Inventory Valuation Under Cyclical Demand: A Modelling Approach

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Abstract

Traditional EOQ models are limited in use because of the constant demand assumption. Firms rarely face constant demand and, therefore, these firms have restrictive use for the EOQ model. The proposed MIPM model in contrast can project orders and schedules for both variable and constant demand. The MIPM model capitalizes on state of the art programming techniques and current computer software. It permits firms to determine the minimum inventory costs in all product demand situations. In addition, it can be used to improve cash planning and to assist in optimized product contribution to firm profits through sensitivity analysis.

Introduction

The need to control costs is of major concern to all types of businesses. One of the primary controllable costs for businesses, both retail and manufacturing, is associated with inventory investment and management. These costs include both acquisition costs and the related holding period costs. Holding period costs includes items such as insurance, storage, and taxes. Holding costs can also include the time value of money. Thus, firm's that finance inventory purchases may have higher inventory holding costs and therefore, gain more by minimizing holding costs. In a competitive market cost containment goals can be attained by keeping inventory to a minimum level. In effect, maintaining inventory levels adequately minimizes total costs.

It is commonplace that managers need to balance the costs of keeping inventory with the cost of being out of stock when a customer wishes to make a purchase, thereby risking both the loss of that customer's loyalty and associated profit with a lost sale. As most retailers know, a great investment of time and effort is made to attract customers. It makes little sense to risk losing customers to the competition simply by not having what that person wants when he/she is ready to buy. On the other hand, keeping inventory that does not meet customer demand is expensive, especially if that merchandise is readily available from suppliers. There exists a need to balance the costs of holding inventory with the costs of being out of stock.

Economic Order Quantity (EOQ) models have been frequently used to balance these conflicting needs.

Recent extensions of the traditional EOQ model (Tersine and Berman, 1991; Masters, 1991; Burwell et al, 1991) rely on two assumptions: (1) Demand occurs at a constant rate over time, and (2) Replenishment orders are for a fixed quantity Q (lot size).

These two assumptions appear too restrictive for real world situations. This paper examines the situation in which the demand for a product is seasonal. Due to the variable demand, the optimal lot size will no longer be fixed. Therefore, there is a need for a new model to determine the economic order quantity and the complete schedule of reordering for the year. Based on the information in this schedule, a manager would also be able to project the cash inflow and outflow for the product. This information should be valuable for product planning, cost minimization and associated cash flow management.

Typical EOQ models are constrained by the assumption of a fixed demand function. This constraint appears to be too restrictive for many real world inventory ordering applications. Our proposed model accommodates a fluctuating demand situation and permits subsequent cost minimization resulting from the solution. The application that will presented is a simple example but the model can readily be adapted to address complex inventory situations. This model views cost minimization to include all inventory holding costs. Implicit in holding costs is the time value of money. This implies that the relationship between the time value of money is dependant on a varying inventory carry. In

other words, the holding cost for an item will include the parameters of carry, security, and condition which will be impacted by the time value of money.

The proposed EOQ model is dynamic in nature but differs from dynamic production models (Swoveland (1975), Newson (1975)). Both types of models allow demand to change over time. However, the objective of the dynamic production model is to determine the production volume at each time period while the dynamic EOQ model tries to solve the retailer's problem of determining the order size and the scheduling of the reordering. Since most retailers obtain quantity discounts from suppliers, a quantity discount schedule will also be incorporated into the proposed model.

For convenience, two simplifying assumptions are imposed on the model: (1) instant delivery of the order to the retailer and (2) no stock shortages at the retail level(this implies that the retailer will not be placed in a situation where product will not be available). The input needed to operationalize the proposed model includes: (i) Demand at each point in time, (ii) The ordering cost per order, (iii) Unit inventory holding cost, and (iv) The quantity discount schedule provided by the supplier. The output includes solutions to the following questions: (i) The best times for retailers to order; (ii) Should the retailer order more to qualify for quantity discounts?; (iii) For each order placed, what quantity should be ordered?; (iv) What is net cash flow for each time period?

Limits of Traditional EOQ Model

The problem encountered with traditional inventory models is that the demand rate is assumed to be constant. This is generally not the situation that exists in the retail market. Although it is possible that some inventory items may have a constant demand, most inventory items will be subject to a cyclical demand structures.

To address these concerns, the next section identifies a new methodology for determining optimal order quantities when demand is not constant and order quantities are allowed to vary. The determination of optimal order quantities is calculated using a dynamic EOQ model.

A Mixed Integer Programming Formulation of the EOQ Model for Cyclical Demand

To illustrate the new dynamic EOQ model, a planning horizon of ten time periods is used. C_h represents the holding cost and C_o is the order cost. Demand at time i is denoted by D_i. Q_i is the order size at time i and I_i is the inventory at the end of time i. N_i (binary variable) represents the decision to order at time i as follows: N_i = 1 if order is entered of size Q_i at the beginning of

time I. N_i = O otherwise.

Instantaneous delivery is assumed so that Q_i units will be available at the beginning of time i. If no order is placed in a particular period, Q_i would be zero.

In this new model, D_i can vary over different time periods, and the demand by period is assumed to be known to the manager. Based on the input of D_i, C_o, and C_h, the new model can be used to construct an optimal schedule of reordering.

The total number that should be placed in 10 periods is: $N_1 + N_2 + ...N10$. The daily average inventory holding at time period i is $(I_{i-1} + Q_i + I_i)/2^i$. Therefore, the total ordering and holding cost is

$$(\Sigma \, N_i) \, C_o + (C_h/10) \, \Sigma \, (I_{i-1} + Q_i + I_i)/2 \qquad (1)$$

where $C_h/10$ is the holding cost for one unit of inventory for one time period.

Relationships among D_i, I_i, and Q_i are given as follows: Beginning inventory + orders placed = demand + ending inventory

i.e. $I_{i-1} + Q_i = D_i + I_i \qquad\qquad i = 1...10 \qquad (2)$

If an order at time i is not placed (N_i = 0), Q_i = 0, is required otherwise Q_i £ annual demand is desired. This implies

$$Qi \leq 5000 \, N_i \qquad\qquad i = 1,...10 \qquad (3)$$

where the total demand in these ten periods is assumed to be 5,000 units. Subsequent EOQ and the schedule of reordering can be determined by minimizing the total cost in (1) subject to the constraints in (2) and (3). This minimization procedure is the standard mixed integer programming problem which can be solved using existing software(Lindo).

A computerized solution to this type of inventory management problem is warranted due to the following: 1) as the number of inventory items increases the computational aspects of the computer will be needed to develop a solution 2) with complex inventory situations the degree of order times will increase as will the relationship to order cycles, this will produce a large matrix of order times by cycles that will be difficult to solve without a computer 3) changing consumer demand for a product will necessitate the need for a dynamic solution to be generated 4) many pricing programs will contain numerous discount options that change rapidly 5) the ability to study the sensitivity of the cash flow implications of the cyclical demand situation by product and cycle time will require the use of a computer solution.

Table 1: Numerical Results

Case 1:   Constant demand, C_o = 49, C_h = 1 (Order Quantity Minimum = 1000)

Period	1	2	3	4	5	6	7	8	9	10	Total
Demand	500	500	500	500	500	500	500	500	500	500	5000
Order	Y	N	Y	N	Y	N	Y	N	Y	N	5
Order size	1000		1000		1000		1000		1000		5000
Beg Inventory	0	500	0	500	0	500	0	500	0	500	2500
End Inventory	500	0	500	0	500	0	500	0	500	0	2500
Avg Inventory	750	250	750	250	750	250	750	250	750	250	5000
Ordering Cost	49	0	49	0	49	0	49	0	49	0	245
Holding Cost	75	25	75	25	75	25	75	25	75	25	500
Purchase Cost	4,850	0	4,850	0	4,850	0	4,850	0	4,850	0	24,250
Cash Outflow	4,974	25	4,974	25	4,974	25	4,974	25	4,974	25	24,995

Cyclical Demand and Quantity Discount

An extended EOQ model when quantity discounts are available is now presumed. For illustration, the following discount schedule is used:

Discount Schedule

Discount Category	Order Size	Discount	Unit Cost
A	0-999	0%	$.500
B	1000-2499	3%	$4.85
C	2500 and over	5%	$4.75

The decisions of no price discount, a 3% discount or a 5% discount at time i are represented by three integer variables A_i, B_i, C_1 which are defined as follows:

A_i = 1 if $Q_i \leq$ 999   i = 1,...10
      = 0 otherwise

B_i = 1 if 1000 $\leq Q_i \leq$ 2499   i = 1,...10
      = 0 otherwise

C_i = 1 if $Q_i \geq$ 2500   i = 1,...10
      = 0 otherwise

Q_i can only be in one of the three discount categories, therefore;

$A_i + B_i + C_i$ = 1   i= 1,...10   (4)

Then, let QA_i, QB_i, and QC_i be the order size corresponding to situations where no price discount is in effect, a 3% discount and a 5% discount respectively, i.e.:

QA_i = Q_i if $Q_i \leq$ 999   (i.e. A_i = 1)
      = 0 otherwise   (i.e. A_i = 0)

QB_i = Q_i if 1000 $\leq Q_i \leq$ 2499   (i.e. B_i = 1)
      = 0 otherwise   (i.e. B_i = 0)

QC_i = Q_i if $Q_i \geq$ 2500   (i.e. C_i = 1)
      = 0 otherwise   (i.e. C_i = 0)

(5)

Since $A_i + B_i + C_i$ = 1, only one of these variables (i.e. QA_i, QB_i, QC_i) is nonzero. Consequently,

Table 2:   Numerical Results

Case 2: Variable Demand (5 cycles) C_o = 49, C_h = 1 (Order Quantity Varies)

Period	1	2	3	4	5	6	7	8	9	10	Total
Demand	250	750	250	750	250	750	250	750	250	750	5,000
Order	Y	Y	N	Y	N	Y	N	Y	N	N	5
Order size	250	1,000		1,000		1,000		1,750			5,000
Beg Inventory	0	0	250	0	250	0	250	0	1,000	750	2,500
End Inventory	0	250	0	250	0	250	0	1,000	750	0	2,500
Avg Inventory	125	625	125	625	125	625	125	1,375	875	375	5,000
Ordering Cost	49	49	0	49	0	49	0	49	0	0	245
Holding Cost	12.5	62.5	12.5	62.5	12.5	62.5	12.5	137.5	87.5	37.5	500
Purchase Cost	1,250	4,850	0	4,850	0	4,850	0	8,487.5	0	0	24,287.5
Cash Outflow	1,311.5	4,961.5	12.5	4,961.5	12.5	4,961.5	12.5	8974	87.5	37.5	25,032.5

$$Q_i = QA_i + QB_i + QC_i \qquad i = 1,...10 \qquad (6)$$

Moreover, the purchase cost at time i is (PC_i):

$$PC_i = \$5 \ QA_i + \$4.85 \ QB_i + \$4.75 \ QC_i \qquad i = 1,...10 \quad (7)$$

From (7), QA_i and A_i are not independent and their relationship is

$$QA_i \leq 999 \ A_i \qquad i = 1,...10 \qquad (8)$$

Similarly,

$$1000 \ B_i \leq QB_i \leq 2499 \ B_i \qquad i = 1,...10$$
$$2500 \ C_i \leq QC_i \leq 5000 \ C_i \qquad i = 1,...10 \qquad (9)$$

Finally, the objective function is expanded to include the total purchase cost (TC) as follows:

$$TC = (\Sigma \ N_i) \ C_o + (C_h/10) \ \Sigma \ (I_{i-1} + Q_i + I_i/2 + \Sigma \ PC_i \qquad (10)$$

Therefore, the EOQ and ordering schedule are chosen to minimize the total cost in (10) subject to the demand constraints (i.e. (2) and (3)) and the quantity discount constraints ((4), (6), (7), (8), (9)).

Results of Mixed Integer Programming Model

For the first case, the mixed integer programming

model (MIPM) was used to develop the EOQ and scheduling for a constant demand situation. The results obtained would have been the same as those developed using the traditional EOQ model.

Table 1 contains the results of the initial case. Constant demand is assumed and order costs are $49 per order. Inventory holding costs are given at $1 per unit and total costs are $24,995. This result was anticipated because the total annual costs in a stable demand situation should be the lowest possible total cost. Conversely as demand fluctuates, costs of holding inventory to meet changing demand would increase total costs.

Another aspect of obtaining results consistent with traditional EOQ models is the implied verification of the adequacy of the proposed MIPM model. Similarly, since constant demand is a special case of product demand, the MIPM model can be viewed as a general model for all EOQ applications.

Since the traditional EOQ model cannot adequately address a fluctuating demand situation, the proposed MIPM model is used in the following sections to illustrate results for various fluctuating demand adaptations. The simplified examples selected are intended to illustrate the MIPM model's ability to handle varying

Table 3: Numerical Results

Case 3: Variable Demand (2 cycles) C_o = 49, C_h = 1 (Order Quantity Varies)

Period	1	2	3	4	5	6	7	8	9	10	Total
Demand	500	500	1,000	500	250	250	500	1,000	500	250	5,000
Order	Y	Y	N	N	N	N	Y	Y	N	N	4
Order size	250	2,500					1,000	1,250			5,000
Beg Inventory	0	0	2,000	1,000	500	250	0	500	750	250	5,250
End Inventory	0	2,000	1,000	500	250	0	500	750	250	0	5,250
Avg Inventory	125	2,250	1,500	750	375	125	750	1,250	500	125	7,750
Ordering Cost	49	49	0	0	0	0	49	49	0	0	196
Holding Cost	12.5	225	150	750	37.5	12.5	75	125	50	12.5	775
Purchase Cost	1,250	11,875	0	0	0	0	4,850	6,062.5	0	0	24,037.5
Cash Outflow	1,311.5	12,149	150	75	37.5	12.5	4,974	6,236.5	50	12.5	25,008.5

demand and cost variables.

In case two, demand is assumed to be variable and includes five distinct cycles. To be consistent with prior examples, the variables of order cost, inventory holding costs and discount schedule are unchanged.

The results obtained in minimizing total annual costs in the variable demand situation increase from $24,995 to $25,033 (see Table 2). Intuitively, variable demand would be expected to increase costs to the firm because of the need to balance inventory, order costs, holding costs, and discounts. Variable demand would force the firm to vary order quantities, thereby, affecting discounts taken. This is the situation because in case two the only change in total costs relates to a discount not taken for an order placed in period one.

Use of the MIPM model in a variable demand situation is also depicted in case three. In this case, costs again are held constant. The only change is that there are two distinct cycles in product demand. The results of scenario 3 are presented in Table 3.

The MIPM model projects the requirement of placing four separate orders for inventory. In contrast, both previous examples resulted in five orders being placed. Total costs for the two cycle demand situations are $25,009. Fewer cycles result in a lower total cost. This finding also supports the initial finding that the lowest inventory cost possible occurs in situations where demand is stable. Cost increases as the variations in demand increases.

To further illustrate the robustness of the MIPM model, case four, another cyclical demand situation is also determined. In this example, order costs are increased to $99 per order. All other variables remain unchanged. The results indicate that 3 orders will be placed during the period of different quantities. Total costs of $25,185 are determined to be the lowest cost possible in this situation. The results indicate that large order costs tend to encourage firms to minimize the number of orders placed (see Table 4).

To contrast the result of example four, case five, a cyclical demand situation, is determined. In this case, holding costs are increased to $2 and order costs remain at $49. Other variables remained unchanged. Results from this case indicate that the number of orders placed will be seven and total costs will be $25,568. These comparative results are shown in Table 5. As holding costs increase, there is a tendency to minimize total inventory by ordering smaller quantities more frequently.

The first case presented in this paper addresses

Table 4: Numerical Results

Case 4: Variable Demand (2 cycles) C_o = 99, C_h = 1 (Order Quantity Varies)

Period	1	2	3	4	5	6	7	8	9	10	Total
Demand	250	500	1,000	500	250	250	500	1,000	500	250	5,000
Order	Y	N	Y	N	N	N	N	Y	N	N	3
Order size	750	0	2,500	0	0	0	0	1,750	0	0	5,000
Beg Inventory	0	500	0	1,500	1,000	750	500	0	750	250	5,250
End Inventory	500	0	1,500	1,000	750	500	0	750	250	0	5,250
Avg Inventory	625	250	2,000	1,250	875	625	250	1,250	500	125	7,750
Ordering Cost	99	0	99	0	0	0	0	99	0	0	297
Holding Cost	62.5	25	200	125	87.5	62.5	25	125	50	12.5	775
Purchase Cost	3,750	0	11,875	0	0	0	0	8,487.5	0	0	24,112.5
Cash Outflow	3,911.5	25	12,174	125	87.5	62.5	25	8,711.5	50	12.5	25,184.5

constant demand situations for EOQ modeling. The results obtained with the MIPM model for constant demand were identical with the results obtained by the traditional EOQ model. Four separate cases which addressed cyclical demand were also presented. The MIPM model is capable of handling these situations whereas the traditional EOQ model is not.

Additional Applications

The MIPM model has the ability to handle at least two additional planning applications. The first application is to calculate net cash flow for the various products for specific demand situations. The second application is to determine sensitivity analysis for various selling prices with their associated demand.

The first application of computing net cash flow is a simple extension to the model. Because of cyclical demand, total annual costs (cash outflows) vary and are difficult to compute manually. Since the MIPM model can determine total annual costs for cyclical demand, the occurrence of projecting cash inflows and cash outflows would be improved over existing methods.

The second application is to use the model to determine sensitivity analysis for different selling prices and demands. The model would then be used to determine the optimum selling price for a product. This optimum selling price would be determined based on contributions to firm profits. In effect, the model would project contributions for each possible selling price and associated demand. The firm would then choose selling price for the product which optimized contributions to firm profits. Although this can be calculated manually, accuracy of the model in projecting total annual costs would ensure decision makers have accurate and timely cost data when making pricing decisions.

Conclusions

The traditional EOQ model is limited in use because of the constant demand assumption. Firms rarely face constant demand and, therefore, these firms have restrictive use for the EOQ model. The proposed MIPM model in contrast can project orders and schedules for both variable and constant demand. The MIPM model capitalizes on state of the art programming techniques and current computer software. It permits firms to determine the minimum inventory costs in all product demand situations. In addition, it can be used to improve cash planning and to assist in optimized product contribution to firm profits through sensitivity analysis.

Table 5: Numerical Results

Case 5: Variable Demand (2 cycles) C_o = 49, C_h = 2 (Order Quantity Varies)

Period	1	2	3	4	5	6	7	8	9	10	Total
Demand	250	500	1,000	500	250	250	500	1,000	500	250	5,000
Order	Y	Y	Y	Y	N	N	Y	Y	N	Y	7
Order size	250	500	1,000	1,000			500	1,500		250	5,000
Beg Inventory	0	0	0	0	500	250	0	0	500	0	1,250
End Inventory	0	0	0	500	250	050	0	500	000	0	1,250
Avg Inventory	125	250	500	750	375	125	250	1,000	250	125	3,750
Ordering Cost	49	49	49	49	0	0	49	49	0	49	343
Holding Cost	25	50	100	150	75	25	50	200	50	25	750
Purchase Cost	1,250	2,500	4,850	4,850	0	0	2,500	7,275	0	1,250	24,475
Cash Outflow	1,324	3,049	4,999	5,049	75	25	3,049	7,524	50	1,324	25,568

This approach to cyclical demand problems is dynamic; which allows managers to address inventory issues, pricing decisions and market variations simultaneously. In extending traditional EOQ and MRP models firm decision makers can be fully informed about quantity discounting and its complete impact on inventory ordering and carry.

A computerized solution that addresses the inventory valuation aspects of a cyclical demand situation is required due to the increasing complexity of the inventory problem as demand changes rapidly and inventory items are quickly added. These actions coupled with varying order time intervals and changing ordering cycles necessitate a computer solution to this dynamic model.

Suggestions For Future Research

Future research issues that may be extensions from this project are (1) using this methodology to address cost minimization aspects of inventory acquisition with just in time inventory practices. (2) The handling of inventory items that have a direct affect on secondary inventory practices, in other words, the maintaining of certain inventory items will have a direct effect on other items. For example, if a firm maintains oil related products for resale purposes, there may be a direct relationship of oil sales to oil filter sales. (3) Various aspects of cash flow management that are associated with inventory aging as it pertains to shelf life consid-

erations may be valued using this type of methodology.   𝒕𝒂

✻✻✻Footnotes✻✻✻

1. Average inventory holding at time period I can also be calculated as follows: H_i = ((B + Q - E) /2) + E where B = beginning inventory; $2H_i$ = B + Q - E + 2E where Q = Quantity ordered; H_i = (B + Q + E) / 2 where E = ending inventory.

✻✻✻References✻✻✻

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