

# A Contemporary Policy-Making Perspective Concerning Supermarket Pricing and Quality

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## Abstract

*This manuscript presents a methodology from which supermarket management who, faced with competition, must provide recommendations for profit-maximizing price, quality, and capacity. It is assumed that service quality and price impact jointly on demand for service, and that both demand and service quality impact on the costs of providing service. A service quality constraint is applied to the mathematical optimization model in order to guarantee that recommended optimal service standards are met. We generate conditions that characterize optimal solutions and provide illustrative results based on empirical data obtained from a supermarket study.*

## Introduction

Although price has traditionally served as a decision variable through which competitive advantage may be attained, service quality has also emerged as another important attribute in the development of strategies employed by firms (Headley and Choi, 1992). In fact, industries previously imbued with monopoly status, such as postal services, have recently been forced to contend with related issues of entry and competition (Dobbenberg, 1993), where service quality is an integral component of competitive strategy.

Quality and price both impact significantly on demand for services (Berry, Zeithaml, and Parasuraman, 1990) and a firm's ability to remain competitive in a market economy (Lewis, 1989). In fact, Starr (1996) reports that consumers' quality expectations, often measured in terms of service time (Umesh, Pettit, and Bowman, 1989) often override price consideration. Although traditional practice encouraging cost reduction and short-term profits has been the norm (Zeithaml, Berry, and Parasuraman, 1988) recent findings indicate that this trend is reversing (Rust, Zahorik, and Keiningham, 1995). For example, Delta Airlines and McDonald's are noted for the establishment of formal goals that relate to service quality: such goals have been shown to improve organizational performance (Ivancevich and McMahon, 1982).

The importance of maintaining a balance between quality as pursued in the marketing literature (Berry, Zeithaml, and Parasuraman, 1990) and more production oriented approaches involving the effective use of technology, capacity and effective delivery systems have been neglected to some extent. Chase and Bowen (1988, 1991) address this omission by proposing the joint consideration of (a) a producer component which considers important production oriented concepts, such as the effective use of capacity, together with (b) a consumer component, where marketing and management oriented concepts regarding price and quality are jointly considered in the development and implementation of system-wide delivery services.

*Readers with comments or questions are encouraged to contact the author via e-mail.*

The model presented here addresses this gap by simultaneously considering a consumer component, where demand is influenced by both service quality and price, and an operations component focusing on the joint impact of demand and quality on total costs, an approach encouraged by Sasser, Olsen, and Wyckoff (1976). The net result of this analysis is the joint determination of both optimal price and service standards, in conjunction with operational policy, including capacity allocation. A service quality constraint in the mathematical optimization model provides the linkage that guarantees that resulting capacity decisions adhere to recommended service quality standards. The model presented here represents a variant of previous work (Boronico, 1998; Crew and Kleindorfer, 1992) focusing on welfare maximization, applied to postal services (Boronico, 1998a), with theoretical underpinnings established in Boronico, Crew, and Kleindorfer (1992). The model also presents a generalization of those presented in Stidham (1992) and Rump and Stidham (1998), whose results build upon foundations established in Dewan and Mendelson (1990), who focus on issues of equilibrium. The primary difference between these established models and the model presented here is that we assume that price and service quality are ex-ante declared and perfectly observable by consumers, which in turn generates demand, as opposed to assuming that consumers have rational expectations of service quality and update these expectations based on particular realizations of service quality encountered through observation.

Optimal policies suggest that price should be increased from quality-constrained marginal cost, and adhere to the well established "inverse-elasticity" rule first noted in the seminal work of Ramsey (1927) and Boiteaux (1956), where the percentage increase is inversely related to the price elasticity of demand. These findings are consistent with those established elsewhere (Boronico, 1998; Dewan and Mendelson, 1990; Crew and Kleindorfer, 1979) and applied to public utilities (Crew and Kleindorfer, 1986). The results here are differentiated from traditional results in that marginal cost is influenced by service quality, and is derived utilizing the envelope theorem (Silberberg, 1978; Crew and Kleindorfer, 1992; Boronico, 1997). In addition to pricing, the following intuitive prescriptions also apply: (1) service quality should be set so as to equate the benefits of increasing service quality to the costs of doing so, and (2) operational variables are so set so as to minimize total costs while adhering to the optimal level of service quality. Although intuitive in nature, the complexity of the simultaneous determination of these important policy components should not be underestimated.

The remainder of the paper is organized as follows: The theoretical model, foundations and prescriptions for optimal solutions are discussed in Section II. Technical/mathematical components of the model are provided in Appendix I, with corresponding proofs available upon request. The application of the methodological findings to supermarket operations are provided in Section III, and a brief comparative statics analysis is pursued in Section IV. Section V offers conclusions and implications for future research.

### **A Model of Quality-Constrained Profit Maximization**

We assume that the objective of the supermarket enterprise is one of profit maximization ( $\Pi$ ), where (a) optimal price ( $p^*$ ) for a typical basket of goods, (b) service quality ( $w^*$ ), and (c) capacity ( $\mu^*$ ) form the set of decision variables. Demand for service ( $X$ ) is assumed to be a function of both price and service quality. The model imposes a mathematical constraint within the formulation to guarantee that capacity decisions are made so as to adhere to the optimal recommendation for service quality. In fact, it may be shown that capacity will be set so as to adhere to the following theoretical prescription:

*Operating Variable Rule:* Capacity is set so as to minimize costs while adhering to the service quality constraint.

The specific form for the service quality constraint may vary across applications. For example, a systems service support helpdesk might require that the mean wait time for service does not exceed a declared service standard (Boronico, Zirkler, and Siegel, 1996). A service repair facility might provide a percentage guarantee that repairs will be effective or alternatively declare the probability for which a given repair's life span will exceed an established period of time. Similar types of probabilistic service quality constraints might be found in emergency medical facilities, where both time to serve and survival rates are critical. For example, ambulatory allocation

problems may focus on meeting a pre-specified mean response time with a certain probability (Klafehn, Weinroth, and Boronico, 1996).

The model utilized here, and presented in Appendix I, involves the maximization of profits, which are modeled as the difference between revenues ( $p \cdot X(p, w)$ ) and costs ( $C(X(p, w), w, \mu)$ ). Note that revenues are simply the product of price and demand, where demand is a function of both price and service quality. Costs (C) are a function of demand, service quality, and capacity. The constrained mathematical model also incorporates a service quality constraint. As noted above, this service quality constraint may assume various forms: for our purposes the service quality constraint will guarantee that the total throughput (number of checkout lanes that are open) is sufficient in order to adhere to and maintain the optimal mean wait time ( $w^*$ ) that is recommended by the model. As noted earlier, in addition to providing a capacity recommendation, the model provides a simultaneous recommendation for optimal price and quality:  $(p^*, w^*)$ . In the spirit of the discussion presented earlier, note that the model embodies both a marketing component in that demand is driven by both price and quality as well as an operational component linking capacity to both pricing and quality considerations.

The optimal price-service quality vector that results from the model presented here may be shown to adhere to the following recommendation:

*Optimal Pricing Rule:* Optimal quality-constrained price is increased from marginal cost, with the resulting percentage deviation of optimal price from marginal cost inversely proportional to the price elasticity of demand. Marginal cost, in this case, represents the cost of meeting an additional unit of demand.

The intuitive rationale for this rule is that in order to maximize profits, prices should be raised for those services with inelastic demand rather than from those which are more sensitive to price. These results owe much to Ramsey (1927), and the more recent synthesis by both Lipsey and Lancaster (1956) and Baumol and Bradford (1970), who consider "second-best" pricing solutions under those conditions where departures from marginal cost pricing are appropriate (competitive equilibrium is not achieved). We note, however, that results here differ from traditional pricing results as price is embodied within marginal costs as influenced by service quality, and are derived utilizing the envelope theorem. With respect to optimal service quality, the following prescription may be shown to apply:

*Service Quality Rule:* Optimal service quality is set so as to equate the marginal benefit of increasing service quality with the marginal cost of doing so.

Theoretical derivations supporting the aforementioned rules are available upon request.

### **A Quality, Price, and Capacity Model for Supermarkets**

The objective here is to illustrate how the theoretical results of section II may be applied to determine the profit-maximizing (a) price to charge for a typical basket of goods ( $p$ ), (b) level of throughput ( $\mu$ ), and (c) level of service quality ( $w$ ), measured by the mean time spent at the checkout counter. Supermarket management can utilize optimal price in the determination of an overall markup strategy for their products, while service quality and capacity recommendations assist in the determination of staffing requirements and shift scheduling.

The capacity decision variable is measured in terms of the overall service rate ( $\mu$ ) necessary to achieve a specified level of service quality, measured by the mean wait time ( $w$ ) at a checkout counter in the checkout facility. The overall service rate represents the total throughput rate at the checkout facility. The number of checkout lanes required may be approximated by dividing this throughput by the empirically measured mean service rate for one checkout clerk; this M/M/1 approximation provides an upper bound on profits, since it is well known that one fast server is superior in terms of throughput than multiple servers providing the same overall service rate

(Hillier and Lieberman, 1995).

Assumptions and data from Ittig (1994) are also maintained: the queuing process operates under the assumption of exponential service time and a Poisson arrival process. The consideration of exact solutions for multiple channel queues, an investigation into the relaxation of the assumptions made for M/M/C queues, and the consideration of general service time processes are left as implications for future research. The markup for a typical basket of goods is approximately \$5.08, or 9% of revenues. Equivalently, this represents a purchase price to the consumer of approximately \$56.44 for a typical basket of goods, with associated cost of  $v = \$56.44 - \$5.08 = \$51.36$ . The clerk cost at the checkout counter is provided by supermarket management and set at  $s = \$.3812/\text{service}$ , based on hourly wage and mean service times.

Demand is governed by an exponential distribution, as discussed in Ittig (1994). This choice for the demand distribution is further supported in the retail literature (Lee and Cohen, 1985; Lilien and Kotler, 1983), and has been found to characterize increasing returns to scale well, in addition to capturing other effects of price, such as sales approaching zero at high levels of price (Bolton, 1989). Alternative choices for the demand distribution form an important consideration for future research. Although linear demand functions are sometimes employed, linearity in terms of the impact of wait time on demand is questionable. For example, Osuna (1985) and Larson (1987) both indicate that psychological costs associated with wait times escalate nonlinearly. The actual determination of how waiting time and price impact on demand (i.e. coefficient determination), and associated multi-variable statistical analyses that incorporate issues of multicollinearity, heteroskedasticity, and the specification of an appropriate set of variables that span the demand space also form open questions that are not addressed here, but serve as significant implications for future research. The major emphasis and contribution of this research involves the ex-post analyses of profit maximization for known demand parameter values. However, a comparative statics analysis, presented in the following section, will illustrate how one might address the issue of solution sensitivity to changes in these parameter values. The standard form for the exponential demand distribution, which is utilized here, is  $X(p, w) = \alpha e^{-\beta w - \gamma p}$ , where  $\alpha, \beta, \gamma$  correspond to the aforementioned demand parameters (coefficients). Utilization of data provided by Ittig (1994) provide the basis from which we may determine that, for this particular data set, the demand parameters assume the following values:  $\alpha = 13,951,325$ ,  $\beta = .40$ , and  $\gamma = .21$ .

The cost function (C) is composed of two components: capacity cost ( $s\mu$ ) and product acquisition cost ( $v \cdot X(p, w)$ ). The service quality constraint provides the appropriate linkage between the service quality variable (mean wait time:  $w$ ), capacity variable (overall throughput at the checkout counters:  $\mu$ ), and demand ( $X(p, r)$ ). The explicit notation for the demand function, model, and solution equations are provided in Appendix II.

For the model utilized here, the optimal price to charge for a typical basket of goods equals:  $p^* = v + s + 1/\gamma$ . Marginal costs for the model are given by  $v + s$ , simply representing the sum of the per basket acquisition and per unit server cost. The third term,  $1/\gamma$ , represents the explicit impact of demand on optimal price. Note that this term is positive (the intuitive prescription that  $\gamma > 0$  may be assumed from the theoretical model) but decreases as  $\gamma \rightarrow 0$ . Noting that in the model  $\gamma$  represents the sensitivity of demand to price, with greater values implying greater sensitivity, it is clear that as demand sensitivity to price increases, optimal price decreases and approaches marginal cost. This is consistent with the previously discussed inverse elasticity tenet, namely, that increases in price from marginal cost are lower when price sensitivity is greater. Interestingly enough, similar marginal cost findings in other contexts have indicated that counterintuitive results often obtain (Boronico, 1998). Moreover, the pricing recommendation that applies here suggests that the monopolist might set price in the area where demand is inelastic, however, this is not unreasonable in light of the fact that the demand curve utilized is nonlinear and exponential in nature, and that the firm simultaneously competes in the domain of

service quality as well as price. The solution for optimal service quality, or mean wait time in the checkout queue ( $w^*$ ) requires an iterative search, as the solution equation is nonlinear and cannot be solved exactly using algebraic manipulation.

Utilizing the pre-specified values for demand sensitivity parameters ( $\alpha, \beta, \gamma$ ), the following optimal solutions/recommendations are obtained from the model:  $p^* = \$56.50$ ,  $w^* = .0455$  hours. The resulting demand, at optimum, may be shown to equal:  $X(56.50, .0455) = 96.27$  purchases per hour. The solution for the operating variable  $\mu^*$  is found through substitution of respective values into the service quality constraint, from which  $\mu^* = 118.2$  units of service/hour is obtained. Consequently, the supermarket should charge \$56.50 for a typical basket of goods, representing an 10% markup over cost. The optimal level of service quality, or mean time spent at the checkout counter, should be set at .0455 hours, or equivalently, 2.73 minutes. The capacity required to simultaneously meet demand at the stated service quality level is 103 services/hour. This figure could be used to approximate the number of checkout clerks required. For example, if the mean service time required for a clerk to process a typical basket of goods is  $1/\mu_c$ , then in order to meet the capacity requirement,  $\mu/\mu_c$  clerks would be required. This would represent an approximation to the optimal solution.

The model presented provides supermarket management with a prescription for pricing their goods, setting a quality standard, and installing the appropriate number of servers in order to maximize profits. The solutions have been generated under the assumption that demand is impacted on by both price and wait time, an extensions of much of the previous work provided in the literature.

**Comparative Statics and Sensitivity Analysis**

This section provides interesting comparative static results that may be derived from the model presented in this manuscript. We consider the sensitivity of optimal solutions with respect to changes in both ex-ante declared demand parameters and capacity cost. That is, we utilize the model to predict how the optimal solutions for the endogenous/choice variables (price, service quality, capacity) will respond to changes in some of the model’s exogenous parameters (cost and demand coefficients). The analysis here is grounded in the seminal work of Silberberg (1978). Details concerning this methodology are provided in Appendix III, which also provides an example for how the methodology is used to derive the results of the model presented here.

Consider how changes in the unit capacity cost  $s$  will impact on both optimal price and wait time. In this case, one expects price to increase as unit cost increases. This may be shown to be true. The comparative static result  $\partial p^*/\partial s = 1$  indicates that each dollar increase in unit cost results in a one dollar increase in price. The impact of a unit change in capacity cost on service quality, however, is not as intuitive. In this case it may be shown that  $\partial w^*/\partial s < 0$ , thus indicating that as capacity cost increases, optimal wait time decreases.

A similar analyses may be conducted for demand parameters. A summary of the results of these comparative static analyses are summarized and presented below, in Table 1:

**Table 1: Sample Comparative Statics Results**

Parameter Increased	Comparative Statics Results	
	Impact on price $p$	Impact on mean wait time $w$
$s$ (unit capacity cost)	increase	decrease
$\alpha$ (base demand parameter)	no change	decrease
$\gamma$ (demand sensitivity to price)	decrease	increase

For example, an increase in base demand ( $\alpha$ ) results in no change in optimal price but a decrease in mean wait time. Hence, if base demand increases, supermarket management responds by decreasing wait time, at optimal. If demand becomes increasingly sensitive to price ( $\gamma$  increases), supermarket management responds by decreasing price, at optimal, but simultaneously compensates for the lower price by increasing the mean wait time. Hence, while consumers pay less for a basket of goods, they wait longer, on average, in the queue.

In summary, comparative statics analyses such as those presented above help provide insight for management when considering market fluctuations that may impact on demand, availability of labor, product acquisition costs or other factors that influence managerial decisions. Finally, specifics concerning the functional forms for the comparative statics analyses presented above are available, upon request.

### **Conclusions and Implications for Future Research**

This research has considered the joint determination of price, service quality, and a local operating variable, such as capacity, for a firm whose objective is profit maximization. Although much attention has been paid to issues of cost and quality, as well as the impact of quality on demand or cost, the joint consideration of these issues has received less attention. The model developed (1) assumes that service quality impacts on both demand for service as well as costs and (2) unifies both marketing and operations oriented system components, an omission in the literature that has been previously noted (Chase and Bowen, 1988; 1991).

While the foregoing analysis has been applied to the supermarket industry, it should be recognized that the general framework presented here may bear upon other related industries as well. For example, Boronico (1998) discusses the implication of reliability constrained modeling to postal services, and Boronico, Zirkler, and Siegel (1996) consider a similar approach applied to systems service support. Concerning supermarket operations themselves, the extent to which service quality and cost have both become salient attributes in the consumer market for food products is documented and discussed in much of the mainstream consumer oriented literature. For example, a recent issue of *Consumer Reports* (September, 2000) rates 39 supermarket chains and 3 warehouse clubs based on both price and quality, including speed at the checkout facility. However, prior to the results presented here becoming operational in practice, an investigation of factors limiting the applicability of this methodology should be considered. For example, empirical studies regarding alternative demand distributions and their impact on profits could be studied. The consideration of exact solutions for multiple channel queues, an investigation into the viability of the M/M/C queuing process, and the consideration of general service time processes are also considerations for future research. The cost function could also be extended to include related inventory and distribution costs, as well. Finally, time varying demand and peak load issues could be investigated as they impact on costs and capacity requirements.

Theoretical results indicate that profits are maximized when price is set greater than marginal cost, but where marginal cost represents the additional cost of meeting a per unit increase in demand. In general, this result is intuitive, but differs from traditional pricing results in that price is embodied within marginal cost as influenced by service quality. Marginal cost is derived utilizing the envelope theorem, within which the impact of service quality is considered. The percentage increase of price from marginal cost is inversely related to the price elasticity of demand, a result that has also been found to apply within a welfare-maximization framework and public utility pricing (Boronico, 1998). Other results are also supported theoretically: optimal service quality should be set so that the marginal costs of increasing service quality are equal to the marginal costs of doing so, and the local operating variable is set so that costs are minimized subject to meeting an ex-ante declared constraint on service quality. Section II provided the specific set of prescriptions that guide the determination of optimal price, service quality, and operational variables.

The methodology was applied to an empirical study of supermarket operations. The analysis provides recommendations for supermarket management interested in determining a markup strategy for a basket of goods together with associated staffing and capacity recommendations. Service quality at the checkout counter is also

jointly considered through the recommendation of an optimal mean wait time. This analysis has important implications to both management and marketing as it assists supermarket management in positioning themselves within the market through both price and quality considerations.

In summary, while a considerable literature exists concerning the impact of service quality on demand or cost, less work has focused on the explicit impact of service quality jointly on both demand and cost. The continued study of how service quality and price jointly impact on demand for services as well as cost is important due to increasing levels of competition found in many industries. These increases are partially attributable to technological advancements, which in turn will continue to contribute to the creation of a global market for many products and services. These issues, as well as related problems, pose a significant and clearly important area for future research. ☐

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## Appendix I

The mathematical optimization problem may be formally stated as follows:

$$\text{Maximize } \Pi(X(p, w), p, w, \mu) = pX(p, w) - C(X(p, w), w, \mu) \quad (1)$$

$$\text{subject to: } H(X(p, w), w, \mu) \leq 0 \quad (2)$$

where  $C(X(p, w), w, \mu)$  represent the costs required to meet demand at the stated price and level of service quality. In order to guarantee that the declared service quality level is met, the service quality constraint (2) is ap-



pended to the formulation (1). A more generalized stochastic variation where demand is impacted by uncertainty would incorporate expectations and is left as an implication for future research.

*Operating Variable Rule:* The local operating variable  $\mu$  is set so as to minimize costs while adhering to the service quality constraint. More specifically:

$$C(X(p, w), w, \mu^*) = \underset{y}{\text{Min}} [C(X(p, w), w, \mu) \mid H(X(p, w, \mu), w, \mu) \leq 0] \tag{3}$$

where  $\mu^*$  represents the optimal value for the local operating variable  $\mu$ . This assertion is easily verified and is available by request. It remains to determine the optimal price-service quality vector  $(p^*, w^*)$  for the problem given by (1)-(2), whose necessary conditions are established in the following theorem:

*Theorem 2:* The optimal price-service quality vector for the problem given by (1)-(2) is characterized by the following:

$$(a) \quad \left\{ p - \frac{\partial C}{\partial X} - \lambda \frac{\partial H}{\partial X} \right\} \frac{\partial X}{\partial p} = -X, \text{ and} \tag{4}$$

$$(b) \quad p \frac{\partial X}{\partial w} = \left\{ \frac{\partial C}{\partial X} + \lambda \frac{\partial H}{\partial X} \right\} \frac{\partial X}{\partial w} + \frac{\partial C}{\partial w} + \lambda \frac{\partial H}{\partial w}, \tag{5}$$

These results may be obtained by associating Lagrange multiplier  $\lambda$  to the constraint (2), and applying the Karush-Kuhn-Tucker conditions. Second-order conditions may be employed to verify that the optimal solution results in a global optimum.

Rearrangement of terms and division by  $p$  results in the following alternative specification for the price characterization (4):

$$\frac{p - \frac{\partial C}{\partial X} - \lambda \frac{\partial H}{\partial X}}{p} = \frac{1}{\eta} \tag{6}$$

where  $\eta = -(p/X)/(\partial p/\partial X)$  and represents the price elasticity of demand. Moreover, for the constrained optimization problem given by (1)-(2), marginal cost may be derived from the envelope theorem for constrained optimization problems (Silberberg, 1978) and is given by:

$$MC = \frac{\partial C}{\partial X} + \lambda \frac{\partial H}{\partial X} \tag{7}$$

Although this basic pricing rule bears the familiar earmarks of typical inverse-elasticity pricing policies discussed in the economic literature, the interpretation of marginal cost representing the cost of meeting one extra unit of demand is non-standard. Direct substitution of (7) into (6) results in:

$$\frac{p - MC}{p} = \frac{1}{\eta} \tag{8}$$

## Appendix II

The particular form utilized for the exponential demand distribution is shown below:

$$X(p, w) = \alpha e^{-\beta w - \gamma p} \quad (9)$$

where  $X$  represents the mean demand for service at the stated price and mean wait time, and  $\alpha$ ,  $\beta$ , and  $\gamma$  represent demand constants. The resulting formulation (1)-(2) for the supermarket optimization problem is shown below:

$$\text{Maximize } \Pi(p, w, \mu) = pX(p, w) - C(X(p, w), w, \mu) = pX(p, w) - \{s\mu + vX(p, w)\} \quad (10)$$

$$\text{Subject to: } H(X(p, w), w, \mu) = \frac{1}{\mu - X(p, w)} - w = 0 \quad (11)$$

The service quality constraint (11) establishes the relationship between mean hourly demand  $X$ , service quality  $w$ , and the local operating variable  $\mu$ . The cost function  $C(X(p, w), w, \mu) = \{s\mu + vX(p, w)\}$  represents the capacity cost of providing service ( $s\mu$ ) in addition to the product acquisition cost ( $vX(p, w)$ ). It may be shown that the solutions to the problem (10)-(11) under the demand distribution (9) are characterized by the following:

$$p^* = v + s + \frac{1}{\gamma} \quad (12)$$

$$w^2 e^{-\beta w - \gamma p^*} = \frac{s}{\{p^* - v - s\} \beta \alpha} \quad (13)$$

Substitution of (12) into (13) allows for the determination of the optimal wait time,  $w^*$ . The solution to (13) may be obtained through an iterative search in conjunction with the utilization of spreadsheet software.

Data from Ittig (1994) allows for the determination of the following empirical values for demand parameters:

$$\alpha = 13,951,325, \beta = .40, \text{ and } \gamma = .21. \quad (14)$$

The numerical solution to the problem (10)-(11) is found by substituting these values into (12)-(13), from which the following are obtained:

$$p^* = \$56.50, w^* = .0455 \text{ hours} \quad (15)$$

Substitution of these values into (9), together with the values from (14), leads to the following mean demand, at optimum:  $X(56.50, .0455) = 96.27$  purchases per hour. The solution for the local operating variable  $\mu^*$  is found through substitution of respective values into the service quality constraint (11), from which  $\mu^* = 118.2$  units of service/hour is obtained.

Appendix III

For an unconstrained optimization model with two choice parameters  $(p, w)$  the impact of a change in exogenous parameter  $\lambda$  on the choice variables  $p$  and  $w$  may be evaluated by signing the following comparative static derivatives:

$$\frac{\partial p^*}{\partial \lambda} = \frac{\begin{vmatrix} f_{p\lambda} & f_{pw} \\ f_{w\lambda} & f_{ww} \end{vmatrix}}{H}, \quad \frac{\partial w^*}{\partial \lambda} = \frac{\begin{vmatrix} f_{pp} & f_{p\lambda} \\ f_{wp} & f_{w\lambda} \end{vmatrix}}{H} \tag{16}$$

where  $f_{ij}$  represents the second order derivative of objective function  $f$  with respect to  $j$  and  $i$ , and  $H$  represents the determinant of the Hessian matrix for the unconstrained optimization problem. Solving (11) for  $\mu$  and substituting into (10) converts the constrained optimization problem (10)-(11) into the following unconstrained problem:

$$\text{Maximize } \Pi(p, w, \mu) = pX(p, w) - C(X(p, w), w, \mu) = pX(p, w) - \left\{ s \left( \frac{1}{w} + X \right) + vX(p, w) \right\} \tag{17}$$

Optimal solutions for (17) concerning price and service quality can be determined from equations (12) and (13) for any choice of exogenous parameter values. Consider how a change in the unit capacity cost  $s$  will impact on both price and service quality at optimum. From (16), the following are obtained:

$$\frac{\partial p^*}{\partial s} = \frac{\begin{vmatrix} f_{ps} & f_{pw} \\ f_{ws} & f_{ww} \end{vmatrix}}{H} = \frac{\begin{vmatrix} -\gamma X & 0 \\ -\beta X - \frac{1}{w^2} & \frac{\beta^2 X}{\gamma} - \frac{2s}{w^3} \end{vmatrix}}{\begin{vmatrix} -\gamma X & 0 \\ 0 & \frac{\beta^2 X}{\gamma} - \frac{2s}{w^3} \end{vmatrix}} = 1 \tag{18}$$

$$\frac{\partial w^*}{\partial s} = \frac{\begin{vmatrix} f_{pp} & f_{ps} \\ f_{wp} & f_{ws} \end{vmatrix}}{H} = \frac{\begin{vmatrix} -\gamma X & -\gamma X \\ 0 & -\beta X - \frac{1}{w^2} \end{vmatrix}}{\begin{vmatrix} -\gamma X & 0 \\ 0 & \frac{\beta^2 X}{\gamma} - \frac{2s}{w^3} \end{vmatrix}} = \frac{-\left( \beta X + \frac{1}{w^2} \right)}{\frac{\beta^2 X}{\gamma} - \frac{2s}{w^3}} < 0 \quad \forall w < \frac{\beta}{2} \tag{19}$$

It is clear from (18) that  $\partial p^* / \partial s > 0$ , indicating that increases in unit capacity cost  $s$  will result in an increase in optimal price,  $p^*$ . Similarly, from (19) we observe that  $\partial w^* / \partial s < 0 \quad \forall w < 2/\beta$ . Hence, any increase in capacity cost  $s$  will result in a decrease in expected time in the checkout line, at optimal. Note that it may be shown that any solution for the optimization problem (17) must satisfy the requirement  $w < 2/\beta$ . In summary, the comparative statics results presented here indicate that an increase in capacity cost will increase service quality, but also increase price, at optimal. Similar comparative statics results may be generated for exogenous demand parameters, and are available upon request.

Notes