Redistributing Rewards to Revitalize Racing

Dr. Donn R. Pescatrice, Economics, Loyola University of New Orleans
Dr. Mary Fragola, Economics, Iona College

Abstract

The thoroughbred horseracing industry is mired in a situation where many of the owners of the typical unheralded racehorse--the staple of everyday horseracing--are experiencing a severe financial strain as a result of an inadequate purse structure. As owners abandon the industry, an undersupply of racehorses is limiting racing opportunities and producing uncompetitive races with few entrants at many racetracks, negatively impacting industry revenue. There is a growing fear that the entire horseracing industry may be irreparably damaged if owners of these unspectacular but useful racehorses are not retained. The model developed here reveals that there exists a self-sustaining level of overall purses for a particular racemeet that can be supported by patron wagering. The proposed plan for distributing this aggregate level of purses embodies a minimum subsistence reward that ensures that owners of better horses of even the lowest quality can remain viable. The purse distribution scheme also offers the owners of higher quality horses the opportunity to earn greater returns on their racehorse investments, providing a strong incentive for owners to upgrade their racing stock. Stemming racehorse undersupply while raising racing quality is critical to enhancing patron support and industry revenue.

Introduction

Although the overall gambling industry encompassing casino gaming, lotteries, and parimutuel wagering is in a growth cycle, the parimutuel horseracing industry is not participating in the upturn. During the 1982 through 1992 period, gross wagering for the overall gambling industry increased at a robust 10.1% average annual rate. The horseracing industry recorded a 0.50% average annual decrease in gross wagering during this period, and managed a 1.9% average annual increase only as a result of the proliferation of off-track betting systems [Christiansen, 1993]. A key contributor to the decline in patron support of horseracing is the intensifying competitive pressure from the alternative gambling forms of casino and riverboat gaming, lotteries, and other forms of government sanctioned games of chance. The impact of competing gambling forms on the horseracing industry has been the focus of previous studies. The effect of lotteries on the business of horseracing has been discussed in [Gulley and Scott, 1989], [Simmons and Sharp, 1987], [Thaler and Ziemba, 1988], and [Thalheimer and Ali, unpublished]; and the impact of off-track betting was analyzed in the works of [Coate and Ross, 1974], [Pescatrice, 1980], and [Thalheimer and Ali, 1992].

Although these external competitive forces have certainly contributed to the downturn of the parimutuel horseracing industry, there are internal factors that have

also precluded the horseracing industry from sharing in the growth spiral of other gambling forms. This paper focuses on one of these intrinsic factors--the imbalanced purse or prize money distributive scheme that rewards racehorse owners for their investments.

Owners of racehorses earn the purse or prize money of horseraces contested at the racetrack. As evidenced by the data presented below, purses are inordinately large for races contested by the few premier top-quality racehorses; yet prize money is inadequate for races comprised of horses of middle and lower quality which include the typical racehorse. Consequently, the fortunate few owners of exceptional racehorses are receiving ample purse money, while the overwhelming majority of owners of the typical unheralded racehorse are receiving an insufficient amount. This top-heavy purse distribution structure has resulted in a severe profitability squeeze for owners and personnel associated with the unspectacular but useful racehorses which are the staple of the horseracing industry [Lawrence, 1992].

As profits erode, owners of middle and lower quality horses are dropping out of the industry, raising concerns that there may soon be a shortage of horses to fill race programs [Simon, 1992]. Indicators of nascent racehorse undersupply include the 25% drop in the 1992 foal crop since the 1986 peak, and the steady decline in the

average number of horses per race from 9.1 in 1986 to 8.6 in 1992. Shrinking racehorse supply reduces both the number of racing opportunities and the number of per race contestants, causing a drop in the level of patron wagering as betting possibilities are diminished. In New York for example, the number of daily live races during the winter season was pared from nine to seven. Further, it is not uncommon for races in New York to be contested by six or fewer horses. In Ohio, the racetracks have also reduced their live race offerings from ten to seven. The lack of wagering opportunities both in the number of race offerings and in the number of contestants per race has dampened patron wagering and the resultant revenues to the horseracing industry, exacerbating the profitability pressure plaguing owners of mid-to-lower quality horses arising from the skewed purse distribution structure.

The reason generally offered for the current top-heavy purse distribution structure is that racetracks are bidding for relatively scarce talent to improve their product-analogous to what is happening in many professional sports. In baseball, for example, the multi-million dollar salaries paid to baseball superstars have pulled the salary of the average starting player up to nearly the \$1 million level. But, unlike professional sports, the trickle-down benefits have not materialized in the horseracing industry. Since aggregate purse money is directly tied to the total wager through state legislated formulas, the prevailing purse "pie" is essentially fixed. Inflating purse shares at the upper quality end naturally inhibits growth elsewhere.

The erosion of the industry at the mid-to-lower quality end has far reaching consequences. Thoroughbred horseracing is estimated to be an \$18 billion industry employing over one-half million people. Furthermore, the ancillary industries providing supplies, feed, transportation, advertising, and veterinary care, etc., constitute another collection of businesses which are critically dependent on the horseracing industry. There is a crucial need to preserve the ubiquitous integral lower end of the thoroughbred industry, or risk economic deprivation of a multitude of people directly and indirectly dependent on the viability of the typical racehorse that will never attain superstar status. Furthermore, without a lower end there would be no upper end of thoroughbred horseracing, as racetracks would not find it feasible to operate by merely offering two or three races a day for better quality horses.

The existence of disproportionate purse distributions and the resultant financial strain on owners of unspectacular racehorses is widely recognized by industry insiders, and is explicitly detailed in [Lawrence, 1992]. Capsuled salient statistics illuminating the extent of the problem include:

- In 1991, the top 0.1% (104 horses) of the 86,483 horses that competed in at least one race (termed "runners") garnered 8.6% of all purse money paid; and the top 0.9% (747 horses) of all runners captured 21.4% of all purse money.
- In 1991, 37.3% (32,200) of all runners earned less than \$1,000, including the 15% (13,390) of all runners that earned nothing.
- Stakes races (races contested by top-quality racehorses) constituted only 3.5% of all races contested in 1991, yet stakes races accounted for 27.0% of all purses paid. The average purse for a stakes race was nearly 8.0 times the average purse of all other horseraces.
- In the 10-year period from 1982 through 1991, nominal overall purses increased 45.0%. However, purses of stakes races were up 73.0%, while purses of non-stakes races increased by only 36.4%.

Evidence of the negative impact of a top-heavy reward scheme on the profitability of owners of the typical racehorse is compelling:

- During the 1982 through 1991 period, the median annual earnings per runner fell 10.0% to \$2,433 in nominal dollars, which translates into a steep 33.0% decline (to \$1,789 per runner) based on inflation adjusted 1982 dollars.
- In 1991, average training costs per runner totaled over \$15,000 (based on an average of 8 races per year per runner, requiring 44 weeks of training at \$350 per week in training expenses). Yet, the average earnings per runner was only \$8,805, and a full 86.0% of all runners failed even to cover operating costs.
- In a recent survey [Clay, 1989], nearly 43.0% of owners leaving the business of thoroughbred horseracing sighted financial losses as the main reason for their withdrawal, and suggested that a better purse structure might have kept them in operation.

Solutions offered by leaders in the horseracing industry [Hollingsworth, 1992] include paring and redistributing purse money from many races which offer prize money exceeding \$1 million, and legislating a maximum purse. The potential solution presented here to the skewed purse distribution problem not only embodies the principle of a purse ceiling, but also provides owners at every quality level the opportunity to at least cover operating costs. Also, the proposed purse distribution plan maintains strong incentives for owners

to upgrade the quality of their racing stock so that they may reap even greater returns on their racehorse investments. With purses redistributed to retain current owners and to attract new owners, the racehorse under-supply situation can be curtailed in the incipient stage.

Fundamental Relationships

A simple overview of the parimutuel thoroughbred horseracing industry will illuminate the relationships of purse money, horse quality, and patron wagering. As revealed in the earlier capsuled data, a select set of exceptional top-quality horses compete for richer prizes and capture an inordinately large share of the purse offerings. It will be empirically verified that racing patrons are more attracted to the premier higher quality horseraces, and consequently wager more on these races. The total wager (termed "handle") is vital to the economic viability of horseracing since purses are derived from the total wager through a legislated formula. Purses are the lifeblood of horseracing as they reward owners for their accomplishments and risk-taking. Owners, in turn, compensate trainers, breeders, and all the support personnel who assist in developing the final product. The causal relationships between purse offerings, horse quality, and patron wagering supporting the purse structure are schematically portrayed below, along with the general relationships which will be empirically supported in the study:

The objective of the model to be developed here embodies the circuitous interrelationships outlined above: Devise a means of distributing purses such that owners have an incentive to invest in a broader range of horse quality, whereby the overall quality of horseracing is maintained at a level which generates a total wager that supports a purse structure which sufficiently rewards owners for their investments.

The general relationships employed in the model are the following:

- The relationship between horse quality and purse offerings. A direct relationship can be expected given the aggregate data presented earlier, and the recognition that larger purses attract higher quality horses.

$$V=f(P,X) \quad (1)$$

where V reflects the overall quality level of a particular racing season (racemeet) conducted by a racetrack, P measures the aggregate purse offerings

of that racemeet, while X represents a collection of exogenous variables.

- The relationship between patron wagering (handle) and racehorse quality, where a direct relationship can also be anticipated given the notion that patrons are more attracted to better performers.

$$H=g(V,Z) \quad (2)$$

where H measures the total handle of a particular racemeet, and Z represents another set of exogenous variables to be identified in the empirical section.

- The dependency of purse money on the total handle, although the percentage of the handle allotted to purses must adhere to a legislated formula which varies across racetracks and states.

$$P=h(H,T) \quad (3)$$

where T represents the type of wagering opportunity (e.g. regular win, place, show; multiple exactas and quinellas; or exotic triples) and is treated as a predetermined variable since it is subject to racetrack management policy. In practice, a racetrack's purse structure is tentatively determined based on projected handle and subject to occasional adjust-

ments as the actual handle deviates from the projected handle.

Through successive substitution of relation (1) into (2) and finally into (3), the following composite function arises (with purely exogenous variables of X, Z and T omitted)

$$P=h\{g[f(P)]\}=F(P) \quad (4)$$

Brouwer's fixed point theorem guarantees that a self-sustaining aggregate purse level (P) can be achieved by the model represented by relation (4).¹ That is, there exists an aggregate purse level which entices horses of sufficient quality so as to attract enough patronage, whereby the resultant total wager supports this purse level.

The actual mechanism for converging to the self-sustaining aggregate level of racemeet purse offerings may in practice involve an iterative procedure, where racetracks adjust their purse offerings until a self-sustaining level materializes. For example, once the

racetrack schedules its spectrum of tentative race and purse offerings (called a condition book), the array of horse quality competing at the meet is essentially set. Subsequent wagering trends may then dictate adjustments to the aggregate level of purse offerings. These purse adjustments may result in a change in the composition of the horse quality array of future racemeets; which, in turn, could impact wagering trends requiring additional fine-tuning of the purse offerings. However, such a purse adjustment process is hardly unusual, since many racetracks frequently alter purse offerings so that payouts adhere to the legislated formula based on wagering trends. Furthermore, most racetracks currently offer an aggregate level of purses that is supported by the level of total wagering, and consequently have an initial aggregate purse level which may only require distributional adjustments based on the purse distribution model developed in the next section.

Given that a feasible, self-sustaining aggregate level of purses exists, what remains is to devise a distribution plan that rewards owners at every quality level for their racehorse investments. This will help ensure the retention of owners of the typical unspectacular racehorses and curtail the advance of racehorse undersupply. Additionally, the proposed purse distribution plan will also maintain incentives for owners to upgrade the quality of their racing stock by acquiring more valuable horses so that they may share in more lucrative purses. An extension of the model to include a purse distribution plan which satisfies these remaining facets of the overall objective is presented in the next section.

The general relationships of horse quality, purse money, and patron wagering reflected in relations (1), (2), and (3) will now be more specifically defined. As noted earlier, the dependency of aggregate purses on the total handle as reflected in relation (3) does not require estimation, as purses must adhere to a legislated formula based on the per race wager. The prescribed relation varies across states and racetracks, and a typical example is provided in the appendix. Greater levels of patron wagering do support larger purse structures.

The interdependency of purse offerings, racehorse quality, and patron wagering reflected in relations (1) and (2) above will be estimated at the individual race level since the racemeet aggregate relationships must mirror the behavior of the individual races which comprise the meet. The data base consists of races contested in the Spring and Summer of 1991 at the following racetracks: Belmont Park (New York), Churchill Downs (Kentucky), Monmouth Racetrack (New Jersey), Calder Racetrack (Florida), and Detroit Racecourse (Michigan). The objective is to estimate relations (1) and (2) based on the purse offerings, horse quality, and patron wagering tendencies at geographically dispersed racetracks of different class levels. Based

on racing industry standards of overall racemeet quality, Belmont is classified as a premier class I racetrack, Monmouth, Churchill, and Calder are of class II, and Detroit is rated as a lower level class III racetrack.

The explanatory variables employed in the estimation process are defined in the appendix in conjunction with the regression results. However, the measure of racehorse quality requires clarification. Horse quality is measured by a horse's market value. The market value of a horse competing in a claiming race is set at the claiming price at which the horse can be bought or sold. The market value of a horse in a non-claiming allowance, handicap, or stakes race is assessed by employing a valuation model developed by racehorse industry analysts Beyer and Oppenheim (1988).

As expected, the regression results reveal that (1) racehorses of higher quality do compete for greater purses, and (2) patrons do wager more on races contested by higher quality horses.

The Model

Given the existence of a self-sustaining aggregate level of purse offerings, what remains is to devise a means of distributing this aggregate purse level to ease the financial strain on owners of the typical racehorse. The purse distribution plan will also preserve incentives for owners to upgrade their racing stock so that they may reap greater rewards. The variables utilized in the model are:

- v_i = the market value of a representative horse of class "i". The market value is based on claiming market prices or on the appraisal formula cited in the previous section. There exists N distinct classes.
- R_i = the number of races contested by class "i" horses during a particular racemeet.
- w_i = the total purses paid during the racemeet to class "i" runners.
- $p_i = \frac{w_i}{R_i}$ = the per race purse offering in a race of class "i" runners.
- $V = \sum R_i v_i \quad i=1,2,\dots,N$, which is a measure of overall racemeet performance quality.
- $P = \sum w_i = \sum R_i p_i \quad i=1,2,\dots,N$, where the aggregate self-sustaining purse level of a racemeet is treated as a predetermined variable in the subsequent model given that its existence was guaranteed earlier, and typically evolves from an iterative

procedure.

n = the number of calendar days in the racemeet.

k = the per day training and maintenance cost per horse.

s = the average number of starts of a typical horse during the racemeet.

Let

$$r_i = \frac{P_i}{8v_i} = \frac{w_i}{8R_i v_i}, \quad i = 1, 2, \dots, N$$

measure the per race rate of return (hereafter just "return") to the market value of a class "i" race; or the per race return to class "i" based on the combined market value of the individual horses of class "i". The scalar eight employed in assessing the combined market value of the horses in a particular race reflects the fact that, on average, there are about eight participants per race [Lawrence, 1992]. This scalar, however, is immaterial to the key implications of the model. Given the tentative schedule of races for each racemeet, the distribution of v_i is, for the most part, determined prior to the onset of the racemeet.

Suppose that the total purses paid to each class of racehorse is proportional to that class's contribution to overall meet performance quality V . If, for all classes, $w_i = P(R_i v_i / V)$ then the per race return to class "i" would be identical for all horse classes ($r_i = P/8V$). If returns were equivalent across all horse classes, owners would generally be indifferent to the class of racehorse they campaigned, and would have little incentive to upgrade their racing stock in search of greater rewards.

To generate an incentive reward purse distribution plan, define an alternative measure of overall meet performance quality:

$$\bar{V} = \sum c_i R_i v_i, \quad i = 1, 2, \dots, N$$

where the c_i 's are class adjustment parameters, with

$$c_i > 0 \text{ and } c_j > c_i \text{ for all classes } j > i.$$

A plausible set of class adjustment parameters are implied in the forthcoming example. Since higher quality horses are assigned larger parameters in , if the total purse paid to each class of racehorse is proportional to that class's contribution to this alternative measure of overall performance quality:

$$w_i = P \left[\frac{c_i R_i v_i}{\bar{V}} \right] \text{ such that } \sum w_i = P, \quad i = 1, 2, \dots, N$$

then the per race return to class "i" is:

$$r_i = \frac{P_i}{8v_i} = \frac{w_i}{8R_i v_i} = c_i \left[\frac{P}{8\bar{V}} \right], \quad i = 1, 2, \dots, N$$

Since the class adjustment parameter is larger at higher class levels, r_i is positively related to v_i such that

$$r_j > r_i \text{ for all } j > i.$$

That is, the per race return to market value is an increasing function of horse quality. Owners of higher quality horses are rewarded with purse offerings providing greater potential returns on their racehorse investments. These greater potential returns will provide the incentive for owners to upgrade their racing stock by acquiring more valuable horses, which in turn should enhance patron appeal.

With an incentive reward system in place, what remains is to ensure that the owners of the lower quality racing stock are not unduly squeezed out of the industry. Define the minimum subsistence reward to class "i" as the amount of total class "i" purse compensation that is equal to a portion "m" of the horse training and maintenance expenses, as well as the foregone opportunity cost of the at-risk capital for the entire group of class "i" horses. It would be unrealistic to set the level of minimum compensation equal to the full amount of each class's operating costs since this would insulate owners at every quality level from a business loss--a protection afforded to virtually no other industry. In general, "m" is a choice variable of racetrack management. However, since the total per race purse money is distributed in declining shares to only the owners of the top four or five finishers in a typical eight-horse race, a plausible range for "m" would be 50% to 60%. This minimum subsistence reward would provide the owners of horses that are frequently among the top finishers in each race with the potential means to cover expenses and opportunity costs. Horses competing at every quality level would have to exhibit occasional ability to generate minimum subsistence rewards for their owners.

The minimum subsistence reward to class "i" is:

$$\bar{w}_i = m \left[\left(\frac{8R_i}{s} \right) kn + \left(\frac{8R_i}{s} \right) v_i \hat{1} \right], \quad i = 1, 2, \dots, N$$

where:

$\frac{8R_i}{s}$ = the approximate number of horses of class "i", determined by dividing the total number of starters in class "i" races by the average number of starts per horse,

kn = the per horse training and maintenance expense for the full racemeet assuming the horse remains

in training for the entire meet, and

$\hat{i}$ = an appropriate measure of the opportunity cost of capital; e.g., a risk-free rate of return during the period of the racemeet.

Note that the per horse cost per day parameter k reflects the cost of feed, supplies, veterinary services, and payment to the trainer. Except for unusual cases where superstar horses receive occasional special treatment, k is a reasonable constant across horse classes. Therefore, the first term within the brackets in the $\bar{w}_i$ relation above approximates the aggregate training and maintenance expenses for all class "i" horses, and the second term reflects the foregone opportunity cost of the combined market value of all class "i" horses competing at the racemeet.

In brief, during a racemeet, the minimum level of purse money earmarked for each racehorse class should at least compensate the owners of the better horses at any quality level for the cost of preparing the "performers", including the foregone opportunity cost of the at-risk capital.

Let $\bar{r}_i$ be the minimum subsistence per race return to class "i":

$$\bar{r}_i = \frac{\bar{p}_i}{8v_i} = \frac{\bar{w}_i}{8R_i v_i} = m \left[\frac{\bar{k}}{v_i} + \bar{i} \right], \quad i = 1, 2, \dots, N$$

where:

$\bar{k} = \frac{kn}{s}$ = the per horse cost per start, and

$\bar{i} = \frac{\hat{i}}{s}$ = the risk-free opportunity cost per start.

The minimum subsistence per race return to each class must cover a portion of the per market value dollar of the cost of starting plus the foregone opportunity cost of competing.

Note that $\bar{r}_i$ is exogenous to the model since it is solely dependent on the outside factors of k , m , n , s , $\hat{i}$, and market forces as reflected by v_i . The minimum subsistence return ($\bar{r}_i$) is inversely related to the market value of the class "i" horse (v_i). That is, the minimum subsistence per race return justifiably decreases as horse quality increases, since training and maintenance expenses are a reasonable constant generally invariant with respect to horse quality. Consequently, the $\bar{k}/v_i$ term in the above relation is analogous to average fixed costs, which steadily decline at higher output levels in a conventional production setting.

Given the incentive reward system developed earlier providing greater per race returns at higher horse

quality levels: $r_j > r_i$ for all classes $j > i$, if $r_1 \geq \bar{r}_1$ for the lowest class of racehorse competing at the meet, then

$r_i > \bar{r}_i$ for all classes $i=1,2,\dots,N$.

This conclusion naturally follows since the minimum subsistence per race return ($\bar{r}_i$) decreases as horse quality increases; whereas the actual per race return (r_i) increases at higher quality levels.

An incentive reward system can be implemented where the purses paid to a particular racehorse class are proportional to that class's contribution to overall meet performance quality. If purses are allocated such that owners of the better horses of the lowest class of racehorse have the potential to at least cover their training and maintenance expenses, as well as the foregone opportunity cost of the at-risk capital, then this level of minimum subsistence is more than guaranteed at every other quality level.

The purse distribution plan based on the model developed here will help retain owners and curtail the erosion of racehorse supply. A growth in the number of racehorses will result in more racing opportunities and larger more competitive fields; which, in turn, will boost patron wagering and the resultant revenues of the horseracing industry.

An Example

Consider one of the lowest ends of the horse quality spectrum--the \$5,000 claiming horse. Suppose a racemeet operates for six months ($n=180$ days) with per horse training expenses of \$50 per day ($k=\50). Assume that the typical horse competes about every two weeks ($s=12$), and generously set the annual risk-free rate of return at 6% ($\hat{i}=.0025$). Let $m = 55\%$. The minimum subsistence per race return to this lowest class of racehorse would be approximately $\bar{r}_1=.085$.

For illustrative purpose, increase the per race rate of return by .0025 for each \$5,000 incremental increase in market value.² Based on the rate of return relation (5), the result would be the following array of purses for various quality classes:

v_i	\$5,000	\$10,000	\$15,000	\$20,000	\$50,000
p_i	\$3,400	\$7,000	\$10,800	\$14,800	\$43,000

These purse/quality pairings are quite plausible when compared to many purse structures currently in existence.

However, a rate of return cap possibly in the range of about 11%, requiring smaller incremental rate of return increases than those employed in the example, may be

required to keep purses realistic. For example, an 11% cap for premier quality (Grade 1) horses valued at \$400,000 by Beyer/Oppenheim would result in a purse at the highest quality level of approximately \$350,000. Currently, purses at the premier quality level often exceed \$1 million. A maximum purse at the highest quality level would be consistent with some of the recent proposals of respected horseracing industry analysts [Hollingsworth, 1992].

Conclusion

There exists a self-sustaining level of overall purses for a particular racemeet that can be supported by patron wagering. This purse money can be distributed across a broad range of racehorse quality proportional to each class's contribution to a measure of overall racemeet performance quality, whereby the owners of higher quality horses can potentially earn greater returns on their racehorse investments. A minimum subsistence reward ensures that owners of better horses of even the lowest quality can potentially cover training and maintenance expenses and the foregone opportunity costs of at-risk capital.

This purse distribution plan satisfies the objective of retaining owners of the typical unheralded racehorses which are the staple of thoroughbred horseracing, and who are crucial to the preservation of the overall horseracing industry. As owners are retained in the industry, the emerging racehorse undersupply situation can be curtailed, providing a boost to patron wagering and the resultant revenues of the horseracing industry. Also, a purse distribution structure that embodies incentives to upgrade horse quality will cultivate additional patron support.

Suggestions For Future Research

This paper suggests a means of distributing an aggregate purse level to retain owners of racehorses of varying quality -- essentially proposing a method of sharing a "fixed reward pie". The next natural step is to develop of means of increasing the overall level of purses -- or enlarging the reward pie. This, of course, requires an increase in patron wagering. Since patron on-track participation offers little room for additional growth, the racing industry's current emphasis is to cultivate additional patron support through off-track betting and simulcasting systems. While efforts in these areas should continue, possibly the mechanism offering the most potential for enlarging the patron base is interactive home television wagering. Interactive home wagering consists of the in-home viewing of live racing accompanied by a telephone betting system. A model optimally integrating an interactive home wagering system with ongoing racetrack operations would provide

the horseracing industry with a means of growing its patron base while also participating in the ever-expanding in-home informational superhighway network.

Footnotes

1. Consider Brouwer's theorem as presented in [Takayama, 1974]: Let S (the domain set) be a non-empty, compact, convex subset of R^n (n-dimensional real space), and let F be a single-valued continuous function from S into itself. Then there exists a P in S such that $P=F(P)$.

Brouwer's theorem, as illustrated below, simply identifies conditions under which a continuous function which maps its domain into itself (termed "onto") must map a point into itself; i.e., cross a 45-degree line or have a fixed point.

Other than a "well behaved" domain set, the most restrictive condition is that the function be "onto," which is certainly satisfied in (4). Continuity of F is, of course, essential; whereas the single-valued condition eliminates more than one possible fixed point.

Brouwer's theorem is satisfied in this model as reflected by relation (4). The set of aggregate purse offerings (P) is certainly non-empty, closed, bounded, and convex. The subsequent empirical results reveal that horse quality is directly influenced by the purse level, and that patrons wager more on races comprised of horses of better quality. The legislated formulas that determine the required amount of the handle that must be paid in purses ensure that purses are an increasing function of the handle. Since there is a direct relationship between the key variables of V , P , and H in relations (1), (2), and (3), the F function of relation (4) is single-valued, and without loss of generality, is assumed to be continuous.

Brouwer's theorem confirms that there exists an

aggregate level of purses that is self-sustaining. That is, a sufficient number of patrons can be attracted by the overall quality of the races that they wager enough money to support this aggregate purse level.

2. Given the total racemeet purse (P) and the quality measure $\bar{V}$, set $c_1=1.0$. With $\bar{r}_1 = .085$; using relation (5), if $r_1 = \bar{r}_1$ then $P/8\bar{V} = .085$. The .0025 incremental increase in returns per \$5,000 increase in market value effectively sets the remaining c_i parameters employing (5). The $r_1 \geq \bar{r}_1$ condition requires that $P/8\bar{V} \geq .085$. This implies that

$$\frac{\sum R_i p_i}{8 \sum c_i R_i V_i} \geq .085$$

which implies that $\sum R_i p_i \geq .68 \sum c_i R_i V_i$, which would be satisfied if $p_i \geq .68 c_i V_i$ for all classes. That is, the essential $r_1 \geq \bar{r}_1$ condition is satisfied if the per race purse at each quality level is approximately 70% of the market value of a representative horse of each class. This condition is satisfied in the hypothetical example.

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Appendix

An example of the legislated relationship between purse offerings and the total wager is based on that which exists at Belmont Park. For regular win, place, and show wagers at the racetrack:

- total parimutuel tax=17.0% -- 5.0% state, 8.5% racetrack, 3.0% purses, 0.5% breeders.

For daily double, exacta, and quinella wagers at the racetrack:

- total parimutuel tax=17.0% -- 4.0% state, 8.5% racetrack, 4.0% purses, 0.5% breeders.

For triple wagers at the racetrack:

- total parimutuel tax=25.0% -- 7.5% state, 13.0% racetrack, 4.0% purses, 0.5% breeders.

The following set of variable are employed in the regression analysis:

h = the per race handle or the total race wager

v = the market value of a representative horse

p = the per race purse offering

x = the number of the race on the daily program of races

z = the number of horses competing in the race

a = the patron attendance at the racetrack

d = an opportunity dummy reflecting races where patrons have the opportunity of making an additional exotic wager such as a "pick-three," trifecta (triple), or late daily double.

The results for each racetrack are presented below with t-values parenthetically included:

Belmont Park:

$$v = -11.8 + 1.6p + 3.1x \quad R^2=.81 \quad F=494 \quad D.W.=2.27 \quad n=225$$

(2.5) (26.0)

$$h = 148.2 + 0.7v + 18.0z + 16.7a + 198.3d \quad R^2=.73 \quad F=151 \quad D.W.=2.15$$

(8.1) (6.2) (7.6) (15.8)

Churchill Downs:

$$v = -1.8 + 0.7p + 2.0x \quad R^2=.63 \quad F=235 \quad D.W.=2.07 \quad n=275$$

(11.7) (4.7)

$$h = -21.8 + 0.8v + 2.6z + 10.9a + 45.9d \quad R^2=.71 \quad F=167 \quad D.W.=2.26$$

(8.1) (2.8) (17.9) (11.7)

Monmouth Racetrack (no added opportunity (d) existed at the racemeet):

$$v = -0.3 + 0.8p + 2.7x \quad R^2=.86 \quad F=789 \quad D.W.=1.77 \quad n=254$$

(32.8) (8.4)

$$h = -0.5 + 0.5v + 5.3z + 13.2a \quad R^2=.60 \quad F=124 \quad D.W.=1.88$$

(9.4) (3.9) (12.5)

Calder Racetrack:

$$v = -3.4 + 0.8p + 2.4x \quad R^2=.71 \quad F=317 \quad D.W.=2.06 \quad n=253$$

(14.0) (7.8)

$$h = -7.9 + 0.9v + 2.9z + 16.1a + 10.9d \quad R^2=.62 \quad F=102 \quad D.W.=2.35$$

(9.1) (3.4) (14.5) (1.9)

Detroit Race Course:

$$v = -0.8 + 0.7p + 1.2x \quad R^2=.85 \quad F=728 \quad D.W.=1.80 \quad n=247$$

(34.6) (8.2)

$$h = 4.7 + 0.7v + 1.5z + 9.9a + 2.2d \quad R^2=.90 \quad F=570 \quad D.W.=2.05$$

(21.3) (6.3) (36.3) (1.8)

There is no theoretical rationale for pooling the data since the regression parameters are not expected to be the same across racetracks off different class levels. Also, although all races were contested in the Spring and Summer of 1991, the actual time period race dates do not perfectly coincide. Negligible cross-sectional contemporaneous covariances deemed pooling to be inappropriate.