu)), Wheaton College

## Abstract

*Financial institutions have, for a long time, used interest rate swaps to address risk in managing assets and liabilities. Variable interest streams can be “swapped” for fixed payment streams (or visa versa) to either increase or decrease the sensitivity of the financial institution’s profits to changes in interest rates. With the advent of outsourcing, all businesses have the ability to do essentially the same thing by swapping fixed or variable cost streams. This article addresses managerial accounting topics of break-even analysis and degree of operating leverage which both have implications on risk and cost of capital. The analysis demonstrates a point that is generally ignored in the treatment of these issues. Conventional understanding says that shifting costs from variable to fixed will increase both the break-even point and degree of operating leverage. The analysis demonstrates that this is not necessarily true and could, indeed, have the opposite effect. The analysis also provides the conditions for determining how a shift in cost will impact break-even points and degree of operating leverage. The analysis is timely and offers insights to managers as to how outsourcing can be used to address the management of risk.*

## Introduction

One of the many dimensions of risk that a firm has to deal with is the variability of its earnings. The swings in a firm’s profits have a direct bearing on the firm’s cost of capital. Increased variability of earnings is generally interpreted as higher risk to investors for which they must be compensated. Managers therefore have a strong interest in understanding how any actions they take impact earnings volatility and cost of capital.

*Readers with comments or questions are encouraged to contact the author via e-mail.*

Theoretically, there are an infinite number of profit points (positive and negative) along the continuum of possible profit levels for a given firm. Managers have historically been schooled to take special note of one of those points in particular. Learning to identify the breakeven point is often highlighted in basic accounting, finance and microeconomics courses, as well as in other business courses. The point of zero profit or loss serves as a useful watershed for managers in the portfolio of tools used to evaluate risk.

These two issues, earnings variability and breakeven point, are related in that both are dependent on the mix between fixed and variable costs. Any change in this mix will alter the breakeven point and impact the variability of profits.

**Analysis**

In recent years, there has been an increasing trend in business to move towards outsourcing. One notable advantage to outsourcing is that it offers firms the ability to change the fixed/variable cost mix. In this respect it is somewhat akin to interest rate swaps among financial institutions. A bank, for example, will swap fixed rate interest streams for variable flows or visa versa depending on the objectives of a particular asset or liability management strategy. In similar fashion, a firm may outsource a given segment of its operations to an outside supplier who agrees to bill on a unit of production basis. If the outsourced operation originally had a significant fixed cost component in the form of rents, salaries, etc., then such a move will have altered the firm's basic cost mix, effectively "swapping" fixed for variable costs. It is also entirely plausible that a firm might have an operation that has experienced a preponderance of variable costs, which it then outsources to a supplier that agrees to bill on a fixed fee contract basis. This also alters the cost mix, but in the other direction.

Assuming that one of the objectives of outsourcing is to reduce risk, then it follows that the firm should use outsourcing to restructure cost patterns in ways that decrease the variability of earnings. The degree of operating leverage, (DOL) is a measure that provides insight into variability of earnings. DOL can be viewed, from the perspective of microeconomics, as an elasticity concept relating the sensitivity of changes in earnings to changes in output. Specifically, DOL reflects the percentage change in earnings divided by the percentage change in

quantity produced and sold. Its calculation is sometimes expressed using the formula: <sup>1</sup>

$$\frac{\text{Total Contribution Margin}}{\text{Net Income}}$$

Total contribution margin is unit contribution margin (ucm) times the quantity of units produced and sold. Unit contribution margin is the difference between unit price and unit variable cost and represents the rate of change in profits on a per unit basis. Every one-unit change in output produced and sold will increase or decrease income by the amount of the unit contribution margin. A shift in cost patterns from fixed to variable will decrease the unit contribution margin and hence reduce the rate of change in income. It is tempting to take this result one step further and conclude that any such swapping of fixed for variable cost will also reduce the variability of earnings. This may not be the case, however, if one measures variability of earnings in terms of percentage change in earnings, as in DOL, rather than in rates of change, i.e., unit contribution margin. The fact is that at a specific level of output, swapping fixed for variable cost may in reality increase the variability of earnings. In other terms, variable for fixed cost swaps have the potential of increasing or decreasing DOL at a given level of output. This can be demonstrated by using differentials to approximate the change in DOL as fixed and variable costs are swapped. <sup>2</sup>

The result leads to a useful decision rule that can be used to predict the effect of a given cost swap. If the percentage change in fixed cost is greater than the percentage change in unit contribution margin, then the degree of operating leverage will increase. Similarly, if the percentage change in fixed cost is less than the percentage change in unit contribution margin, then degree of operating leverage will decrease.

$$\text{If } \frac{\Delta F}{F} > \frac{\Delta \text{ucm}}{\text{ucm}}, \text{ then DOL will increase.}$$

If  $\frac{\Delta F}{F} < \frac{\Delta ucm}{ucm}$ , then DOL will decrease.

It is important to note that in applying the conditions above, the sign indicating direction of change, is critical. A variable for fixed cost swap would mean that the change in fixed cost,  $\Delta F$ , and the change in unit contribution margin,  $\Delta ucm$ , both represent decreases and thus, bear a negative sign. A 25% decrease in fixed costs would be mathematically greater than a 50% decrease in contribution margin and would imply an increase in DOL. Correspondingly, a 25% increase in fixed costs is less than a 50% increase in unit contribution margin and would lead to a decrease in DOL.

Changes in DOL reflect changes in the variability of earnings and have an impact on that dimension of risk. But cost swaps will have an impact on another dimension of risk. Trading fixed cost for variable costs will also change a firm's breakeven point. If a firm could lower its breakeven point, then it is reasonable to interpret such an outcome as a reduction of risk. It is tempting to think that a firm could reduce its breakeven point by a cost swap that lowers fixed costs and increases variable costs. That, however, is not always the case. Changing to a cost structure that includes less fixed cost and greater reliance on variable costs may just as well increase the breakeven point for a given firm. Consider Exhibit 1 on the next page.

Assume that the firm depicted in the above exhibit begins with the cost structure labeled TC1 and the revenue pattern corresponding to TR1. The breakeven quantity would be Q1. Should that firm change its cost structure to TC2, then given the same pattern of revenues, the corresponding breakeven quantity would decrease to Q2. This is what is normally expected when fixed costs are lowered relative to variable costs.

But given a different pattern of revenues, the situation changes. If the revenues follow the pattern depicted by TR2, then a shift in cost structure TC1 (high fixed-low variable costs) to TC2 (low fixed-high variable costs) would, in fact, increase the breakeven quantity from q1 to q2.

At least three different decision rules can be developed to determine whether a shift in the fixed variable cost mix will increase or decrease a firm's breakeven point. The first focuses on relative percentage changes in fixed and variable costs. The second decision rule involves comparing relative levels of output. The third rule employs the variables of price and average cost.

The breakeven point expressed in units can be expressed in generalized form as:

$$Q_* = \frac{F_i}{p_i - v_i}$$

where  $p_i$  is the unit selling price and  $v_i$  is the unit variable cost.

The first decision rule is developed by differentiating the general breakeven quantity formula with the goal of defining the mathematical relationships for which the change in  $Q_*$  will be either positive or negative, indicating an increase or decrease, respectively, in the breakeven point. This leads to a result that is strikingly similar to the decision rules developed for analyzing the effect of cost swaps on the degree of operating leverage. The breakeven point will increase whenever the percentage change in fixed cost is greater than the percentage change in unit contribution margin. But if the percentage change in fixed cost is less than the percentage change in unit contribution margin, then the breakeven point will decrease.<sup>3</sup>

$$\text{If } \frac{\Delta F}{F} > \frac{\Delta ucm}{ucm},$$

then the breakeven point will increase.

If  $\frac{\Delta F}{F} < \frac{\Delta ucm}{ucm}$ ,

then the breakeven point will decrease.

A casual inspection of Exhibit 2 suggests that the crossover point, where the two alternative cost patterns TC1 and TC2 intersect, should provide a useful reference point for developing a second and relatively simple decision rule to predict what will happen to the breakeven point when the cost mix changes by swapping fixed and variable costs.

At  $Q^*$ , total costs for TC1 and TC2 are the same even though the underlying mix between fixed and variable components is different.

What happens to the breakeven point will

depend on whether the initial breakeven quantity is below or above  $Q^*$ .<sup>4</sup> If a firm wants to lower to the left of  $Q^*$ , then it could do so by swapping fixed costs for variable. This would be akin to shifting from TC1 to TC2. But the strategy would change if the firm's initial breakeven level of output was greater than, i.e., to the right of,  $Q^*$ . In that case the appropriate strategy to reduce the breakeven point would be to shift the cost pattern to one that employed relatively greater fixed and less variable costs. This would be akin to shifting from TC2 to TC1.

The decision rule can be represented by the matrix in Table 1.

The third decision rule focuses on the variables, price and average cost. Referring again to Exhibit 2, a reference line has been drawn from

Exhibit 1

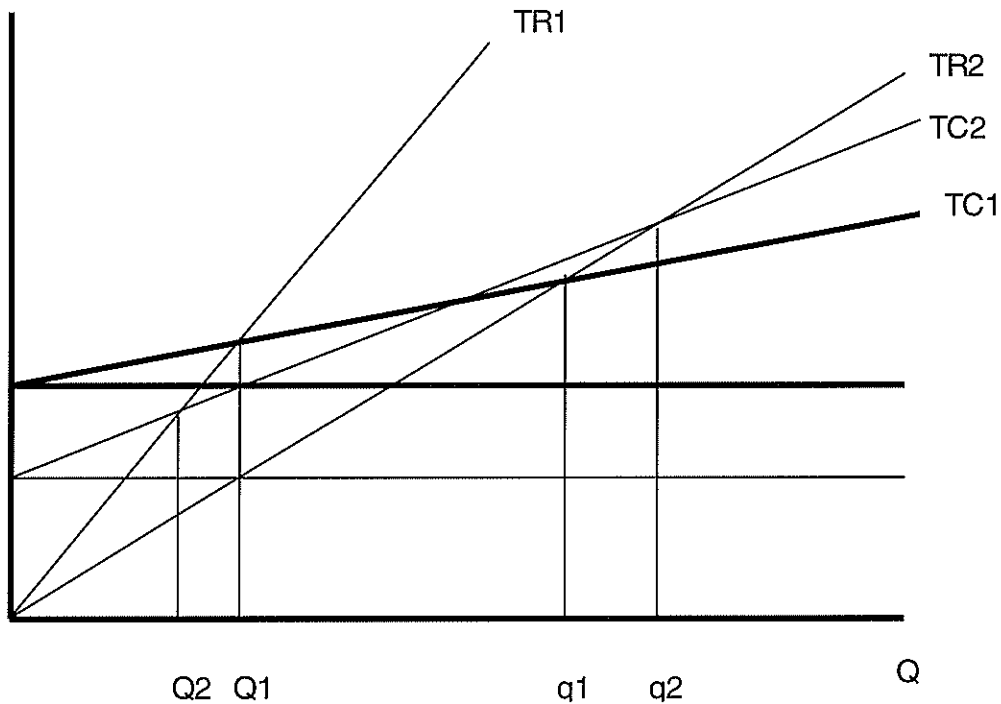
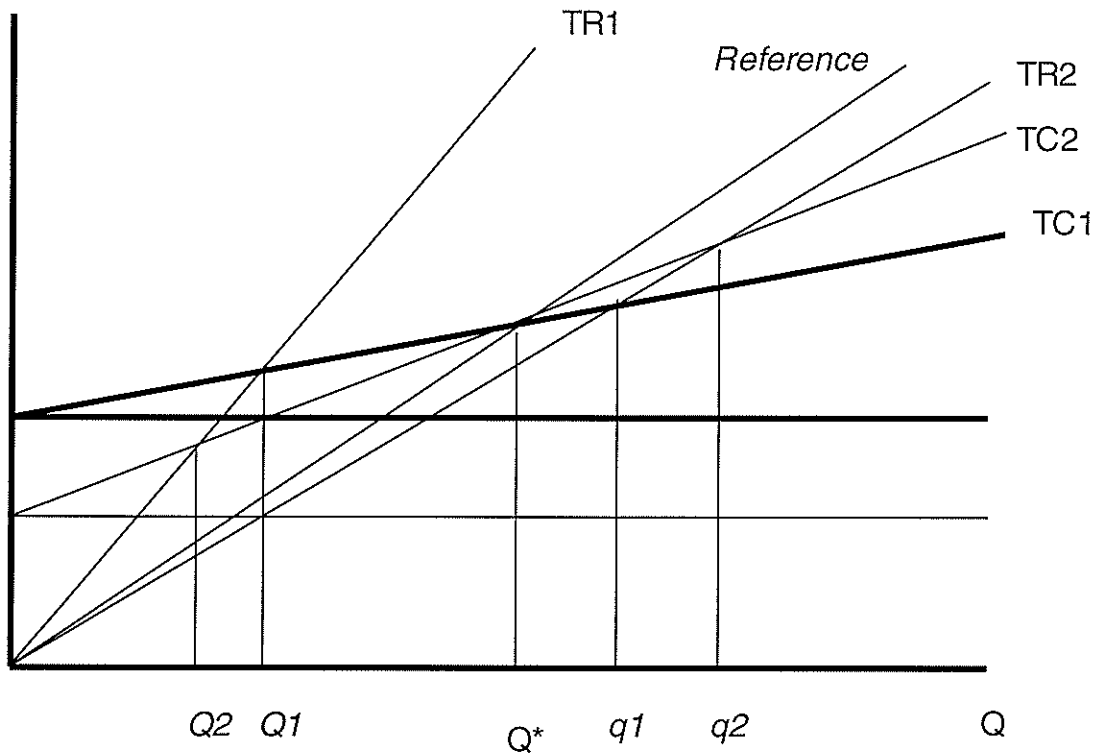


Exhibit 2



the origin through the point where TC1 and TC2 crossover. The slope of this reference line represents the average cost for both TC1 and TC2 at quantity  $Q^*$ . The slope of the total revenue function is simply the price of the product. By comparing the slope of the total revenue function, i.e. price, with the slope of the reference line from the origin through the crossover point, i.e., where average cost for both TC1 and TC2 are

equal, the outcome of a shift in the cost mix can be determined.

If the current price is greater than average cost at the crossover point, then a shift in the cost structure away from fixed and more to variable will decrease the breakeven quantity. Conversely, if the current price is less than average cost at the crossover point, then introducing

Table 1

	Initial breakeven quantity is below the crossover quantity:	Initial breakeven quantity is above the crossover quantity:
Shifting costs from fixed to variable	LOWERS THE BREAKEVEN POINT	INCREASES THE BREAKEVEN POINT
Shifting costs from variable to fixed	INCREASES THE BREAKEVEN POINT	LOWERS THE BREAKEVEN POINT

**Table 2**

	If current price $P > AC^*$ , where $AC^*$ represents average cost at the crossover point:	If current price $P < AC^*$ , where $AC^*$ represents average cost at the crossover point:
Shifting costs from fixed to variable	LOWERS THE BREAKEVEN POINT	INCREASES THE BREAKEVEN POINT
Shifting costs from variable to fixed	INCREASES THE BREAKEVEN POINT	LOWERS THE BREAKEVEN POINT

more variable and less fixed costs will in fact increase the breakeven quantity.<sup>5</sup> This decision rule can be represented by the following matrix:

**Summary**

Changes in the mix between fixed and variable costs will change both the variability of earnings, as reflected by the degree of operating leverage, and the breakeven point. Cost swaps of this nature have the potential to either increase or decrease DOL and the breakeven point. Firms can take advantage of outsourcing as a means of decreasing risk by reducing swings in earnings and lowering the breakeven point. Outsourcing can be used to create fixed/variable cost swaps, which in turn, will affect DOL and breakeven points. It cannot be assumed that the variability of earnings and breakeven dimensions of risk will be reduced by swapping fixed cost patterns for ones that vary with output. Such trades have the potential to either increase or decrease risk depending on the given level of output. By comparing the relative percentage changes in both unit contribution margin and fixed costs, it can be determined whether a proposed fixed/variable cost swap will increase or decrease risk as measured by changes in the degree of operating leverage and breakeven point.

**Suggestions for Future Research**

Outsourcing is an important trend in current business. There may be several reasons for this. It would be interesting to know the

extent to which firms are employing this technique with the express purpose of influencing the degree of operating leverage and/or breakeven point. If that indeed is the primary motivation, it would also be interesting to know if the majority of firms engaged in outsourcing techniques are doing so in order to either, 1) increase or 2) decrease DOL and/or breakeven points. It would be important to understand if there is a predominant form of cost structure in outsourcing contracts and whether these are being arranged in a manner that converts fixed costs to variable or variable to fixed. It would also be important to learn if outsourcing activity is having the desired effect. It could very well be that the answers to these questions vary from firm to firm, industry to industry, region to region and country to country. Even within a large multinational firm, the answers to these questions might vary among product lines or geographical regions. Finally, it would be important to learn if outsourcing activity is having an impact on the variability of earnings. □

**Endnotes**

1. The derivation of the above formula for DOL is straightforward. By definition, degree of operating leverage (DOL) is the percentage change in income (NI) divided by the percentage change in quantity (Q) of output produced and sold, expressed as follows:

$$DOL = \% \Delta NI / \% \Delta Q = (\Delta NI / NI) / (\Delta Q / Q)$$

Net income can be computed by the difference between Total Contribution Margin ( $ucm \cdot Q$ ) and fixed costs ( $F$ ). The change in net income will be the change in quantity times the unit contribution margin. Making these substitutions yields:

$$DOL = [(\Delta Q \cdot ucm)/(Q \cdot ucm - F)]/[\Delta Q/Q]$$

Canceling like terms and rearranging terms in expression above yields the desired result.

$$DOL = (Q \cdot ucm)/(Q \cdot ucm - F) =$$

$$\frac{\text{Total Contribution Margin}}{\text{Net Income}}$$

- At any given level of output,  $Q$ , the approximate change in DOL can be determined by differentiating equation (1) with respect to unit contribution margin ( $ucm$ ) and fixed cost ( $F$ ), remembering that  $Q$  is held constant. The objective in holding quantity constant is to afford the opportunity to focus on what will happen to variability in earnings as measured by changes in DOL if cost patterns are swapped. The result of doing so is shown below:

$$dDOL = d[(Q \cdot ucm)/(Q \cdot ucm - F)] =$$

$$\frac{Q(ucm \cdot dF - F \cdot ducm)}{(Q \cdot ucm - F)^2}$$

Inspection of the above result shows that the outcome can either be positive or negative depending on whether the numerator is greater or less than zero. That in turn depends on the relative values of  $ucm$ ,  $F$ ,  $dF$  and  $ducum$ . The numerator will be zero whenever unit contribution margin times the change in fixed cost is equal to fixed cost times the change in unit contribution margin. It follows that the numerator will be zero if the percentage change in fixed costs is equal to the

percentage change in unit contribution margin. These are demonstrated below:

$$dDOL = 0, \text{ when } (ucm \cdot dF) = (F \cdot ducm) \Leftrightarrow$$

$$\frac{dF}{F} = \frac{ducum}{ucm}$$

This result can be extended to develop conditions for determining whether or not DOL will increase or decrease by comparing the relative percentage changes in fixed costs and unit contribution margin.

If  $\frac{dF}{F} > \frac{ducum}{ucm}$ , then DOL will increase.

If  $\frac{dF}{F} < \frac{ducum}{ucm}$ , then DOL will decrease.

- The breakeven point expressed in units can be expressed in generalized form as:

$$Q_* = \frac{F_i}{p_i - v_i}, \text{ where } p_i \text{ is the unit selling price}$$

and  $v_i$  is the unit variable cost.

Taking the total differential of the above expression yields:

$$dQ_* = \frac{1}{p_i - v_i} \cdot dF - \frac{F_i}{(p_i - v_i)^2} \cdot dp + \frac{F_i}{(p_i - v_i)^2} \cdot dv$$

The next step derives the conditions for which a change in the fixed/variable cost mix would not alter the breakeven point  $Q_*$ . This result will then provide the basis for determining whether the breakeven point will increase or decrease.

If price is held constant, i.e.,  $dp=0$ , then it follows that the conditions for no change in breakeven quantity will be:

$$\frac{1}{p_i - v_i} \cdot dF = - \frac{F_i}{(p_i - v_i)^2} \cdot dv$$

Rearranging terms and simplifying yields:

$$\frac{dF}{F_i} = - \frac{dv_i}{(p_i - v_i)}$$

$$\text{If } \frac{dF}{F_i} > - \frac{dv_i}{(p_i - v_i)},$$

then the breakeven quantity will increase.

$$\text{If } \frac{dF}{F_i} < - \frac{dv_i}{(p_i - v_i)},$$

then the breakeven quantity will decrease.

Recognizing that an increase in unit variable cost would necessarily imply a decrease in unit contribution margin, it then becomes evident that the above conditions are equivalent to those previously derived for determining whether or not a cost swap will increase or decrease DOL.

$$\text{If } \frac{dF}{F_i} > - \frac{dv_i}{(p_i - v_i)} \Leftrightarrow \frac{dF}{F} > \frac{duc_m}{ucm},$$

then the breakeven quantity will increase.

$$\text{If } \frac{dF}{F_i} < - \frac{dv_i}{(p_i - v_i)} \Leftrightarrow \frac{dF}{F} < \frac{duc_m}{ucm},$$

then the breakeven quantity will decrease.

In other words, if the percentage change in fixed costs exceeds the percentage change in unit contribution margin, then the unit breakeven point can be expected to increase.

Conversely if the percentage change in fixed costs is less than the percentage change in the unit contribution margin, then the breakeven point in units will decrease.

- An actual numerical value for  $Q^*$  (the reference quantity corresponding to the crossover point) can be calculated by setting the two cost equations equal to each other and then solving for quantity  $Q^*$ .

$$TC_1 = TC_2 \Leftrightarrow v_1 \cdot Q^* + F_1 = v_2 \cdot Q^* + F_2$$

Solving for  $Q^*$  yields the following result:

$$Q^* = \frac{F_1 - F_2}{v_2 - v_1} \text{ or } \frac{F_2 - F_1}{v_1 - v_2}$$

The initial breakeven quantity,  $Q_1$ , should be compared to the results shown in the equation above to determine if the new breakeven quantity under the different cost regime will increase or decrease.

- The actual numerical value for average cost at the crossover point can be calculated by finding the slope of this reference line representing average cost for both  $TC_1$  and  $TC_2$  at  $Q^*$  as follows:

$$TC^*/Q^* = (v_1 \cdot Q^* + F_1)/Q^* = v_1 + \frac{F_1}{Q^*}$$

Substituting the results shown in note 4 for determining  $Q^*$  into the equation above yields:

$$TC^*/Q^* = v_1 + \frac{F_1(v_2 - v_1)}{F_1 - F_2} = v_2 + \frac{F_2(v_1 - v_2)}{F_2 - F_1}$$

This decision rule can be represented by the following matrix:



	If the current price, $P > v_1 + \frac{F_1(v_2 - v_1)}{F_1 - F_2}$ or $v_2 + \frac{F_2(v_1 - v_2)}{F_2 - F_1}$	If the current price, $P < v_1 + \frac{F_1(v_2 - v_1)}{F_1 - F_2}$ or $v_2 + \frac{F_2(v_1 - v_2)}{F_2 - F_1}$
Shifting costs from fixed to variable	LOWERS THE BREAKEVEN POINT	INCREASES THE BREAKEVEN POINT
Shifting costs from variable to fixed	INCREASES THE BREAKEVEN POINT	LOWERS THE BREAKEVEN POINT

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Notes