Note On A Stochastic Present Value Model Arising In Investment Analysis

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Abstract

Several stochastic present value models have been adopted in many areas of financial theory and practice. The purpose of this paper is to introduce a new stochastic present value model and offer an application in investment evaluation.

1. Introduction

Present value models have been widely adopted in financial theory and practice and play a very important role in capital budgeting and profit planning. Financial literature in the past appears to have concentrated on developing present value models which were essentially deterministic. Stochastic present value models aim to provide management with more detailed and more realistic criteria upon which to base financial decisions.

Several probabilistically-based analytical methods have been used by Zinn et al [6] to deal specifically with present value models under conditions of uncertainty. Most of these methods concentrate only on the establishment of several moments of a stochastic present value model [4], [5]. Principal among these probabilistic methods are those which derive explicitly the distribution function of a stochastic present value model [3]. Artikis and Jerwood [1] have considered the application of a wide class of probability distribution functions in stochastic discounting. Moreover Artikis et al [2] have considered the application of the same class of probability distributions in discounting binomial random sums under random timing.

The main purpose of this paper is to introduce a new stochastic present value model and provide an application in investment programs.

2. A Stochastic Formulation Of A Present Value Model

Let Y represent a payment to be paid at some future time T . Considering continuous discounting of Y when the nominal continuous compound interest rate is r , it can be shown that the present value V of Y is given by

$$V = Ye^{-rT} \quad (2.1)$$

Artikis and Jerwood [1] considered a stochastic formulation of the above present value model by assuming that r is constant and Y , T are continuous, positive and independent random variables with Y arbitrarily distributed and T decomposed into a geometrically distributed number of exponentially distributed random variables. Properties and applications of this stochastic formulation of (2.1) in risk and insurance management, machine failure and replacement policies, and administration of investment programs have also been established by the authors.

Artikis et al [2] considered another stochastic formulation of (2.1) by assuming that r is a constant, Y and T are continuous, positive and independent random variables with Y decomposed into a binomially distributed number of random variables and T exponentially distributed. The authors also provided applications of this stochastic formulation of (2.1) in replacement of investments and quality control.

In this paper we offer a new stochastic formulation of (2.1) by assuming that r is constant and Y , T are continuous, positive and independent random variables with Y decomposed into a binomial random sum and T decomposed into a geometric random sum. This stochastic formulation of (2.1) is mathematically equivalent to the sequential assessment of a number of investment proposals presented for appraisal in order to find an acceptable investment, which will be financed by the liquidation of other investments.

3. An Economic Application

Suppose that individual investment proposals arise at discrete time epochs

$$T_n = X_1 + X_2 + \dots + X_n$$

where $\{X_n : n = 1, 2, \dots\}$ is a sequence of continuous positive, independent and identically distributed random variables. Let W_n represent the internal rate of return of the n -th investment proposal and suppose $\{W_n : n = 1, 2, \dots\}$ is again a sequence of continuous, positive, independent and identically distributed random variables with distribution function $F_W(w)$ and that each W_n is independent of X_n , $n=1,2,\dots$. An acceptable investment proposal may be one whose internal rate of return exceeds some critical value w_0 , with probability

$$p = P[W_n > w_0] \\ = 1 - F_W(w_0)$$

If the discrete random variable N denotes the number of investment proposals assessed until the first acceptable investment, then N is geometrically distributed with probability distribution function

$$P[N = n] = p q^{n-1}, \quad q = 1 - p, \quad n = 1, 2, \dots,$$

and the first acceptable investment proposal arises at time

$$T = X_1 + X_2 + \dots + X_N \quad (3.1)$$

Now, consider a portfolio of a fixed number of investments, m , and let C_k denote the replacement cost of the k -th investment, $k = 1, 2, \dots, m$ at time T defined by (3.1). It is assumed that $\{C_k : k = 1, 2, \dots, m\}$ are continuous, positive, independent and identically distributed random variables with distribution function $F_C(c)$. If C_k exceeds some critical value c_0 then the k -th investment will be immediately liquidated. Taking

$$\pi = P[C_k > c_0] \\ = 1 - F_C(c_0)$$

to be the probability of liquidation, then K , the number of investments which will be liquidated at time T follows the binomial distribution $B(m, \pi)$. Let S_k denote the market value of the k -th investment at time T . It is assumed that $\{S_k : k=1, 2, \dots, m\}$ are continuous, positive, independent and identically distributed random variables. The market value of the liquidated investments at time T is given by the binomial random sum

$$Y = S_1 + S_2 + \dots + S_K \quad (3.2)$$

Considering continuous discounting of the amount Y in (3.2), which occurs at time T given by (3.1), we conclude that the model (2.1) gives the present value of the market

value of those investments in the portfolio, which are liquidated at time that the first acceptable investment proposal arises. From the fact that the market value of the liquidated investments can be used in financing the first acceptable proposal it follows that the above stochastic formulation of the present value model (2.1) can be of some practical importance in investment programs.

4. Concluding Remarks

The distribution function $F_V(v)$ which corresponds to the present value model (2.1), with Y and T defined by (3.2) and (3.1) respectively, is extremely complicated, but the possibility of applying numerical methods and approximations to $F_V(v)$ facilitates the use of this present value model in financial decision making. Moreover, Monte Carlo simulation techniques provide a viable means to verify, in some practical way, the consequences of the stochastic present value model provided by the paper.

In general, any analytical determination of the distribution function $F_V(v)$ is intrinsically difficult. However, it can be shown that this distribution is related to certain known classes of distribution functions. In particular, if we suppose that the random variables $\{X_n : n = 1, 2, \dots\}$ are exponentially distributed with parameter λ then it can be shown that the distribution function $F_V(v)$ belongs to the class of unimodal distribution functions with $\hat{a} = p\lambda / r$ [2]. In this case, for certain values of the parameters p , λ and r , explicit determination of the distribution function $F_V(v)$ is possible.

5. Suggestions For Future Research.

It seems that further study should be carried out on the properties and the financial applications of the present value model (2.1) under the following assumptions for r , Y and T . The nominal continuous compound interest rate r is constant, the payment Y and the timing T are continuous, positive and independent random variables with Y decomposed into a Poisson random sum and T decomposed into a geometric random sum.

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Evidence On The Behavior Of Bid-Ask Spreads Surrounding Stock Split Announcements

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Abstract

We test for the bid-ask spread reaction to the announcement of stock splits. If managers use signaling to convey private information to the market, then capital market participants engage in trading activity that utilizes a more symmetric information set after corporate announcements. Consequently, share prices more accurately reflect their true equilibrium value; adverse selection risk of security dealers is lower; percentage bid-ask spreads become more narrow and the liquidity of the market increases. The results of this study provide evidence of a reduction in percentage bid-ask spreads in reaction to a split announcement. Managers use stock splits as signaling devices to inform market participants of stock undervaluation.

I. Introduction

Stock splits are fairly common events for publicly traded corporations and are reported daily in the Wall Street Journal and other business publications. Splits cosmetically change the equity portion of the balance sheet, but otherwise do not change the fundamental value nor the ownership structure of a firm, yet it has been shown that stock prices change around *announcements* of forthcoming splits. The well-documented share price reaction to split announcements has at least been partially explained in a signaling model developed by Brennan and Copeland (1988) and a theory of managerial preference for an optimal price trading range by McNichols and Dravid (1990).¹ Whether share prices react to firm undervaluation signals or to the prospect of a more affordable price range, financial economists generally agree that split announcements increase the proportion of outstanding shares traded daily. The potential profit of making a market in stocks whose shares are expected to split attracts more dealers and higher trading volume. As the number of market makers increases, the competition for order flow increases. Each of these market forces should reduce the magnitude of bid-ask spreads. Ferris, Hwang, and Sarin (1993) show evidence of an increase in the number of liquidity traders in the post split period.

Corporate announcements will generally impact the mechanics of secondary market trading. Bid and ask prices are sensitive to changes in available information.² In this paper, the bid-ask spread reaction to the

announcement of stock splits is tested. If managers actively use financial decisions to signal firm undervaluation, then share prices will more accurately reflect the true value of a firm. If signaling is a motivation for corporate announcements, then managers, shareholders, and other capital market participants will engage in trading activity that utilizes a more symmetric information set. Consequently, the component of the bid-ask spread that depends on the quality and availability of information will be lower. Dealers will be exposed to a lower level of adverse selection risk and will adjust spreads accordingly to a more narrow range.

Evidence consistent with a reduction in percentage bid-ask spreads in reaction to split announcements is uncovered. The managerial decision to signal stock price undervaluation causes a reduction in spreads due to at least two factors: (1) a reduction in the adverse selection risk faced by the market makers and (2) an increase in the liquidity of shares traded.

II. Behavior of Bid-Ask Spreads

Bid and ask prices bracket the true frictionless share value and are an implicit transaction cost faced by investors. As presented by Stoll (1989), spreads are a function of three component costs: 1) adverse information; 2) inventory holding; and 3) order processing.

Inventory costs represent the dealer's tradeoff between holding too many shares and holding too few shares, with opportunity costs being a large fraction of inventory costs. Order processing costs are comprised of such costs as computer time, telephone calls, and monthly statements.

Glosten and Milgrom (1985) have developed a highly intuitive model describing adverse selection and its affect on bid-ask spreads. Dealers trade with two basic types of investors; those with superior information sets (information traders) and those with inferior information sets (liquidity traders). Dealers expect to profit from trading with liquidity traders while trying to minimize losses from trading with informed investors. The risk of incurring losses in trades with better informed investors is referred to as adverse selection risk.

Since dealers do not often know whether a trade is being executed due to liquidity needs or an information advantage, spreads are adjusted to reflect the amount of risk facing dealers. When the volume of shares traded by information-based traders in the marketplace exceeds the volume of liquidity traders, dealers widen spreads. As managers release information through public announcements, information flows to both dealers and uninformed investors who can now compete with informed traders. This reduction of information asymmetry decreases dealers adverse selection risk exposure, and allows dealers to narrow bid-ask spreads.

Since an information release event is being investigated in this study, adverse selection risk is assumed to change and both inventory costs and order processing costs are assumed to be held proportionally constant (relative to the percentage bid-ask spread) during the period surrounding a stock split announcement. The dynamic nature of adverse selection risk provides an opportunity to test for shifts in spreads due to relative changes in information sets.

III. Review of Stock Split Literature

Johnson (1966) was one of the first to note that the announcement of stock splits drove prices upward. Grinblatt, Masulis, and Titman (1984) attribute the positive stock market reaction to financial signaling, but show that since average cash dividends do not increase after splits, the split factor is the source of the signal and not an expected increase in dividends.

Ferris, Hwang and Sarin (1993) perform an analysis of trading activity following stock splits. Their focus is not on the announcement, but on post-split market characteristics. They find an increase in bid-ask spreads after splits, as well as a specific increase in the adverse selection

component of spreads. Trading frequency increases while the average trade size decreases. They conclude that there is an increase in the proportion of liquidity traders in the post-split period and that the positive *announcement* returns documented in the literature are due to signaling rather than price correction.

Conroy, Harris and Benet (1990) also study the relationship between spreads and splits. They gather a sample of stock splitting firms and a sample of non-stock splitting firms from the years 1981 to 1983. They compare the cross-sectional mean of closing bid-ask spreads using a simple difference of the means test. Their estimation period is the two months prior to the split announcement; the event period is the two month period following the ex-date of the split. They find a significant decrease in the absolute spread and a significant increase in the percentage spread. They conclude that shareholder liquidity is reduced after a split as traders are confronted with a larger percentage spread. This evidence is inconsistent with the notion that managers' objective is to move share prices into an optimal trading range.

It should be noted, however, that percentage spreads increase as stock prices fall unless spread are also reduced by the same percentage, that is, the split factor. It is not always possible for spreads to decrease by the same proportion as stock prices due to the discrete nature of trade and quote prices, which is typically in 1/8th fractions.

IV. Data Description

All firms that split their stock during 1991 and 1992, and are listed on the NYSE, AMEX, or NASDAQ are included in the original sample.³ Splitting firms are identified using the Institute for the Study of Security Markets (ISSM) database, which contains transactions data and information such as number of outstanding shares, dividend dates and amounts, and split factors.

The final sample consists of 226 firms that split their common shares according to the following criteria: (1) Daily *bid and ask prices* are available for the period from day -65 to day +5, with day 0 being the announcement day of the corresponding stock split; and (2) Daily *stock returns* are available on the CRSP tapes for the period -270 days to +45 days for splitting firms in 1991.⁴ This portion of the study used the 120 firms from 1991 only.

Once the sample is identified, daily closing quotes from the New York Stock Exchange, the American Stock Exchange, and the over-the-counter market are collected during the period (-65 to +5) as are CRSP daily returns. For the organized exchanges (NYSE and AMEX), the

daily closing quote is identified by a "closing" condition code. For the NASDAQ stocks, the last inside quote of the trading day is used. The inside quote refers to a quote that is part of the NASD best bid and offer calculation. Quotes that are not inside quotes do not reflect true market value and were therefore ignored.

V. Methodological Design

Both stock price and bid-ask spread reaction to the announcement of stock splits are tested. The CRSP tapes are used to verify previous stock price research on positive announcement date returns. The market model and an estimation period of 225 days are used to calculate parameters required for the calculation of firm prediction errors. The announcement date is reported by ISSM and is confirmed by examining the Wall Street Journal and newswire services. The prediction errors are determined as follows:

$$PE_{j,t} = R_{j,t} - (a_j + \beta_j R_{m,t}) \quad 1)$$

Where

$R_{j,t}$ is the return for firm j on day t , $R_{m,t}$ is the CRSP value-weighted market return on day t , a_j and β_j are the market model parameters.

The appropriate test statistic, developed by Mikkelsen and Partch (1988), is the z statistic. The Mikkelsen and Partch statistic guarantees that standard errors are unbiased even in the presence of serial dependence. Z is calculated as follows:

$$Z = \sum SCPE / \sqrt{J} \sim N(0,1) \quad 2)$$

Where

SCPE is the standardized cumulative prediction error and J is the total number of firms.

The bid-ask spread reaction to the announcement of stock splits is tested using the percentage spread as follows:

$$PS_{j,t} = (\text{Ask} - \text{Bid}) / [\frac{1}{2}(\text{Ask} + \text{Bid})] \quad 3)$$

A simple deviation from the mean model is used to calculate abnormal percentage spread performance surrounding a stock split. An estimation of the true bid-ask spread is calculated over a 60 trading day interval that ended exactly 6 days before the announcement of a split. For firm j , the estimate of the abnormal spread is calculated as follows:

$$APS_{j,t} = PS_{j,t} - PS_j \quad 4)$$

Where

$PS_{j,t}$ is the reported percentage spread for firm j on day t , PS_j is the estimate of the true spread for firm j .

The mean abnormal percentage spread is simply the average of each firm's percentage spread in cross section:

$$MAPS_t = \sum (1/J) APS_{j,t} \quad 5)$$

The appropriate test statistic is the student's t statistic calculated as follows:

$$t_t = MAPS_t / \sigma(MAPS_s) \quad 6)$$

Where

$\sigma(MAPS_s)$ is the standard deviation of the estimation period mean abnormal spread in cross section.

In addition, the MAPS for various intervals around the event date are tested. The MAPS over an interval from t_1 to t_2 during the event period is defined as:

$$MAPS_{t_1,t_2} = [1/(t_2 - t_1 + 1)] \sum MAPS_t \quad 7)$$

The test statistic for establishing a relevant significance level is defined as:

$$t(MAPS_{t_1,t_2}) = MAPS_{t_1,t_2} / [\sigma(MAPS_{t_1,t_2})] \quad 8)$$

This methodology has also been used by Tripathy and Rao (1992).

Finally, a t test for the difference between means procedure is performed to test the hypothesis that the means of the *sample* and *test* periods are equal.⁵

VI. Empirical Results

The preliminary results show a positive stock market reaction to the announcement of stock splits. This is consistent with prior studies and the notion that a decision to split shares is interpreted as a positive signal by investors. The sample results are shown in Table 1. Share prices appear to experience a significant run-up in the one month trading period (-20,-2) prior to a split. It is important that the price appreciation is not quickly reversed after the split announcement. The lack of significance in the cumulative prediction errors after the announcement provides weak evidence in favor of the validity of the signal.

Table 1
Stock Price Reaction to Stock Split Announcements

Standardized cumulative prediction errors (SCPE) around the announcement of stock splits are shown below using a sample of 120 firms that announced stock splits during 1991. The percentage of time the SCPE is positive is reported in the third column, with significance being determined by a sign test.

Standard Cumulative Prediction Errors			
Interval	SCPE (%)	Z-statistic	% Positive
(-45,-2)	3.03	0.52	54.12
(-20,-2)	3.17	2.20*	61.46**
(-1, 0)	0.69	1.92^	56.88*
(+2,+20)	-1.09	-1.08	44.94
(+2,+45)	-0.55	-0.42	50.04
Significant at 10% (^); 5% (*); 1% (**).			

One interpretation of the positive share price reaction is that managers are signaling private information. Shareholders expect earnings to be greater in the future, creating a greater demand for the splitting firms. If this scenario is true, then informed traders and uninformed traders are using an information set that is similar. A reduction in information asymmetry should be reflected in the level of bid and ask prices. The magnitude of bid-ask spreads should be reduced because dealers are exposed to a lower degree of adverse information risk after split announcements. This is known as an information effect.

Even though Grinblatt, Masulis and Titman (1984) find that cash dividends do not increase in the months following stock splits, effective signaling by managers can still occur. A significant narrowing of percentage spreads is shown and it is believed that managers are conveying private information about future earnings by splitting their stock. The size of the spread is reduced because managers, security dealers, liquidity traders, and information traders are now utilizing a more symmetric information set. The dealer is exposed to less adverse selection risk and allows the spread to narrow.

The main results of this study confirm the information effect. Managers do in fact signal private information, and the percentage spreads narrow on the announcement date.

The mean abnormal percentage spreads calculated for several days and over several windows, along with their t-statistics are presented in Table 2.

One important result shown in Table 2 is that percentage spreads continue to remain more narrow through the five trading days following the split announcement. The mean abnormal percentage spread over the interval (+2,+5) is negative and statistically significant. Not only do spreads narrow on the split announcement date, but the lower risk of information trades allows dealers to keep them at that narrow level. This is consistent with the hypothesis that information flows from managers to capital market participants and results in the reduction of the information advantage enjoyed by informed traders.

The final results are shown in Table 3 in which the difference of the means test is reported. These results support those presented in Table 2. The differences in the means of the sample and test periods are statistically significant and show further support that the percentage bid-ask spread narrows around the announcement of a stock split.⁶ The evidence supports the notion that dealers are able to reduce spreads due to the reduction of information asymmetry among the various market participants around the time of stock split announcements.

Table 2
Bid-Ask Spread Reaction to Stock Split Announcements

Mean abnormal percentage spreads (MAPS) for a sample of 226 firms that split their shares during 1991 and 1992. True spreads for each firm are estimated over a 60 day sample period prior to the split announcement. T statistics are calculated for both individual days (Panel A) and event windows (Panel B).

Mean Abnormal Percentage Spread			
Panel A:	Day	MAPS (%)	t-statistic
	-2	-0.019886	-0.706
	-2	-0.087923	-1.114
	0	-0.123686	-2.295*
	1	-0.112195	-1.096
	+2	-0.007229	-0.545
Panel B:	Window	MAPS (%)	t-statistic
	(-5,-2)	-0.042963	-1.498
	(-1, 0)	-0.193761	-3.165*
	(+2,+5)	-0.068145	-2.207*
Significant at 5% (*)			

VII. Suggestions for Future Research

One area of potential future research would be to determine the source of the decision to split the stock. The conclusions reached in this paper confirm the impact that managerial decisions have on security prices and bid-ask spreads. The splitting of shares is used by managers as a valid signaling device. It is important for future research to focus on the quality of the signal in determining if the signal relates to expected dividends, expected earnings, or some other corporate policy. Increased understanding in this area would provide a better understanding of managers' motivations, and therefore, allow better prediction of this type of behavior.

In addition to determining the motivations behind a stock split, there may be other factors affecting the percentage bid-ask spread that are not considered in this work such as changes in volume, the number of market

makers for a particular stock, and variance in returns. Utilizing a linear programming approach to hold some of these factors constant may lead to better isolation of the components of the spread. Another area that has not been investigated is the agency cost of stock splits. Since split executions increase the number of outstanding shares and presumably the number of shareholders, the further separation of ownership and control may reduce firm value in the long run. Future research on the corporate side of stock splits should focus on the use of debt or dividends subsequent to a split in reducing the agency cost of ownership dilution. Future work in both the areas of stock returns and market microstructure may provide insights into the motivations for, and the affects of stock splits.

*** Footnotes ***

1. See the seminal article by Fama, Fisher, Jensen, and Roll (1969) which shows significant wealth effects

Table 3
Difference Between Sample and Test Period Mean Abnormal Percentage Spreads

Mean abnormal percentage spreads (MAPS) for a sample of 226 firms that split their stock during 1991 and 1992 are used to perform a difference of the means test. The two periods being compared are the 60 day sample period and the 10 day test period. The means of the entire period, the sample period, and the test period are given.

Difference of the Means		
Sample	MAPS Mean (%)	t-statistic
Entire Period	-0.01031436	
Sample Period	0.00000018	
Test Period	-0.07073271	
Difference of Means		-2.974**
Significant at 1% (**)		

surrounding stock split announcements. Their original results have been confirmed over the years. See Szewczyk and Tsteksekos (1993) for a recent survey of splitting firms.

2. See Howe and Lin (1992) for a study on the relationship between bid-ask spreads and dividend policy.
3. The rules that govern accounting procedures define stock splits to be any distribution of shares of 25% or more. That convention is used in this study.
4. Since the focus of this study is to test for the bid-ask spread reaction to split announcements, approximately fifty percent of the sample is used to confirm the stock price reaction to the announcement of stock splits.
5. The computation of the t statistic is based on the assumption that the variances of the two periods are equal. A folded F statistic is computed to test for the equality of the variances in accordance with Steel and Torrence (1980).
6. The assumption of variance equality is supported with the calculation of the folded F statistic of 1.89 and a Prob. > F' = 0.3065.

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