Unit Price Contracts: A Practical Framework For Determining Competitive Bid Prices

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Abstract

A general analytical framework within which to solve the competitive bidding problem is developed by considering a unit price contract. By viewing the problem in the standard capital budgeting framework and exploiting the linearity of the firm's objective function and constraints, the problem can be formulated as a standard linear programming (LP) application whose solution is the optimal bid. We also investigate a so-called unbalanced bidding strategy as an effective way for bidding firms to hedge the risk, or uncertainty, inherent in many unit price contracts.

Introduction

The process of capital budgeting for evaluating the financial viability of proposed projects, new pieces of equipment, new product lines, etc. is well known to many firms and has been studied extensively by researchers. A less prevalent but equally important financial problem for many businesses such as construction firms, suppliers, and government contractors is that of submitting a competitive bid to supply a particular product or perform a particular job.

Much of the academic research in the area of bidding focuses on theoretical issues and the gaming aspect of bidding and auctions. The economics literature is replete with theoretical studies that develop and analyze probabilistic bidding strategies, models, and outcomes for different types of auctions. In the area of finance, studies concerning merger bids, such as Asquith,

Bruner and Mullins (1983) and Niden (1993), are the common examples of research in the area of bidding. However, they remain largely theoretical, or concern themselves with the financial ramifications and consequences of the outcome of the merger. The result is that issues of practical importance in determining a competitive bid price (such as the amount and timing of cash flows, required return, and capital investment) are often ignored. Consequently, little guidance is available in the way of practical or applied models that bidding firms can use to help them establish an optimal bid.

In addition, much of the work in the economics and finance literature focuses on high price, common value auctions. The value of the auctioned object is the same for all bidders, the true value is unknown at the time of the auction, and the highest bid price wins the auction. The classic example is the auctioning of oil leases. The true value of the oil field is the same for each bidder, but different estimates of its value

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and different costs of extraction lead to different bids. Bidding in a low price auction of the type considered here is different from a common value auction. In this case, the value of the project is not the same for each bidder since the value is established by the individual firm's bid. The firm's cost structure and required return determine its bid. Hence, while the tools remain the same, the usual valuation processes must be applied from a different perspective.

In contrast to the economics and finance literatures, segments of the engineering literature have addressed some of the more practical aspects of competitive bidding. Beginning with Friedman (1956), more and more studies, including Gates (1967), Shaffer and Micheau (1971), Fuerst (1977), Teicholz and Ashley (1978) and Farid and Boyer (1985), have sought to establish more applied approaches and models to the bid pricing problem.

In this paper we begin to bridge the gap between these two lines of research by showing how the problem of setting a competitive bid price is closely related to the standard capital budgeting decision and can be viewed in the same analytical framework. The important difference in the competitive bidding problem is that the ultimate accept/reject decision of the project does not lie with the firm's management, but with the agent who requested the bids. Therefore, it is crucially important for the firm to bid a price that will be both profitable for the firm and competitive with the other bids. In other words, the firm should bid the lowest possible price that is compatible with earning their required rate of return on the project. Restated in the standard capital budgeting framework, the firm should bid a price such that the project's net present value (NPV), evaluated at the required rate of return, is equal to zero.

The purpose of this paper is to illustrate a general analytical framework within which to solve the competitive bidding problem. We illustrate the concept by considering a specific type of contract, the unit price contract. We also explore the usefulness and limits of a so-called

unbalanced bidding strategy as a way of hedging the risk in a unit price contract in order to increase the likelihood of winning the contract while earning the desired rate of return. One of the key elements in the quantitative solution to the problem is recognizing and exploiting the linearity of the firm's objective function and constraints. In so doing, we are able to formulate the problem as a standard linear programming problem whose solution is the firm's optimal bid. Implementation of the solution is illustrated with an example.

The Unit Price Contract

There are many different types of contracts that firms bid on, including fixed cost, cost-plus-fee, and unit price. Each type has its own characteristics that impact a firm's bid. Because of the way they are structured and administered, unit price contracts can be especially difficult for firms to bid successfully; that is, win the bid and actually earn their required rate of return on the project. A unit price contract consists of a specified set of bid items. Based on the owner's estimates of the requirements for the job, the contract proposal calls for a specific quantity of each of the bid items to be delivered over a given amount of time; for example, 2000 cubic yards of concrete, or 300 feet of pipe. The bidding firm first determines per-unit bid prices for each of the individual bid items in the contract. Given the individual bid prices, the subtotal for each bid item is the unit bid price times the proposal quantity, and the total contract bid is the sum of the individual bid item subtotals. The per-unit bid prices are the contractually obligated prices that will be paid for each of the bid items. As such, it is important that they accurately reflect the relevant costs (both fixed and variable), capital investment, taxes, and profit (required rate of return) for the bidding firm as well as the timing of each of the relevant cash flows.

The solution to the unit price competitive bidding problem turns out to be a natural application of linear programming (LP) since: 1) the firm's objective is to minimize their total bid

(a linear function), and 2) the fundamental bid pricing relationship that reflects the true costs and required rate of return on the project provides a linear constraint that must be satisfied for the project to provide a zero NPV. When solved, the LP solution provides the firm's optimal bid.

The Competitive Bidding Framework

Consider a unit price contract consisting of r bid items. The bidding firm must determine per-unit bid prices for each of the individual bid items. Their total bid is then the sum of the individual bid item totals; that is, the per-unit bid prices for each of the bid items times the estimated quantities of each item:

$$Z = \sum_{j=1}^r u_j BP_j$$

where Z is the firm's total bid, u_j is the number of units of bid item j called for in the contract, and BP_j is the firm's bid price for bid item j .

As mentioned previously, setting a bid price can be viewed in the standard capital budgeting framework. The important difference is that in a bidding scenario the ultimate accept/reject decision for the project lies not with the firm's management, but with the agent who requested the bids. Therefore, the firm must be careful to bid a price that would, in a capital budgeting sense, result in an "accept" decision. That is, the firm must bid a price that will be profitable for the firm and yet competitive with the other bids. A higher price would increase the firm's rate of return, but may take it out of the competition. Bidding a lower price will certainly increase the firm's chances of winning the contract, but it may also render the project unprofitable -- the so-called winner's curse. Hence, the firm's objective is to bid a price such that the project's net present value, evaluated at the bid price and required rate of return, is equal to zero. This will insure that the firm bids the lowest possible price that is compatible with

earning the specified periodic rate of return on the project.

The net present value (NPV) of the project is the sum of the discounted total cash flows (TCF) over the T periods. At a periodic required rate of return, k :

$$NPV = \sum_{t=0}^T \frac{TCF_t}{(1+k)^t}$$

The standard capital budgeting approach is to express the total cash flow in period t as:

$$TCF_t = OCF_t - ANWC_t - CS_t$$

where OCF_t is operating cash flow in period t , $ANWC_t$ is additions to net working capital in period t , and CS_t is net capital spending in period t . Therefore, we measure the firm's periodic total cash flow from the project as the net cash flow generated from the project's revenues, adjusted for any periodic investment in working capital items, less any periodic capital investment in the project.

The bid price that is submitted for the project determines the amount of revenue generated and, therefore, the periodic level of operating cash flow. For a project that requires delivery and payment for the r bid items, we can define operating cash flow in period t as:

$$OCF_t = \left(\sum_{j=1}^r n_{jt} BP_j - \sum_{j=1}^r m_{jt} VC_{jt} - FC_t \right) (1 - t_c) + (1 - t_c) + t_c D_t$$

where n_{jt} is the number of units of item j delivered in period t ($j=1, \dots, r$), BP_j is the per-unit bid price of item j , VC_{jt} is the per unit variable cost of item j incurred in period t , m_{jt} is the number of units of item j paid for in period t , FC_t is fixed costs for period t , D_t is period t depreciation, and t_c is the applicable corporate tax

rate.

Making the appropriate substitutions, setting NPV equal to zero, and solving for the r bid prices yields the following linear equation that relates the individual bid prices to the appropriate time-adjusted costs of the project and quantities of bid items:

$$N_1 BP_1 + N_2 BP_2 + \dots + N_r BP_r = \sum_{t=0}^T \left[\frac{TVC_t}{(1+k)^t} + \frac{TFC_t}{(1+k)^t} + \frac{(1-t_c)^{-1} (ANWC_t + CS_t)}{(1+k)^t} - \frac{(1-t_c)^{-1} (t_c D_t)}{(1+k)^t} \right]$$

where TVC_t is the total variable cost of the bid items in period t , TFC_t is the total fixed costs, and $t_c D_t$ is the depreciation tax shield in period t . BP_j is the bid price for bid item j and $N_j = \sum_t n_{jt}/(1+k)^t$ is the sum of the discounted periodic unit deliveries for bid item j .

The structure of the unit price contract bidding problem makes it a natural application of linear programming. Given that the firm's objective is to submit the lowest possible bid, the objective function of the LP problem can be expressed as:

$$\text{minimize } Z = u_1 BP_1 + u_2 BP_2 + \dots + u_r BP_r$$

where Z is the firm's total bid as defined earlier.⁽¹⁾ The primary linear constraint for the LP problem is provided in the bid-pricing relationship defined by equation (5), which expresses the need to determine per-unit bid prices such that the project's net present value (NPV), evaluated at the bid prices and required rate of return, is equal to zero. Combinations of bid prices that satisfy this equation should yield a zero NPV on the proposed project. Therefore, to earn its required rate of return on the contract the firm's objective function is constrained by equation (5), referred to as the zero-NPV constraint.

If the only other constraint was that bid prices be nonnegative, the optimal solution would tend to be extremely unbalanced; that is, large unit prices would be assigned to those items expected to be completed (and billed) relatively early, and zero bid prices would be given to the remaining items. Hence, the firm's unit bid prices must also be constrained by other factors referred to as "formality" and "cost" constraints (Stark, 1968, 1974). Formality constraints establish relative bid prices between the bid items. For example, if generally accepted cost factors dictate that the bid price for item i should be no more than the bid price for item j , then the two bid prices should be constrained by the inequality $BP_i - BP_j \leq 0$. Recognizing that there should be some reasonable relationship between the cost of an item and its bid price, cost constraints simply provide upper and lower bounds on each item's bid price, e.g. $L_j \leq BP_j \leq U_j$. Together, the formality and cost constraints are necessary to avoid any real or perceived pricing errors or anomalies, and possibly to conceal any firm-specific pricing policies.

Implementation

An example replicating an actual unit price contract provides the best illustration of how the model is implemented. The example is taken from Stark (1974) and is modified only slightly to compensate for data that are not available. While the example is dated, it retains the character of a true unit price contract. As shown in Table 1, the contract calls for the delivery of fifteen bid items in the completion of a road construction project. The expected quantities of each item are listed as well as a proposed completion schedule.

We assume revenues are collected based on the completion schedule. In particular, following Farid and Boyer (1985), we assume the firm submits progress billings at the end of a period for the items completed in that period, and that payment is received one period later. We also assume that variable costs (i.e., those that vary directly with the number of units delivered) are incurred based on the completion schedule.

Table 1
Proposal Quantities And Completion Schedule

Bid Item	Proposal Quantity	Description	Time Period*					
			1	2	3	4	5	6
1	1	Clearing and grubbing	0.80	0.20				
2	280 C.Y.	Channel excavation	0.05	0.10	0.20	0.15	0.45	0.05
3	800 C.Y.	Excavation for pipe trenches	0.10	0.30	0.40	0.20		
4	600 C.Y.	Select borrow		0.20	0.30	0.30	0.10	0.10
5	18 tons	Coarse aggregate					0.70	0.30
6	210 gal.	Rc-70 asphalt					0.65	0.35
7	295 gal.	Rc-250 asphalt					0.75	0.25
8	312 L.F.	60 in. R.C. pipe		0.20	0.30	0.50		
9	516 L.F.	72 in. R.C. pipe		0.25	0.25	0.50		
10	1050 L.F.	Wire rope guard fence		0.20	0.40	0.40		
11	16 each	End post attachments		0.10	0.20	0.70		
12	0.6 A.	Seeding					0.60	0.40
13	2700 S.Y.	Mulching					0.75	0.25
14	1200 S.Y.	Riprap			0.50	0.50		
15	1	Removal of existing structures	0.90	0.10				

*In this example, the time period is one month. The number shown in each column represents the fraction of the total quantity expected to be delivered in that month. For each item, the sum of the fractions equals one.

Specifically, we assume that the cash outflows related to these items will occur one period prior to the delivery date. The result is a uniform two-period lag between the incurrence of a variable cost and the related revenue receipt. While this assumption is made for simplicity, a firm should use its best estimate of the magnitude and timing of all cash flows in calculating the zero-NPV constraint. For example, to obtain a quantity discount, a firm may plan to purchase all of the pipe to be used on a project at one time, even though the pipe will be used (and billed) over an extended time period. In this case, cost and revenue would not “match-up” as in our example; instead, there would be a single large cost incurred early during the project, and multiple smaller inflows received as billings occur.

A project may also involve certain costs which vary at the project level, rather than with the number of individual items delivered. One

example might be the cost incurred in setting up and preparing for the job. In our example, this might include moving machinery and equipment to the project site. We refer to these costs as mobilization costs, and have included \$1000 of these expenses, incurred at the start of the project, in our example. Because we specifically include all cash flows, including those related to working capital items, there is no separate investment in net working capital. Finally, because of the short-term nature of the project, in this example we disregard capital spending and depreciation. However, the overhead costs of \$2000 per period which are allocated to the project could include an allowance for items of this type.

Table 2 indicates the unit cost for each item and the upper and lower bounds (i.e., cost constraints) on each of the item bid prices. In addition, assume the following formality con-

Table 2
Bid Item Data

Bid Item	Proposed Quantities	Unit Cost	Cost Constraints		Bid Price
			Lower	Upper	
1	1	\$1000.00	\$1000.00	\$4000.00	\$4000.00
2	280	1.25	1.25	3.00	1.25
3	800	1.40	1.40	4.00	4.00
4	600	1.75	1.75	3.50	1.75
5	18	15.00	15.00	50.00	15.00
6	210	0.30	0.30	0.80	0.30
7	295	0.30	0.30	0.80	0.50
8	312	26.50	26.50	35.00	30.24
9	516	44.00	44.00	53.00	53.00
10	1050	2.10	2.10	2.60	2.60
11	16	82.00	82.00	100.00	82.00
12	0.6	500.00	500.00	1000.00	500.00
13	2700	0.10	0.10	0.20	0.10
14	1200	10.00	10.00	12.00	10.00
15	1	2000.00	2000.00	5000.00	5000.00
				Total Bid	\$67,475.41

straints exist:

$BP_2 - BP_3 \leq -0.15$

$BP_6 - BP_7 \leq -0.20$

$BP_8 - BP_9 \leq -18.00$

These constraints establish relative prices between different types and sizes of excavation, asphalt and pipe. Table 2 also shows the proposal quantities as well as the resulting bid prices.

We assume that the quantities and the delivery schedule specified in the proposal correspond to the bidding firm's best estimates, and that little variability in these amounts is expected. In this situation, the contract is one of low risk so the required rate of return on the project should be relatively low. We assume a monthly discount rate of 1%. The tax rate is assumed to be 34%.

In sum, the LP problem may be stated

as:

$$\text{Minimize } Z = BP_1 + 280BP_2 + 800BP_3 + 600BP_4 + 18BP_5 + 210BP_6 + 295BP_7 + 312BP_8 + 516BP_9 + 1050BP_{10} + 16BP_{11} + 0.6BP_{12} + 2700BP_{13} + 1200BP_{14} + BP_{15}$$

$$\text{subject to: Zero-NPV constraint(2): } 0.98BP_1 + 266.43BP_2 + 771.11BP_3 + 573.20BP_4 + 16.91BP_5 + 197.14BP_6 + 277.22BP_7 + 298.94BP_8 + 494.65BP_9 + 1007.05BP_{10} + 15.28BP_{11} + 0.56BP_{12} + 2537.23BP_{13} + 1147.47BP_{14} + 0.98BP_{15} = \$64,820$$

Cost constraints: (see Table 2)

Formality constraints:

$BP_2 - BP_3 \leq -0.15$

$BP_6 - BP_7 \leq -0.20$

$BP_8 - BP_9 \leq -18.00$

The optimal bid prices are indicated in Table 2. This set of bid prices minimizes the firm's objective function with a total bid of \$67,475. The zero-NPV constraint is satisfied indicating that this set of bid prices should allow the firm to earn its required rate of return on the project. The importance of the cost and formality constraints should be clear. With the exception of items seven and eight, all bid prices lie at either the upper or lower end of the cost constraint. Those items priced at the high end tend to be those for which payment will be received relatively early during the contract, and vice versa. The main point is that these constraints should be realistic and defensible, since it is likely that the bid price for a given item will be at an endpoint of the constraint.(3)

Hedging Cash Flow Uncertainty

An important problem inherent in many unit price contracts is that the final compensation for the job is unlikely to be equal to the firm's total bid. The quantities specified in the proposal are simply the owner's estimates of the necessary quantities. If any change in the job becomes necessary or if the owner's estimates are wrong, the actual quantities delivered may be different. Nevertheless, payment for each of the bid items is still made at the agreed upon per-unit bid price for the actual quantity delivered, not for the proposed bid quantities. Hence, the actual revenue generated from the contract is likely to be different from the firm's total bid. For this reason, unit price contracts can be risky ventures for bidding firms.

To reduce the risk associated with bidding on these contracts, firms must anticipate any changes in the structure of the contract (e.g., unit quantities or delivery dates) and factor these changes into the calculation of the bid. Naturally, each firm will set their unit bid prices to yield the lowest total bid given their cost estimates. However, while each firm will determine their bid using their private estimate of the total costs of the contract, the total bid will be determined based upon the quantities specified in the proposal, which are the same for all bidders.

The result may be bid prices for individual items that vary widely across firms. Even for a single firm, the bid may be "unbalanced" in the sense that the bid price for some or all items may differ significantly (either higher or lower) from what is considered "normal".

Anticipated changes in some part of the contract may be reflected in a firm's cash flows either directly or indirectly. In the first case, any differences between the bid quantities and the actual quantities naturally imply a direct change in the firm's revenues and costs. Recall that even though the actual quantities delivered may be different from the bid quantities, payment for each of the bid items is still made at the agreed upon per-unit bid price. Assuming the bid price exceeds the variable cost, a reduction in actual quantities relative to bid quantities will reduce the project's profitability. Of course, if the actual quantities delivered turn out to be greater than the bid quantities, the firm will see a direct gain. But in either case, if the firm bases their bid on the contents in the proposal, they will have under-bid or over-bid the contract, resulting in a project that earns less than the desired rate of return or in a bid that is less competitive than it could have been.

Indirect cash flow effects are also possible. For example, suppose the firm anticipates probable changes in the contract that involve significant redesign or re-engineering work. In this case, even if the actual bid quantities are not expected to change, additional indirect costs must be incurred in the form of engineering time and effort, and/or additional overhead costs. The magnitude of these indirect cash flows and their expected variability should be considered in formulating the bid.

The value of the bidding model and LP solution developed here is that it allows a firm to adjust its bid in order to hedge the cash flow uncertainty. To begin with, the zero-NPV constraint should be based on the *expected* magnitude and timing of the project's cash flows, which may not agree with the amounts specified in the proposal. In fact, bidders may be more

knowledgeable than the owner regarding the resources necessary to complete a project, and it would be imprudent to disregard their own expertise and rely solely on the owner's estimates. A second way in which uncertainty should be reflected in the bid is through adjustment of the discount rate; as the variability in cash flows increases, so should the required rate of return. Developing a bid is as much an art as a science, and requires "a combination of intuition, experience, and a measure of clairvoyance" (Nadel 1991, p. 63). Still, given the firm's anticipated changes, the LP model provides an analytical framework to incorporate these expectations in the determination of an optimal bid.

To illustrate how a firm might adjust for uncertainty, we modify the example used to implement the model. In a hedging scenario, assume the bidding firm anticipates that the quantities of each item which will actually be delivered under the contract correspond to the amounts listed in the "Actual" column of Table 3.(4) We assume there is no change in the expected delivery schedule, nor in the mobilization costs or overhead costs assigned to the project. However, because of the increased risk associated with the project, the firm requires a monthly rate of return of 2%. Although these changes have no effect on the objective function (which is still based on the proposal quantities), they do affect the zero-NPV constraint, as indicated below:

$$0.96BP_1 + 196.34BP_2 + 722.59BP_3 + 1829.85BP_4 + 29.66BP_5 + 238.11BP_6 + 529.29BP_7 + 284.18BP_8 + 458.05BP_9 + 802.96BP_{10} + 14.61BP_{11} + 0.40BP_{12} + 1931.86BP_{13} + 708.14BP_{14} + 0.95BP_{15} = \$60,818$$

The resulting individual bid prices for this scenario are shown in Table 3 along with the previous bids for comparison. Note that the bid prices have changed for eight of the fifteen items, although most still lie on the boundary of the cost constraint. The total bid has dropped from \$67,475 to \$65,825, a decline of \$1,650 (2.4%). In this example, using the firm's own estimates results in a more competitive bid

which, if the changes occur as anticipated, will also earn the required rate of return. Of course, adjusting for anticipated changes could result in a higher bid, depending on the nature of the changes. However, regardless of whether the bid increases or decreases, the resulting bid is optimal because it maximizes the chances of winning the contract while maintaining the desired rate of return.

Summary and Conclusions

The competitive bid problem is an often overlooked but very important financial problem applicable to many firms. However, its conceptual closeness to the standard capital budgeting problem makes it a tractable problem to solve. The purpose of this paper has been to provide a general analytical framework within which to solve the competitive bidding problem for a unit price contract. The solution is simplified since the problem is a natural application of linear programming. The firm's objective function is to minimize the firm's total bid, and the primary constraint is the fundamental condition that a firm's optimal bid must provide a zero net present value. The zero-NPV condition provides the necessary constraint that reflects the true costs (both fixed and variable), capital investment, taxes, and required rate of return on the project as well as the timing of each of the relevant cash flows. The LP solution provides the individual bid prices that minimize the firm's total bid.

The model also provides an effective way for bidding firms to hedge the risk, or uncertainty, inherent in many unit price contracts. Given that bidding firms are able to anticipate probabilistic changes in the contract, the resulting unbalanced bid reflects the expected changes in the contract to produce a more accurate and competitive bid.

Suggestions For Further Research

We identify two important avenues for further research on this topic. The first would be to apply the same basic solution framework to

Table 3
Bid Item Data - Hedging Scenario

Bid Item	Bid Item Quantities		Unit Cost	Cost Constraints		Bid Price	
	Proposed	Actual		Lower	Upper	Original Scenario	Hedging Scenario
1	1	1	\$1000.00	\$1000.00	\$4000.00	\$4000.00	\$4000.00
2	280	216.7	1.25	1.25	3.00	1.25	1.25
3	800	777.4	1.40	1.40	4.00	4.00	4.00
4	600	2003.8	1.75	1.75	3.50	1.75	3.50
5	18	33.6	15.00	15.00	50.00	15.00	50.00
6	210	270	0.30	0.30	0.80	0.30	0.60
7	295	599	0.30	0.30	0.80	0.50	0.80
8	312	309.4	26.50	26.50	35.00	30.24	29.29
9	516	498.2	44.00	44.00	53.00	53.00	47.29
10	1050	872.5	2.10	2.10	2.60	2.60	2.10
11	16	16	82.00	82.00	100.00	82.00	100.00
12	0.6	0.45	500.00	500.00	1000.00	500.00	500.00
13	2700	2186.3	0.10	0.10	0.20	0.10	0.10
14	1200	774.1	10.00	10.00	12.00	10.00	10.00
15	1	1	2000.00	2000.00	5000.00	5000.00	5000.00
					Total Bid	\$67,475.41	\$65,825.27

other types of contracts such as fixed cost or cost-plus-fee. As stated earlier, treatment of the practical aspects of this important problem is absent from the business literature. Extending these results to other contract types would help bridge that gap, and expand the set of practical financial models for addressing competitive bidding problems.

The other promising area for further research involves the use of simulation analysis to more accurately assess and control for risk. Hedging the cash flow uncertainty involves estimating probabilistic changes in the contract. Hence, a set of distributional assumptions regarding delivery of each bid item could be used to generate simulated project NPVs and measures of the level of risk in the project. Appropriate adjustments in the discount rate to reflect the level of risk should then allow the firm to fine tune the likelihood of falling prey to the "winner's curse." While expensive initially, if

the firm frequently bids on similar jobs, such an analysis could be cost effective in the long run.

Endnotes

1. We assume that the winning bid will be determined based solely on the lowest bid as indicated in equation (6). In reality, other factors, such as the reputation of the bidding firm, may also be considered. Furthermore, some owners may select the winner based on the lowest bid in a present-value sense; that is, they may consider not only the bid prices but also the timing of the payments to be made under the contract. While such a scenario could be modeled, it would result in a more complicated objective function and is beyond the scope of this paper.
2. The left-hand side of this equation represents the present value of the revenue in-

- flows, while the right-hand side is the present value of all costs related to the contract. These amounts must be equal for the project to have an NPV of zero.
3. If the cost and formality constraints are replaced by the less restrictive requirement that bid prices be simply nonnegative, the optimal solution is a zero bid price for all items except item 15, which has a unit bid price of \$66,188. Taken at face value, bidding the contract in this manner would increase the likelihood of winning the contract, since the total bid of \$66,188 is \$1,287 less than the results from scenario one. However, such a bid would certainly arouse suspicion from the owner, and would greatly increase the risk to the bidder if the quantity to be delivered may vary.
 4. These amounts are the actual final pay quantities on this project. While a bidding firm would not be able to perfectly predict these amounts, we use them to demonstrate the large degree by which the actual quantities may differ from the proposed quantities.
 7. Niden, Cathy M., "An Empirical Examination of White Knight Corporate Takeovers: Synergy and Overbidding," *Financial Management*, Vol. 22, No. 4, pp. 28-45, 1993.
 8. Shaffer, L. R. and J. W. Mischeau, "Bidding with Competitive Strategy Models," *Journal of the Construction Division*, Vol. 97, pp. 113-126, 1971.
 9. Stark, R. M., "Unbalanced Bidding Models - Theory," *Journal of the Construction Division*, Vol. 94, pp. 197-209, 1968.
 10. Stark, Robert M., "Unbalanced Highway Contract Tendering," *Operational Research Quarterly*, Vol. 25, No. 3, pp. 373-388, 1974.
 11. Teicholz, P. and D. Ashley, "Optimal Bid Prices for Unit Price Contracts," *Journal of the Construction Division*, Vol. 104, pp. 57-67, 1978.

References

1. Asquith, P., R.F. Bruner and D.W. Mullins, Jr., "The Gains to Bidding Firms from Merger," *Journal of Financial Economics*, April, pp. 121-139, 1983.
2. Farid, F. and L.T. Boyer, "Fair and Reasonable Markup (FaRM) Pricing Model," *Journal of Construction Engineering and Management*, Vol. 111, pp. 374-390, 1985.
3. Friedman, L., "A Competitive Bidding Strategy," *Operations Research*, Vol. 4, pp. 104-112, 1956.
4. Fuerst, M. J., "Theory for Competitive Bidding," *Journal of the Construction Division*, Vol. 103, pp. 139-152, 1977.
5. Gates, M., "Bidding Strategies and Probabilities," *Journal of the Construction Division*, Vol. 93, pp. 75-107, 1967.
6. Nadel, Norman A., "Unit Pricing and Unbalanced Bids," *Civil Engineering*, Vol. 61, pp. 62-63, 1991.