Market Timing For Active Asset Allocation: A Discrete Regression Model Approach

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Abstract

A discrete regression model (DRM) approach to timing the asset class markets for stocks, bonds, and cash in active asset allocation is presented. The technique involves investing in the asset class whose return is predicted to exceed the other asset class return after observing a sequential signal of estimated probabilities. The empirical results show that the DRM approach provides enhanced portfolio performance when compared to more passive fixed-weight portfolio strategies.

I. Introduction

Active asset allocation involves adjusting the percentage of funds invested in various assets (limited to stocks, bonds, and cash in this research) as both the investor's preference toward risk and return and expectations on future asset class risk and return parameters change. Market timing is a form of active asset allocation management that entails shifting an investor's funds between asset classes, depending on the perception of their short-term relative performance, even if there is no change in the investor's long-term attitude toward risk and return. Financial theory does not dictate a singular method for forming expectations, selecting weights, or market timing in active asset allocation.[1] Whether or not a specific active asset allocation strategy can enhance portfolio performance is an empirical issue.[2]

Previous research concludes that asset class returns and differences in asset class returns can be predicted by various economic and financial variables that have no special standing in asset-pricing theory. Chen, Roll, and Ross (1986) examine the correlation between stock returns and several macroeconomic variables that they hypothesize proxy for systematic risk factors. These include unexpected changes in the borne risk premium measured by the spread between returns on low-grade corporate bonds and U.S. Government bonds, the monthly growth rate in industrial production, unexpected inflation, and unexpected changes in the term premium. Their results imply that stock returns are positively correlated with the bond return spread and industrial production, and negatively correlated with inflation and the term premium.

Keim and Stambaugh (1986) and Campbell (1987) show that stock and bond returns are predictable from a common set of stock market and term structure variables which include: (1) the spread between yields on low-grade corporate bonds and one-month Treasury bills, and (2) minus the logarithm of the ratio of the real Standard and Poor's Index to its previous historical average. Keim and Stambaugh (1986) further find that the variables used in their study have predictive power in estimating differences between asset class returns.

Black (1987) argues that the dispersion of stock returns can be used to forecast subsequent investment expenditures which are positively correlated with stock returns. Fama and French (1988) show that the dividend yield on the NYSE value-weighted portfolio is useful in forecasting the returns of corporate bonds and common stocks. Fama and French (1989) argue that any model employing stock market and term structure variables to predict asset class returns must be continually respecified over time. They emphasize that although variation in expected returns with business conditions is plausible and consistent with asset-pricing theory, any evidence of in-sample predictability should be confirmed by out-of-sample tests.

These empirically determined economic and financial variables and discrete regression model (DRM) techniques are used in this study to model the probability of differences between asset class returns. The DRM procedure provides a process for market timing in active asset allocation based on the likelihood that one asset class return exceeds another. Following Fama and French (1989), the procedures used to select the DRM specification allow the model to vary over time as economic conditions change. The procedures are also able to capture a fixed set of factors when economic conditions are stationary, as in Chen, Roll, and Ross (1986).

The remainder of the paper is organized as follows: The DRM approach is presented in Section II; the data and empirical model specifications are presented in Section III; Section IV contains the empirical results and analysis of the risk-adjusted performance of the DRM portfolios relative to more passively managed fixed-weight portfolio strategies; the conclusions are presented in Section V; and suggestions for future research are addressed in Section VI.

II. Discrete Regression Model Methodology

The Dichotomous (Two Asset) Model

The dichotomous DRM method relies on the existence of a qualitative differential in the expected returns between two asset classes. The DRM can be a linear probability, logit, or probit model. It is assumed that the qualitative differential in asset class returns is a function of the economic and financial variables described in the previous section. For a two asset case, the predicted probability that the first asset outperforms the second is $\hat{P}_t$. The predicted probability that the second asset outperforms the first is $1 - \hat{P}_t$.

The DRM methodology is illustrated using the two asset case of stocks and cash and the linear probability model as an example. The dependent variable is

$$Y_t = \begin{cases} 1 \text{ if } R_S > R_C \\ 0 \text{ otherwise,} \end{cases}$$

where R_s is the monthly total return on stocks and R_c is the monthly total return on cash (i.e., the risk-free return on three-month T-bills with one month to delivery). The linear probability model takes the form

$$Y_t = b_0 + b_1 L^1(X_t) + b_2 L^2(X_t) + \ldots + b_j L^j(X_t) + e_t$$

where

$b_1, \ldots, b_j$ are j element coefficient vectors,

L is a lag operator on X, with $L^j X_t = X_{t-j}$, and j is the appropriate lag length,

X_t is a matrix of economic and financial variables, and

e_t is a vector of independently distributed error terms with zero mean and constant variance.

The model is written as

$$Y_t = b_0 + \sum_{i=1}^{j} b_i L^i X_t + e_t$$

$$Y_t = B(L)X_t + e_t .$$

Thus, taking expectations gives

$$E(Y_t) = B(L)X_t ,$$

where E is the expectations operator.

Given the dichotomous nature of Y_t, $E(Y_t)$ measures the expected probability that $Y_t = 1$. Furthermore, as a conditional expectation of Y on X, $E(Y_t \mid (L)X_t)$ can be interpreted as the conditional probability that $R_s > R_c$ given $(L)X_t$. Subsequently, the value of

$$\hat{P}_t = E(Y_t) = \hat{B}(L)X_t \tag{6}$$

is the estimate of this conditional probability. The well-known statistical problems associated with the linear probability model are potentially serious (Pindyck and Rubinfeld (1991)). In particular, it is inappropriate to use the linear probability model to make predictions where fitted values fall outside the unit (0,1) interval. Probit and logit probability models solve those problems by transforming the original model. The lag structure allowed in the DRM permits the forecasting of future probabilities (out-of-sample) using current information.

The estimated probability that $Y_t = 1$ is

$$\hat{P}_t = \text{Prob}[Y_t = 1] = \text{Prob}[R_S > R_C] =$$

$$\text{Prob}[e_t > -B(L)X_t] = 1 - F((L)X_t, B)$$

where F is the cumulative distribution function (CDF). The likelihood function is

$$o = \prod_{Y_t=0} F(X_t, B(L)) \prod_{Y_t=1} [1 - F(X_t, B(L))] .$$

The choice of the probability distribution of e_t determines the functional form of $F(\cdot)$. The probit model is associated with the cumulative normal probability function, where

$$F(X_t, B(L)) = \int_{-\infty}^{B(L)X_t} (2\pi)^{-\frac{1}{2}} \exp(-v^2/2)dv.$$

It then follows that

$$F^{-1}(\hat{P}_t) = B(L)X_t,$$

where F^{-1} is the inverse of the normal CDF, $\hat{P}_t$ is the probability that stocks outperform cash in period t, and $F^{-1}(\hat{P}_t)$ is the probit of the estimated probability of stocks outperforming cash in month t. An alternative to the probit model is the logit model, which is based on the logistic CDF. If the cumulative distribution of e_t is logistic,

$$\text{Prob}[Y_t = 1] = \text{Prob}[R_S > R_C] =$$

$$[\exp(B(L)X_t)] \cdot [1 + \exp(B(L)X_t)]^{-1},$$

and

$$\text{Prob}[Y_t = 0] = \text{Prob}\ [R_C \geq R_S] = [1 + \exp(B(L)X_t)]^{-1}.$$

The log odds of $R_s > R_c$ are

$$\log[\hat{P}_t / (1 - \hat{P}_t)] = B(L)X_t,$$

where

$$\log[\hat{P}_t / (1 - \hat{P}_t)]$$

is the logit representing the estimated odds that stocks will outperform cash in month t based on information available at time t - 1.

Maximum-likelihood estimation procedures are applied to derive consistent coefficient estimators for the probit and logit probability models.[3] With probit and logit CDF's being close to each other, the choice between the probit and logit model is somewhat arbitrary. In the empirical analyses for the two asset cases, the predicted probabilities of the logit and probit models are found to be similar. Because the conclusions of this study are not significantly affected by the specification of the cumulative distribution of e_t only the results from the logit model are presented.

Sequential Signal Rule For Market Timing

The DRM approach can be used as a market timing strategy that may not require monthly adjustments as in a passive fixed-weight strategy such as a 50/50 stocks-cash portfolio. The potential for reducing the number of transactions, relative to fixed-weight strategies, would make the DRM approach attractive even if performance is stationary. The empirical results of this study, however, support a DRM portfolio strategy that has improved performance relative to fixed-weight strategies, in addition to reducing the number of transactions.

In order to implement the DRM approach to market timing in the stocks-cash (bonds) portfolio, the DRM probabilities are analyzed for trends. Since differences in asset class returns cannot be predicted with certainty on a monthly basis, it is assumed that a trend (sequential signal) in the predicted probabilities will give a better indication of the likelihood of differences in asset class returns in subsequent time periods. A vector of monthly stocks-cash (bonds) asset weights is generated based on the following rule:

Let 100 percent of the available funds be the maximum allowable percentage invested in common stocks ($WMAX_S$) and zero percent be the minimum allowable percentage invested in cash (bonds) ($WMIN_{CB}$). Also, let the minimum percentage of funds invested in common stocks ($WMIN_S$) be zero percent. Consequently, the maximum percentage invested in cash (bonds) ($WMAX_{CB}$) is 100 percent.

For each monthly time period:

If $\hat{P}_t < .5$ and $\hat{P}_{t-1} < \hat{P}_{t-2}$, then set weights at ($WMIN_S$) and ($WMAX_{CB}$).

If $\hat{P}_{t+1} \geq .50$ then set weights at ($WMAX_S$) and ($WMIN_{CB}$).

Otherwise, do not change asset class weights from the previous month's settings. The market timing sequential signal strategy results in being either totally invested in common stocks or totally invested in cash (bonds) during any given month, depending on the predicted probabilities generated by the DRM model.[4]

III. Data and Empirical Model Specification

Data

The data in this study are monthly observations for the period of 1970.1 through 1992.12. The dependent variables in the DRM are monthly total returns for the three major asset classes of stocks, bonds, and cash. Total return data for common stocks, bonds (long-term

U.S. government bonds), and cash (T-bills with one month to maturity) are taken from the SBBI/PC file of Ibbotson & Associates (IA). The stock market, term structure, and macroeconomic explanatory variables are derived from the Compustat Price-Dividend-Earnings (PDE) file and the St. Louis Federal Reserve Bank's Federal Reserve Economic Data Base (FRED).

The dependent variable) (Y_t) for the DRM approach to forecasting probabilities is constructed from the following (the data source is noted in parentheses):

R_S = total return on common stocks (IA)

R_B = total return on long-term U.S. government bonds (IA)

R_C = total return on T-bills with one month to maturity (IA)

The choice of the empirical definitions of the explanatory variables (X_t matrix) considered in the DRM, based on the findings cited in Section I, are as follows (the data source and previous research motivation are noted in parentheses):

GOVTBYLD = the spread between yields on long-term U.S. government bonds (FRED) and three-month T-bills (FRED; Chen, Roll, and Ross (1986))

GOVBAAYLD = the spread between yields on long-term U.S. government bonds (FRED) and long-term Baa corporate bonds (FRED; Chen, Roll, and Ross (1986))

BAATBYLD = the spread between yields on long-term Baa corporate bonds (FRED) and three-month T-bills (FRED; Keim and Stambaugh (1986))

SPTBYLD = the spread between the earnings yield on the S&P 500 Index (PDE) and three-month T-bills (FRED)

SPVAR = the variance of the S&P 500 Index during the previous 24 months (PDE; Black (1987))

SPTREND = minus the natural logarithm of the ratio of the real S&P 500 Index to its 24-month previous average (PDE; Keim and Stambaugh (1986))

INDPROD = industrial production measured by the percentage change in the Federal Reserve Board's Index of Industrial Production (FRED; Chen, Roll, and Ross (1986))

INFL = inflation measured by the percentage change in the Consumer Price Index (FRED; Chen, Roll, and Ross (1986))

DIVYLD = the dividend yield of the S&P 500 (PDE; Fama and French (1989))

Empirical Model Specification

Model selection and the performance of the corresponding market timing portfolio, relative to the more passive fixed-weight portfolios, are interrelated issues in the development of the DRM approach. While previous studies have identified influences of the stock market, term structure, and trend variables on asset return differentials, a criterion for selecting regressors must be adopted in order to focus on the potential of the DRM. In addition, since expectations are captured by the lag specification on the explanatory variables, an appropriate lag structure must also be chosen.

In order to make the process of selecting models to generate out-of-sample forecasts of asset class returns more tractable, the number of alternative models considered is reduced by imposing several arbitrary decisions. Hence, the DRM method and the models selected are not claimed to be optimal. First, only models with at least two financial variables and at least one macroeconomic variable are included in the set of alternative models. Second, the maximum lag length is limited to 12 periods (months), and only 48, 72, 96 and 120-month estimation periods are considered. Using data from 1970.1 through 1972.12 to construct the lagged, trend, and risk variables, the data for the first estimation period begins in 1973.1. This provides a sufficient number of observations for model estimation and for examining the out-of-sample forecast performance of the DRM. Even with the imposed modeling restrictions, the DRM approach is found to capture the expected probabilities of qualitative differentials in asset class returns.

A criterion for selecting an appropriate model specification from the alternatives is chosen from the many procedures available (see Judge, et al (1988)). When models are estimated by methods of maximum likelihood, as in the DRM approach, a commonly used information measure for selecting a subset of potential explanatory variables and an appropriate lag structure is Akaike's (1973, 1981) information criterion (AIC). As a measure of fit, the AIC rewards precision of estimation and parsimony in the parameterization (gains in the degrees of freedom) of the model. The AIC is

$$\text{AIC} = -(2/N)\log\ o\left(\hat{B}_1(L)\middle|\,Y\right) + (2k_1/N),$$

where N is the number of observations, $o(\hat{B}_1 \mid Y)$ is the value of the likelihood function, and $\hat{B}_1$ is the partition of the coefficient vector corresponding to the partition of the design matrix $(L)X$, $(L)X_1$. The criterion is to minimize the AIC among the linear hypotheses $RB(L) = 0$. With $R = (0I_s)$, 0 being the null matrix of order $s \times (k - s)$, and the s element identity matrix, I_s, the hypothesis is that the last s elements in $B(L)$ are jointly zero. Even if it were feasible to specify all possible models, the AIC does not assume the existence of a "true" model. As previously stated, other model specifications and techniques for generating market timing decisions may exist which outperform those developed and tested in this study.

Three sets of out-of-sample forecasts of the probability of differential returns in stocks and cash (bonds) will be constructed. To allow the influence of explanatory variables to change over time, forecasts of DRM probabilities are computed in six-month, three-month, and one-month intervals. The procedures applied in selecting the model specifications based on the AIC are as follows:

(1) Alternative models were fitted to the data in 48, 72, 96, and 120-month estimation periods for 1973.1-1976.12, 1973.1-1978.12, 1973.1-1980.12, and 1973.1-1982.12, respectively. The AIC are computed for each model estimated.

(2) The span of observations for each estimation period is shifted forward in six-month intervals.[5] The alternative models are fitted to data for those periods and AIC are computed.

(3) Comparisons of the AIC for the models of differences in stocks and cash returns in the alternative estimation periods show that the models from the 48-month estimation period generally have smaller AIC than the models from the other estimation periods. For stocks and bonds, the 96-month estimation period models were found to have AIC as low or lower than the models in the other periods. The models with the lowest AIC are then used to compute the sequence of six-month out-of-sample forecasts.[6]

To permit the effect of more frequent parameter estimation, sequences of three-month and one-month out-of-sample forecasts can also be computed from the models selected to compute the six-month out-of-sample forecasts. Here we let the span of observations for each of the estimation periods be shifted forward in three-month and one-month intervals, respectively. Consequently, the model specifications are held constant within the six-month intervals, but the coefficients of the variables are allowed to change over time.[7]

IV. Empirical Results

The empirical results indicate that the DRM approach to market timing in active asset allocation enhances portfolio performance relative to more passive fixed-weight portfolio strategies over the time period studied. Table 1 provides an example of the estimated probabilities, $\hat{P}_t$, generated by the DRM technique for the stocks-cash case from 1987.1 to 1992.12. Notice that the estimated probability of stocks outperforming cash drops from 100 percent in July 1987 to 40.8 percent in September 1987. The estimated probability of stocks outperforming cash is 2.6 percent in October 1987 (the crash) and then rebounds in November 1987 to 100 percent. The sequential signal from the DRM model ahead of the October 1987 crash is taken as empirical evidence of its usefulness. Similar sequential signals are observed prior to the July 1990, June 1991, and November 1991 declines in the S&P 500 Index.

The overall mean monthly returns $(\overline{R})$, standard deviation of monthly returns (σ), and coefficients of variation $(\sigma/\overline{R})$ for fixed-weight (passive) portfolio strategies and the DRM active asset allocation strategy using a six-month estimation period are presented in Table 2. The results from the market timing DRM strategy show significant improvement in performance for both the stocks-cash and stocks-bonds portfolios. Over the period 1977.1 through 1992.12, the DRM strategy for the stocks-cash portfolio produces a mean monthly return of 1.42 percent (18.4 percent annually). This is compared to the mean monthly return of 1.19 percent (15.3 percent annually) for the 100 percent stocks strategy. A 3.1 percentage point increase in annualized average return over the S&P 500 Index (100 percent stocks strategy), as well as a reduction in portfolio risk, are realized. The 3.1 percentage point difference in yearly returns is significant at the 5 percent level in a two-tailed t-test.[8] Based on Bartlett's test for differences in variance, the standard deviation of monthly returns of 3.27 percent for the market timing DRM strategy is less than the standard deviation of monthly returns of 4.49 percent for the 100 percent stocks strategy at the 5 percent level of significance.[9] The coefficient of variation of 2.29 for the market timing DRM strategy is the lowest of all the risky stocks-cash strategies examined. The market timing DRM approach, therefore, outperforms the 100 percent stocks (buy-and-hold) and the 50/50 fixed-weight strategies considered. Other fixed-weight strategies, such as 60/40, 40/60, 75/25, and 25/75, are also investigated and similar results hold, but are not presented for space consideration.

Over the period 1981.1 through 1992.12, the DRM strategy for the stocks-bonds portfolio produces a 1.55 percent (20.1 percent annually) mean monthly return. This is compared to the mean monthly return of 1.25

Table 1

An Example of the Estimated Probabilities Produced by the DRM Method for the Stocks-Cash Portfolio Over the Period 1987.1 to 1991.12

Month	R_S	R_C	P	$\hat{P}$	W_S
1987.1	13.18	0.42	1.00	1.00	1.00
1987.2	3.69	0.43	1.00	1.00	1.00
1987.3	3.38	0.47	1.00	1.00	1.00
1987.4	-1.15	0.44	0.00	1.00	1.00
1987.5	0.60	0.38	1.00	1.00	1.00
1987.6	5.57	0.48	1.00	1.00	1.00
1987.7	4.82	0.46	1.00	1.00	1.00
1987.8	3.50	0.47	1.00	1.00	1.00
1987.9	-1.74	0.45	0.00	0.41	0.00
1987.10	-21.76	0.60	0.00	0.03	0.00
1987.11	-8.53	0.35	0.00	1.00	1.00
1987.12	8.25	0.39	1.00	1.00	1.00
1988.1	4.04	0.29	1.00	0.20	1.00
1988.2	4.18	0.46	1.00	1.00	1.00
1988.3	-2.49	0.44	0.00	1.00	1.00
1988.4	0.94	0.46	1.00	0.90	1.00
1988.5	0.32	0.51	0.00	1.00	1.00
1988.6	5.29	0.49	1.00	1.00	1.00
1988.7	-0.54	0.51	0.00	0.00	1.00
1988.8	-3.86	0.59	0.00	1.00	1.00
1988.9	4.91	0.62	1.00	0.00	1.00
1988.10	2.60	0.61	1.00	0.00	0.00
1988.11	-1.89	0.57	0.00	0.00	0.00
1988.12	2.37	0.63	1.00	0.00	0.00
1989.1	7.11	0.55	1.00	1.00	1.00
1989.2	-2.89	0.61	0.00	1.00	1.00
1989.3	2.95	0.67	1.00	0.00	1.00
1989.4	5.01	0.67	1.00	0.00	0.00
1989.5	3.51	0.79	1.00	0.07	0.00
1989.6	0.10	0.71	0.00	1.00	1.00
1989.7	8.84	0.70	1.00	0.66	1.00
1989.8	1.55	0.74	1.00	0.65	1.00
1989.9	0.15	0.65	0.00	0.63	1.00
1989.10	-2.52	0.68	0.00	0.43	0.00
1989.11	1.65	0.69	1.00	0.66	1.00
1989.12	2.97	0.61	1.00	0.70	1.00

Table 1 (continued)

Month	R_S	R_C	P	$\hat{P}$	W_S
1990.1	-6.88	0.57	0.00	0.70	1.00
1990.2	0.85	0.57	1.00	0.72	1.00
1990.3	3.26	0.64	1.00	0.70	1.00
1990.4	-2.69	0.69	0.00	0.60	1.00
1990.5	9.20	0.68	1.00	0.63	1.00
1990.6	0.00	0.63	0.00	0.52	1.00
1990.7	-0.52	0.68	0.00	0.42	0.00
1990.8	-9.43	0.66	0.00	0.43	0.00
1990.9	-4.19	0.60	0.00	0.55	1.00
1990.10	-0.67	0.68	0.00	0.76	1.00
1990.11	5.99	0.57	1.00	0.77	1.00
1990.12	3.45	0.60	1.00	0.68	1.00
1991.1	4.15	0.52	1.00	1.00	1.00
1991.2	6.73	0.48	1.00	0.83	1.00
1991.3	2.98	0.44	1.00	0.77	1.00
1991.4	0.03	0.53	0.00	1.00	1.00
1991.5	3.86	0.47	1.00	0.60	1.00
1991.6	-3.96	0.42	0.00	0.00	0.00
1991.7	4.49	0.49	1.00	0.00	0.00
1991.8	1.96	0.46	1.00	0.00	0.00
1991.9	-1.12	0.46	0.00	0.29	0.00
1991.10	1.19	0.42	1.00	0.00	0.00
1991.11	-4.39	0.39	0.00	0.00	0.00
1991.12	11.97	0.38	1.00	1.00	1.00

percent (16.1 percent annually) for the 100 percent stocks strategy and 1.14 percent (14.57 percent annually) for 100 percent bonds. The 4.0 percentage point increase in annualized average return over common stocks is accompanied by a reduction in portfolio risk. The standard deviation of monthly returns of 3.84 percent for the market timing DRM strategy is less than the standard deviation of monthly returns of 4.56 percent for the 100 percent common stocks strategy. The coefficient of variation of 2.48 is the lowest of the risky stocks-bonds strategies considered.

The overall mean monthly returns ($\overline{R}$), standard deviation of monthly returns (σ), and coefficients of variation ($\sigma/\overline{R}$) for the DRM active asset allocation strategy with changing estimation periods are presented in Table 3. The results from the market timing DRM strategy show that allowing for more frequent estimation periods permits some improvement in the stocks-cash portfolio performance, but it is not statistically significant. Over the period 1977.1 to 1992.12, the DRM strategy for the stocks-cash portfolio produces a mean monthly return of 1.42 percent (18.4 percent annually) when the parameters are re-estimated every six months, 1.47 percent (19.1 percent annually) when the parameters are re-estimated every three months, and 1.49 percent (19.4 percent annually) when the parameters are re-estimated on a monthly basis. The corresponding coefficients of variation for these three estimation techniques are 2.29, 2.42, and 2.44, respectively. Practically no improvement is noted in the stocks-bonds portfolio, as shown in Table 3, Panel B. The most important results are, however, that all of the DRM strategies investigated outperform the 100 percent stocks, 100 percent bonds, and 50/50 fixed-weight portfolio strategies. More frequent parameter estimation does not

Table 2

A. Comparative Return and Risk Measures for Stocks-Cash Portfolios (Six-Month Estimation Period)

There are 192 months of data extending over the time period 1977.1 to 1992.12. The monthly returns for the DRM portfolio result from the market timing decision using the DRM approach discussed in the paper. The monthly returns for the fixed-weight portfolios result from the weights remaining constant each month.

	Portfolio			
	(1) 100% Stocks	(2) 100% Cash	(3) 50% Stocks-50% Cash	(4) DRM Method
Average Monthly Return $(\overline{R})$ (%)	1.19	.65	.92	1.42
Standard Dev. of Monthly Returns (σ) (%)	4.49	.23	2.24	3.27
Coefficient of Variation $(\sigma/\overline{R})$	3.77	-----	2.44	2.29

B. Comparative Return and Risk Measures for Stocks-Bonds Portfolios (Six-Month Estimation Period)

There are 144 months of data extending over the time period 1981.1 to 1992.12. The monthly returns for the DRM portfolio result from the market timing decision using the DRM approach discussed in the paper. The monthly returns for the fixed-weight portfolios result from the weights remaining constant each month.

	Portfolio			
	(1) 100% Stocks	(2) 100% Bonds	(3) 50% Stocks-50% Bonds	(4) DRM Method
Average Monthly Return $(\overline{R})$ (%)	1.25	1.14	1.19	1.55
Standard Dev. of Monthly Returns (σ) (%)	4.56	3.48	3.37	3.84
Coefficient of Variation $(\sigma/\overline{R})$	3.65	3.06	2.83	2.48

seem to cause the DRM portfolio performance to improve. This may be due to economic and financial factors being somewhat stable over six-month increments of time.

V. Conclusions

A discrete regression model (DRM) approach for market timing in active asset allocation portfolio management has been presented. The principal finding of this study is that the DRM approach provides portfolio performance superior to passive fixed-weight strategies of 100 percent stocks and 100 percent bonds over the sample time period. This enhanced performance is a result of both the reduction in monthly return variance and an increase in return associated with the actively-managed DRM portfolio. The ability to reduce the risk is especially useful in portfolio management. If the higher risk levels of the passive strategies can be tolerated by investors, then leverage may be employed to capture even higher levels of return under the active asset allocation strategy. The market timing DRM approach, due to the fewer number of trades required, also offers the potential to reduce the number of monthly transactions associated with the fixed-weight strategies, since most fixed-weight strategies require monthly rebalancing.

Table 3

A. Comparative Return and Risk Measures for Stocks-Cash Portfolios (Changing Estimation Periods)

There are 192 months of data extending over the time period 1977.1 to 1992.12. The monthly returns for the DRM portfolio result from the market timing decision using the DRM approach discussed in the paper.

	Portfolio		
	(1) DRM (6 Mo. Est.)	(2) DRM (3 Mo. Est.)	(3) DRM (1 Mo. Est.)
Average Monthly Return $(\bar{R})$ (%)	1.42	1.47	1.49
Standard Dev. of Monthly Returns (σ) (%)	3.27	3.56	3.55
Coefficient of Variation $(\sigma/\bar{R})$	2.29	2.42	2.44

B. Comparative Return and Risk Measures for Stocks-Bonds Portfolios (Changing Estimation Periods)

There are 144 months of data extending over the time period 1981.1 to 1992.12. The monthly returns for the DRM portfolio result from the market timing decision using the DRM approach discussed in the paper.

	Portfolio		
	(1) DRM (6 Mo. Est.)	(2) DRM (3 Mo. Est.)	(3) DRM (1 Mo. Est.)
Average Monthly Return $(\bar{R})$ (%)	1.55	1.52	1.55
Standard Dev. of Monthly Returns (σ) (%)	3.84	3.87	3.81
Coefficient of Variation $(\sigma/\bar{R})$	2.48	2.54	2.46

VI. Suggestions for Future Research

The basic goal of this research is to improve the performance of a portfolio of common stocks and cash by considering a dynamic market timing strategy based on a DRM technique. Areas for future research include: (1) testing the DRM technique for alternative stock indices-cash portfolios, (2) expanding the DRM technique to handle more than two asset classes in the portfolio, and (3) investigating the effectiveness of the DRM technique in international markets. Since previous literature primarily focuses on the statistical significance of the predictable variation in security returns, additional empirical research is needed to measure and test the economic significance of this variation.

****Endnotes***

1. The traditional Markowitz (1959) formulation of this problem is stated in terms of the expected return and variance of an asset class. In order to achieve the most efficient portfolio, asset classes are combined so as to minimize variance for a given level of expected return. While the principle of identifying portfolios of various asset classes with the required risk and return characteristics is theoretically well defined, it is not clear just how to predict these parameters.
2. We do not claim that the DRM approach is optimal. Other probability choice models and strategies may outperform the specific methodology presented.
3. For the general case, exact (small sample) properties of the MLE (unbiasedness, efficiency, normality) cannot be established. Under conditions typical-

ly encountered, however, those properties hold approximately, with the quality of the approximations improving as the sample size grows. That is, the MLE exhibits the asymptotic (large sample) properties of unbiasedness, efficiency, and normality. Also, since the likelihood equations for probit and logit are nonlinear in the parameters to be estimated, algebraic solutions are not obtainable. Rather, approximations by standard iterative algorithms are used. For a more complete discussion of maximum-likelihood estimation of probit and logit, see Greene (1990).

4. The focus of this research is on the short-term relative performance between asset classes and not on the investor's long-term attitude toward risk and return. A strategy of being totally invested in a particular asset class is intended to make this focus more clear.
5. The second 48, 72, 96, and 120-month estimation periods correspond to 1973.7-1977.6, 1973.7-1979.6, 1973.7-1981.6, and 1973.7-1983.6, respectively.
6. The stocks-cash models and the stocks-bonds models generate 192 (1977.1-1992.12) and 144 (1981.1-1992.12) monthly forecasts, respectively.
7. The model specifications for each estimation period are available from the authors upon request.
8. The hypothesis to be tested is that two populations have the same mean (variances not equal). The sampling distribution of the statistic

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/N_1) + (\sigma_2^2/N_2)}}$$

is normal with mean 0 and variance 1. This is approximately true for sufficiently large values of N_1 and N_2 even if the populations are not normal. In this research, $N_1 = N_2 = 192$ for stocks. If the values of σ_1 and σ_2 are not known, but the experimenter feels that there is evidence that each population is normally distributed, then he may wish to use the statistic obtained by substituting the observed s_1 and s_2 for σ_1 and σ_2, obtaining the statistic

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{(s_1^2/N_1) + (s_2^2/N_2)}}$$

which, if the assumptions of normality are correct, has approximately a t distribution with f degrees of freedom where

$$f = \frac{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}\right)^2}{\frac{\left(\frac{s_1^2}{N_1}\right)^2}{N_1+1} + \frac{\left(\frac{s_2^2}{N_2}\right)^2}{N_2+1}} - 2$$

for
$$\bar{X}_1 = 18.4\%;\ s_1^2 = 12 \cdot 10.69 = 128.28\%^2$$
$$\bar{X}_2 = 15.3\%;\ s_2^2 = 12 \cdot 20.16 = 241.92\%^2$$
$$N_1 = N_2 = 192$$
$$t = 2.23$$
(significant at the 5 percent level).

9. The hypothesis to be tested is that the variances of k normally distributed populations are equal. The samples are of size n_i where

$$\Sigma n_i = N.$$

The variance of the ith sample is denoted by s_i^2. Let

$$M = (N-k)\ln(s_p)^2 - \Sigma[(n_i - 1)\ln(s_i)^2]$$
$$s_p^2 = [\Sigma(n_i - 1)(s_i)^2]/(N-k)$$
$$A = 1/(3(k-1)) \cdot [\Sigma(1/(n_i - 1)) - (1/(n-k))]$$
$$v_1 = k - 1$$
$$v_2 = (k+1)/A^2$$
$$b = v_2/(1 - A + (2/v_2))$$

Then the sampling distribution of

$$F = (v_2 M)/(v_1(b - M))$$

is approximately $F(v_1, v_2)$. Although this is an approximate formula, it gives sufficient accuracy for practical purposes for any size, n_i, samples.

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