A Chaos Approach to Bankruptcy Prediction

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Abstract

Chaotic systems, although deterministic and predictable over short horizons, appear to be random. This study applied chaos theory to bankruptcy prediction using a pair-matched sample of bankrupt firms. Given that healthy systems exhibit more chaos than unhealthy ones, it was hypothesized that the returns of firms nearing bankruptcy would exhibit significantly less chaos, measured with Lyapunov exponents, than at an earlier period. Successful univariate and multivariate bankruptcy prediction models were then constructed using the obtained Lyapunov exponents.

Introduction

Creditors and investors are greatly concerned with the possibility of a firm's bankruptcy. To aid these decision makers, various bankruptcy prediction models have been developed by accounting researchers. The authors of this study call upon chaos theory [Yorke, 1975] and use non-linear dynamic methodology to develop a new model for individual firm bankruptcy prediction.

Chaotic systems appear to be random when in actuality they are deterministic and predictable over short periods of time. They are feedback systems in which more than one equilibrium can exist. They are fractal, meaning that self-repetition exists on smaller and smaller scales; and they are extremely sensitive to initial conditions [Lorenz, 1963]. It is this sensitivity, sometimes called the "butterfly effect," which limits the predictive ability of chaos models to relatively short periods of time [Peters, 1991].

Etheridge and Sriram [1993] argue persuasively that economics and finance researchers have already successfully used chaos theory to study systems such as the stock market, and that it is time for accounting researchers to begin using the methodology. Therefore, this study applies chaos theory to a question that has engaged accounting researchers for decades: the prediction of firm bankruptcy. The study consists of five parts. This Introduction is followed by sections covering Literature Review, Methodology, Results, Summary and Conclusions.

Literature Review

Foster [1986] reviews some 44 journal articles dealing with the problem of bankruptcy prediction. The application of financial statement analysis for objective bankruptcy prediction started with univariate models which relied on the predictive value of a single financial variable [Beaver, 1966; Zmijewski, 1983], and led to multivariate models which used a set of such variables [Altman, 1968; Altman et al., 1977; Ohlson, 1980; Marais et al., 1984].

When evaluating predictive models, it is not enough just to look at the total percentage of correct predictions. Two very different sorts of misclassification are possible: Type 1 errors and Type 2 errors. One is much more costly to the decision maker than the other [White, Sondhi, and Fried, 1994]. In the case of a Type 1 error, the model misclassifies a firm which actually will go bankrupt as one which will not go bankrupt. Type 2 errors occur when the model misclassifies a firm which will not go bankrupt as one which will. A Type 1 error would lead a lender, for example, to make an inappropriate loan leading to the potential loss of principal. A Type 2 error would, on the other hand, preclude a lender from making an appropriate loan leading to a less costly loss of potential interest earnings. Type 1 errors have been estimated to be 35 times more costly to decision makers than Type 2 errors [Altman et al., 1977].

In all of the models, the longer the time period preceding bankruptcy, the less accurate the results. Fortunately, decision makers' greatest need is to predict which firms will face insolvency in the relatively near future [White, et al., 1994].

To date three different types of statistical techniques have been used to generate bankruptcy prediction models: discriminant analysis [Altman et al., 1977], logit or probit analysis [Ohlson, 1980], and recursive partitioning [Marais et al., 1984]. Recently, a fourth methodology, neural networks, has been tried [Boritz et al., 1993]. No one has yet attempted to use non-linear dynamics to generate a bankruptcy model.

It is interesting to note that most of these prediction models are based on cross-sectional analysis which compares different firms on the basis of financial variables reported at a specific point in time. A number of different financial variables have been used. With respect to the cross-sectional studies, Zmijewski [1983] isolated the 75 individual financial ratios most often used in distress prediction studies. No theory has been successfully proposed that suggests why some variables would be preferable to others [Foster, 1986, p. 560]. Only occasionally has a study combined a market-based variable with ratios derived from financial statements [White et al., 1994, p. 1040]. There is, furthermore, no theoretical reason why a longitudinal or time series approach could not be used.

Non-linear dynamic models have proven quite successful in the prediction of certain endogenously determined catastrophic system failures. With such models it is possible to exploit the characteristic of chaotic behavior that it can be deterministic and predictable over short periods of time [Etheridge and Sriram, 1993]. Goldberger [1990] cites nine articles in the medical literature which argue that non-linear dynamic models can predict cardiac events, such as myocardial infarction, which other medical methodologies cannot. Goldberger concludes that use of nonlinear dynamic models may "extend the diagnostic and prognostic utility of the electrocardiogram." Firm bankruptcy can also be considered a catastrophic event. Clearly, it would be so regarded by a firm's principal investors and creditors. In modeling bankruptcy, as in modeling heart failure, the cost of Type 1 errors far exceeds the cost of Type 2 errors.

Stock returns, which may bear information predictive of bankruptcy, exhibit chaotic behavior [Peters, 1991]. Returns data on thousands of firms for long periods of time are readily available in data tapes from the Center for Research into Security Prices (CRSP). A model using non-linear dynamics to predict bankruptcy can be expected to be predictive for only a short advance period. This time limitation is not a major drawback for bankruptcy prediction, however, since the greatest need of financial decision makers is to predict which firms will face insolvency in the near future, in one to two years. Any deterministic model which predicts imminent bankruptcies would be of enormous value to the financial community. This paper proposes to initiate the development of such a model by comparing a measure of chaos of a sample of bankrupt firms, the Lyapunov exponent, just prior to insolvency to the same statistic calculated at a far earlier point.

Chaotic systems are deterministic, but are only predictable over short periods of time, due to extreme sensitivity to initial conditions. The Lyapunov exponent measures the degree of sensitivity to initial conditions by measuring the average exponential rate of divergence or convergence of nearby orbits in phase space. By definition, any system containing a positive Lyapunov exponent is chaotic. The larger the exponent the sooner the system becomes unpredictable.

The interested reader will find a detailed discussion of the formula used to compute the Lyapunov exponent in Wolf et al. [1985], an except from which is quoted below:

"The i th one-dimensional Lyapunov exponent is defined in terms of the length of the ellipsoidal principal axis $p_i(t)$:

$$\lambda_i = \lim_{t\to\infty} \frac{1}{t} \log_2 \frac{p_i(t)}{p_i(0)}$$

where the λ_i are ordered from largest to smallest."

Several chaos statistics in addition to the Lyapunov exponent exist. Two alternative measures of the fractal dimension of the strange attractor (Henon attractor) are the capacity dimension and the correlation dimension. The capacity dimension, also known as the Hausdorff dimension, is calculated by using a heuristic "box-counting" procedure [Grassberger et al., 1984]. The correlation dimension is calculated by plotting the log of the radius of a hyperdimensional sphere against the log of the number of data points found within that sphere. Another commonly used chaos statistic is the Fast Fourier Transformation (FFT). This statistic measures the frequency at which the power spectrum has its maximum value [Sprott and Rowlands, 1992].

Methodology

The primary objective of this study is to initiate the development of a bankruptcy prediction model using non-linear dynamics. The research question is, "Can a non-linear dynamic model of bankruptcy prediction be constructed which results in a low rate of Type 1 mispecifications?"

This study takes a time series approach to bankruptcy prediction. Goldberger's [1990] argument that healthy systems exhibit more chaos than unhealthy systems, leads directly to the hypothesis of this paper:

H_0: The returns of firms approaching bankruptcy will exhibit significantly less chaos than the returns of the same firms from an earlier time period.

For the years 1983 through 1992, all firms which filed for Chapter 11 bankruptcy protection, as well as the date they filed, were identified by reference to the *Wall Street Journal Index*. Any bankrupt firm which was not also included on one of the CRSP daily returns tapes (listed or NASDAQ) was omitted from the sample.

A control sample was constructed by randomly pair-matching each firm in the bankruptcy sample with a non-bankrupt firm with the same 4 digit SIC code. If no such firm was available, a match was made to a firm with the same 3 digit code. To qualify as a pair-match, CRSP data had to be available for the firm.

For each firm in both samples, CRSP return data were collected for an early two-year window of time, the period 7-5 years prior to filing for bankruptcy. A two-year window is needed to generate sufficient observations to apply fractal procedures, about 500 data-points. No previous study detailed bankruptcy indicators as early as five years before the event. The data were also collected for a late two-year window of time, the period 3-1 years prior to filing for bankruptcy. It was felt that, to be useful, bankruptcy information would need to be available to decision-makers at least one year in advance of the event. As is illustrated in Exhibit 1, four sub-samples of data existed: early bankrupt, late bankrupt, early control, and late control.

Exhibit 1
The Four Data Subsets

A Bankrupt Firms 7-5 years prior	**B** Bankrupt Firms 3-1 years prior
Pair-Match Firms 7-5 years prior **C**	Pair-Match Firms 3-1 years prior **D**

For each firm, for both the early and late window of time, the CRSP time series return data were processed using a program called the *Chaos Data Analyzer*[1992]. This program computed the Lyapunov exponent, for each firm, in each window. A random selection of one third of the bankrupt firms and their pair-matches was then set aside for later use to test the model. The remaining two thirds constituted the sample used to generate the model. Given the emphasis on time series or longitudinal analysis, the Lyapunov exponent estimated for the early window of time was subtracted from that for the late window of time for each of those bankrupt and matched firms used to generate the model. The study hypothesis leads to the expectation that the early and late Lyapunov exponents would differ for bankrupt firms but not for the others. A t-test of the differences was calculated for the bankrupt and control samples.

T-tests reflecting cross sectional considerations were also performed. The Lyapunov exponent estimated for the early window of control firms was subtracted from the same measure for the early window of bankrupt firms. The two measures for the late windows were also tested. Nonparametric tests of both the longitudinal and cross-sectional differences were run in addition to parametric t-tests.

Results

CRSP daily returns for both the early and late time windows were obtained for 69 bankrupt firms and their pair matches. The names of these firms are listed in Table 1. Using this return data the *Chaos Data Analyzer* generated a Lyapunov exponent for an early and a late window for each firm. A random selection of 23 of the bankrupt firms, and their pair matches was set aside. The remaining 46 firms, and their pair matches, constituted the samples used to generate the model.

For both the control sample and the test sample, Table 2 shows the differences obtained when the Lyapunov exponent estimated for the early window of time was subtracted from that measured for the late window of time. For each of the two windows of time, the same table also shows the differences obtained when the exponent estimated for the test group is subtracted from the same statistic estimated for the control group. These differences constituted the data to be used in the t-tests.

Table 3 shows the results of the t-tests of the differences (i.e. whether the calculated differences diverged significantly from zero). For the sample of bankrupt firms, the difference between Lyapunov exponents for the late measure and the early measure (DIF1) was both negative and significant at the .05 level. For the control sample the difference between the late measure of the exponent and the early measure (DIF2) was not statistically significant. These results are consistent with the hypothesis of this paper.

Table 3 also shows the results of the cross sectional tests. The difference between the early measure of the exponent for the bankrupt sample and the early measure of the exponent for the control sample (DIF3) was not significant. Likewise, the difference between the exponent calculated from the late measure for the bankrupt sample and for the control sample (DIF4) was not significant.

The nonparametric Wilcoxon signed ranks test was then applied to the four sets of difference measures. The test uses the direction and relative size of differences in matched pair data [Siegel and Castellan, 1988, p. 87]. By relaxing the assumption of normality required by the t-test, the Wilcoxon test is robust to the presence of possible outliers [Snedecor and Cochran, 1989, p. 136]. The underlying data used for the relatively small sample of difference measures had been subjected to a lengthy process to accumulate market returns and calculate Lyapunov exponents, a process that could yield an outlier or extreme data point. The nonparametric test is appropriate for data sets that may include outliers and sets drawn when the underlying population is highly skewed.

The Wilcoxon signed ranks test echoed the findings of the parametric t-test which showed that the difference between the Lyapunov exponents for the late and early measures for bankrupt firms (DIF1) was significantly different than zero to the

Table 1
Bankrupt Firms and Their Pair-Matches

Bankrupt Firm	Pair-Match Firm	Bankrupt Firm	Pair-Match Firm
Marion	Petrol	Midway	USAir
Roberts	Xerox	Leisure Technology	Royal Palm Beach
Flame	Portec	Harvard Industries	Rauch
Manville	Reserve	Enstar	La Petite
Branch	Lynch	University Graphics	Harland
Transcontinental	Atwood	Lionel	Toys R Us
Berry	Digicon	Iroquois Brands	E & B Marine
Wheeling	Proler	America West	Atlantic Southeast
LTV	Quanex	Columbia Gas	K N Energy
Texaco	Hess	Transcisco	PS Group
Cramer	Hon	Newmark	Good Guys
Eastern	AMR	IBC	Century Park
Resorts	Club Med	Colorocs	Eastman Kodak
Miniscribe	System	Floating Point	Stratus
Jumping Jack	Walker	Russ Togs	Cherokee
Gibraltar	American Capital	Sprouse-Reitz	Price Co.
Integrated Resources	ASA	Voplex	Decorator Industries
Doscocil	Sara Lee	Bonneyville Pacific	Long Lake
National Lumber	Wolohan	Orion	Sandy
Siliconix	LSI	Sterling Optical	Valken
Mortgage Reality	American Holdings	El Paso Electric	Houston Industries
GF Group	Tab Products	Home Centers	Pier 1
Wilfred American	Flight Safety	Stuarts	Mercantile
Circle K	Casey	International Consumer	SPX
Salent	VF Corp	NVR	Lennar
Federated	Sears	Child World	Tandy Crafts
Daisy Systems	Jetronics	Alexander's	Pubco
Prime Motor Inns	Kahler	Alliant	Pyramid Technology
UMM	Hallwood	Alloy computer	Ciprico
Pharma Kinetics	Syntex	Autodie	SPX
Lone Star	For Better Living	Wang	Hewlett
Pan Am	AMR	Highland Superstores	Magnetic Technologies
Insilco	Margaux	Savin	VWR
Interco	Mcrae	Brendle's	Woolworth
Barton	Panatech		

.05 level of probability. The longitudinal test of the nonbankrupt firms (DIF2) and the cross sectional test of the early measures (DIF3) were not significant in both the nonparametric and parametric tests. For the cross sectional comparison of the late measures (DIF4), however, the nonparametric test indicated a difference significant to the .10 level, but not to the .05 level in contrast to the t-test which indicated no significance at either level. The parametric and nonparametric tests agreed for the DIF1 sample, then, but not the DIF4 sample.

The Kolmogorov-Smirnov goodness-of-fit test was then performed on the two samples to see if either diverged substantially from a normal distribution [Siegel and Castellan, 1988, p.51]. Sample DIF1 was found consistent with a normal distribution to the .01 level while sample DIF4 did not exhibit a normal distribution. Therefore, the parametric test can be expected to be reliable for

Table 2
The Differences of the Exponents
From the Various Subsamples in Exhibit 1

OBS	DIF1	DIF2	DIF3	DIF4
1	0.019	-0.046	0.016	-0.030
2	-0.042	-0.389	-0.167	-0.556
3	-0.184	0.119	-0.117	0.002
4	-0.168	0.050	-0.064	-0.014
5	-0.018	0.063	-0.069	-0.006
6	-0.032	0.012	-0.030	-0.018
7	-0.047	-0.039	-0.013	-0.052
8	-0.091	0.034	-0.194	-0.160
9	-0.190	-0.124	0.040	-0.084
10	-0.001	0.036	0.035	0.071
11	-0.113	-0.060	-0.086	-0.146
12	-0.106	-0.053	0.000	-0.053
13	0.198	-0.071	-0.014	-0.085
14	-0.097	-0.066	-0.031	-0.097
15	-0.035	-0.063	0.025	-0.038
16	0.404	-0.002	0.130	0.128
17	0.073	-0.101	0.157	0.056
18	-0.371	-0.241	0.018	-0.223
19	-0.200	-0.284	0.285	0.001
20	-0.035	-0.587	0.503	-0.084
21	-0.020	0.355	0.054	0.409
22	0.192	-0.082	0.139	0.057
23	-0.106	0.206	-0.128	0.078
24	-0.126	-0.049	-0.067	-0.116
25	-0.001	0.069	0.008	0.077
26	-0.086	-0.165	0.042	-0.123
27	-0.050	0.003	-0.136	-0.133
28	-0.093	-0.026	-0.018	-0.044
29	0.005	0.000	0.035	0.035
30	0.034	0.221	-0.161	0.060
31	0.012	0.088	-0.117	-0.029
32	0.274	0.249	-0.029	0.220
33	-0.159	-0.051	0.010	-0.041
34	0.031	0.703	0.007	0.710
35	-0.052	-0.084	0.040	-0.044
36	-0.045	0.060	0.038	0.098
37	-0.104	-0.155	0.087	-0.068
38	0.024	-0.127	0.096	-0.031
39	-0.134	0.005	0.033	0.038
40	-0.028	-0.166	0.105	-0.061
41	-0.191	-0.016	-0.058	-0.074
42	-0.017	-0.357	0.071	-0.286
43	-0.093	-0.016	-0.003	-0.019
44	-0.036	-0.065	0.030	-0.035
45	-0.075	0.128	-0.099	0.029
46	0.014	0.166	-0.030	0.136

sample DIF1 but not for sample DIF4, a possible explanation for the divergent findings.

The mean and the standard deviation calculated for the t-tests of DIF1 and DIF2 were then used to generate a decision rule to predict whether or not a firm would go bankrupt (see Exhibit 2). The decision rule is based on the parameters of samples DIF1 and DIF2, therefore a Kolmogorov-Smirnov goodness-of-fit test was performed for both and both exhibited a normal distribution. Thus the decision rule is derived from two samples which exhibited a normal distribution. The resulting rule is that if the difference between the exponent for the late period and the exponent for the early period is less than -0.03, the prediction is that the firm will go bankrupt. If the difference is greater than this value, it is predicted that the firm will not go bankrupt. This decision rule was then tested on the set aside sample, and the incidence rates of Type 1 and Type 2 errors were measured.

A Type 1 error occurs when the model misclassifies a firm which will go bankrupt as one which will not go bankrupt. A Type 2 error occurs when the model misclassifies a firm which will not go bankrupt as one which will go bankrupt. As previously mentioned, Type 1 errors are much more costly to decision makers than Type 2 errors [Altman et al., 1977]. At this stage of development, the model treats both types of errors as equally likely.

Of the 23 bankrupt firms in the set aside sample, the model correctly predicted bankruptcy in 15 of the cases, yielding a 65% accuracy rate. Similarly, of the 23 non-bankrupt firms in the set aside sample the model correctly predicted that 15 would not go bankrupt. Consequently, the model exhibits both Type 1 and Type 2 error rates of 35%. This single factor model compares favorably as to Type 1 error with the best univariate predictor chosen by Beaver in an early cross-sectional bankruptcy prediction study whose purpose was to identify the most efficient single predictors. That study also used a matched pair design which equated the number of bankrupt and non-bankrupt firms [cited in White et al., 1994]. A number of the individual variables tested by Zmijewski who used realistic proportions in a cross-sectional test achieved similar results [cited in Foster, 1986].

Table 3
T-test of the Differences

	Mean	Std Error	T	Prob>|T|
DIF1	-0.0405652	0.0187877	-2.1591347	0.0362
DIF2	-0.0199565	0.0292308	-0.6827213	0.4983
DIF3	0.0081087	0.0174735	0.4640555	0.6448
DIF4	-0.0118478	0.0258585	-0.4581786	0.6490

It has not yet been shown that chaos statistical data can be used to improve the power of "traditional" bankruptcy prediction models. Beaver [1966] demonstrates that the total debt to total asset ratio is an important predictor of firm bankruptcy. Foster [1986] argues that firm size is an important variable related to bankruptcy. Data from the Compustat Industrial Annual tape were used to generate the debt to total asset ratio as well as the natural log of sales for each firm in each sample two years prior to the year of bankruptcy filing. Discriminate analysis was then used on the 46-firm model generation sample to develop another prediction model to be tested subsequently on the set aside sample. This simple two factor model exhibited a 29.27% Type 1 error rate and a 19.51% Type 2 error rate.

Exhibit 2
Computation of the Decision Model

	DIF1	DIF2
mean	-0.04057	-0.01996
s.d.	0.127425	0.198253

1. Distance between the two means = 0.02061
2. Sum of the two s.d.'s = 0.325678
3. Weight = (s.d. of DIF1/sum of s.d. of DIF1 & DIF2) (0.127424/0.325678) = 0.3912576
4. Midpoint of the means weighted by standard deviation -0.04057 + 0.02061*0.3912576 = -0.0325062

This simple model was then expanded by adding the following chaos statistics: the "early" measure of the Lyapunov exponent, the difference between the late and early measures of the Lyapunov exponent, the difference between the late fast Fourier transformation (FFT) and the early FFT, the late measure of the correlation dimension and the late measure of the capacity dimension. The addition of these statistics reduced the Type 1 error to 19.51% and the Type 2 error to 12.20%, a dramatic increase in model accuracy.

Summary and Conclusions

Etheridge and Sriram [1993] have argued that it is time for accounting researchers to begin using chaos methodology. This study has attempted to do just that. The Lyapunov exponent, the primary chaos statistic, was calculated for two different time periods for a sample of bankrupt firms and their pair matches.

Goldberger [1990] argued that healthy systems exhibit more chaos than unhealthy systems. Similarly, this paper hypothesized that the returns of firms approaching bankruptcy would exhibit significantly less chaos than they had exhibited earlier. The hypothesis was tested by computing the differences between the Lyapunov exponents calculated for a period 3-1 year prior to filing for Chapter 11 (late) versus the exponents calculated for a period 7-5 years prior to filing (early). Consistent with the hypothesis, the t-test and Wilcoxon signed ranks test of these differences was significant at the .05 level. Similar tests of the differences for a sample of pair match firms was found not to be significant.

The means and the standard deviations of these two sets of differences were then used to construct a single-variable bankruptcy prediction model. This model was tested on a set aside sample of 23 bankrupt firms and their pair matches and exhibited both Type 1 and Type 2 error rates of 35%. While these error rates are high, they compare favorably with other single-factor bankruptcy prediction models that have been previously proposed. Significantly lower error rates were then demonstrated as the Lyapunov exponent was augmented by additional chaos and traditional financial variables.

Suggestions for Future Research

Future applications of non-linear dynamics to the problem of bankruptcy prediction are likely to replicate the progress of model-building based on traditional financial measures and ratios. A critical next step is the application of chaos-derived models to test samples that reflect realistic base rates in which most firms endure and relatively few firms experience insolvency. A further no less critical step is the development of a theoretical understanding of the chaos-derived model based on more than the simple health analogy used here.

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