International Competition in Research and Development: Incentives to Subsidize and Retaliate

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Abstract

Is laissez faire optimal in international trade under conditions of imperfect competition? Possible economic profits may tempt governments to intervene in trade in order to enhance the national income. An intriguing intervention is subsidizing a firm's research and development (R&D) efforts. This paper presents Bertrand competitors, in an international duopoly, who invest in R&D directed towards process innovation. Ironically, a subsidy may not be welfare-enhancing.

Introduction

This paper identifies when it is in a country's interest to intervene in trade. Intervention is in the form of a subsidy to a firm's research and development efforts (R&D). The subsidy is meant to enhance the firm's chances of attaining a cost-reducing innovation[1]. Once a firm is able to lower costs, it can lower price and thus capture the market from its foreign rival who is constrained by higher costs.

A firm will invest in R&D for the potential profit but the government has broader interests. While the government values profitable firms, it is also concerned with the welfare of consumers. Thus an important divergence of incentives arises. Investment in R&D will benefit the firm with profits only when the firm is the sole innovator. If the foreign rival and the domestic firm both innovate simultaneously, price competition will drive economic profits for the firm to zero yet the country will benefit by the lower prices which increase the consumer surplus. As a result, a government has incentive to subsidize the firm to invest in R&D beyond the level the firm might have chosen.

From the firm's optimization problem, the optimal level of investment in R&D is found as well as a Nash equilibrium for two firms in competition. A similar optimization problem, along with the Nash equilibrium, is solved for a government to determine the optimal subsidy to the firm's R&D. For ease of exposition, when the government subsidizes investment, the government as a decision maker will be referred to as "country". Then the Nash equilibria are found for when a firm is competing with a government subsidized firm, referred to as "firm vs. country competition".

The Model

International duopolists produce homogeneous products, engage in Bertrand type competition, and face constant marginal costs. As soon as one firm successfully develops a cost-reducing innovation, that firm lowers the price by epsilon and captures the entire market because the rival is constrained by higher marginal costs.[2] The difference between the lower marginal cost and the price which remains at the rival's higher marginal cost creates positive profits but only if the firm is the sole innovator. If both firms innovate, prices fall to the new marginal cost and profits are eradicated.

The probability of successful innovation, $\phi(k)$, depends on k, the amount invested in R&D. Similarly, the rival's level of investment, k', determines her probability of innovating, $\phi(k')$. Thus, the expected pay-off is the product of the profit, λ, the probability of success, $\phi(k)$, and the probability of the other's failure, $1-\phi(k')$, minus the cost of the R&D, C(k). Each firm sets its level of investment to maximize the expected pay-off.

$$\text{Firm:} \qquad \max_{k} \; \phi(k)[1-\phi(k')]\lambda - C(k) \qquad (1)$$

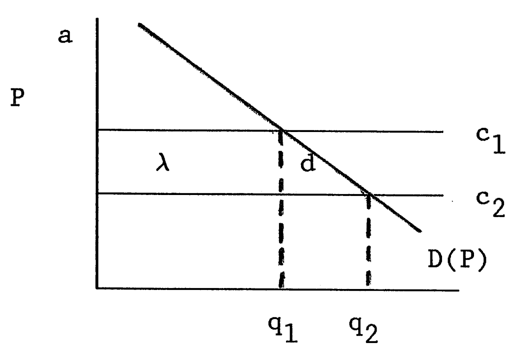
The probability of success is assumed to be a linear function of the level of investment such that $\phi(k) = k$, and the probability of failure is $1-\phi(k) = 1-k$; and as a probability, $k \in [0,1)$. An extension of the model uses an exponential probability function where $\phi(k) = 1-e^{-\mu k}$; this strengthens some results discussed in a later section.

The cost of R&D is a quadratic function of the level of investment, $C(k) = mk^2$, and once the investment is made, it is a sunk cost. In this case, m acts as an efficiency parameter. The profit to the singularly successful firm, λ, is determined by the difference between the marginal cost which set the price before the innovation, c_1, and the marginal cost resulting from a successful process innovation, c_2. This difference is multiplied by the quantity demanded in the world market at the original price. The market demand is linear, price is set at marginal cost, $q_1 = (a-c_1)/b$, thus the profit is $\lambda = (c_1-c_2)[(a-c_1)/b]$.

Whether or not subsidization is welfare enhancing depends on the size of d. This d is the difference between the profit area and the increase in consumer surplus which is determined by the elasticity of demand. The increase in consumer surplus is thus $\lambda+d = \int_{q1}^{q2} P^{-1}(q)dp$, or in the linear case, $(c_1-c_2)[f(c_1) + f(c_2)]/2$. This is only obtained when both firms succeed, thus with probability: $\phi(k)\phi(k')$.

Graph 1:

World Market with a Cost Reducing Innovation



$P = a - bq$

$q_1 = (a-c_1)/b$

$d = (c_1-c_2)^2/2b$

Two firm competition

The firm decides the level of investment to maximize the expected pay-off:

$$\text{Firm's objective: } \max_k \; k(1-k')\lambda - mk^2 \tag{2}$$

The first order conditions[3] give one the reaction function.

$$k = \lambda(1-k')(2m)^{-1} \tag{3}$$

The Nash Equilibrium occurs when the firms invest the same amounts, k=k'.

$$k = \lambda(2m+\lambda)^{-1}$$

$$\text{which satisfies } k \in [0,1). \tag{4}$$

Comparative statistics for the two firm Nash equilibrium yield the expected signs: the level of investment is decreasing increasingly with costs and increasing decreasingly with profits.

Following is an illustration of the best response functions for each firm and the resulting equilibrium. The most the firm will invest will be $\lambda/2m$ which occurs when the rival does not invest in R&D. The level of investment decreases to zero as the rival increases its investment.

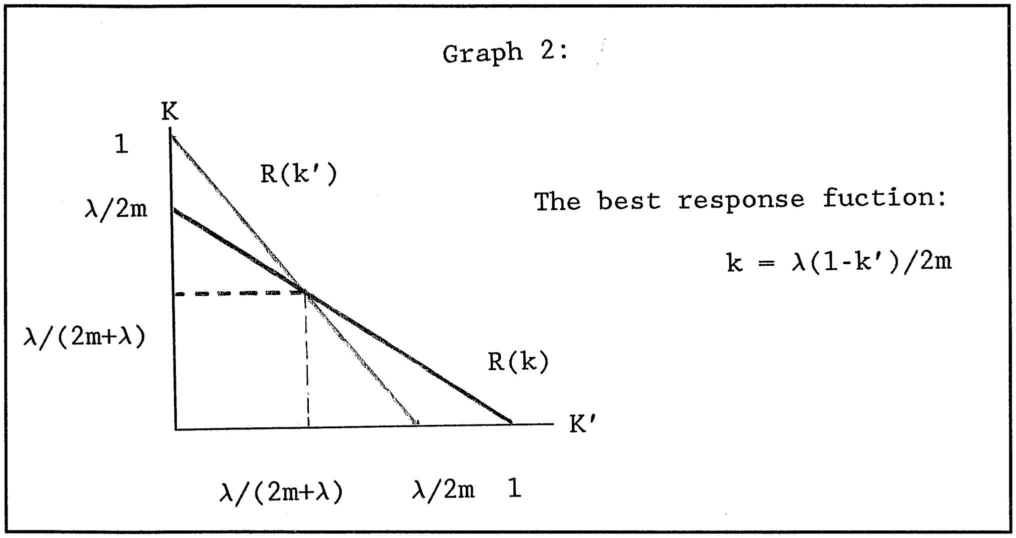
The downward sloping best response function indicates that the firm invests less as the rival firm invests more. The rival's increased investment increases the probability of the rival alone winning the innovation race as well as the probability of a tie. Both of these outcomes reduce the firm's expected profit. In response to less expected profits, the firm decreases its level of investment. Thus an active rival does not spur the firm to invest more in order to win but rather to retract.[4]

Two country competition

When a country subsidizes a firm's R&D, it takes into account the consumer surplus, $(\lambda+d)$, available to the domestic residents if the rival innovates simultaneously. This benefit accrues to that country only in proportion to the consumers that reside domestically, ω. Additionally, the country values the pay-off to the successful firm, λ, according to the proportion of the domestic firm's shareholders that reside domestically, α, as well as the proportion of the foreign firm's profits going to domestic shareholders, δ. The maximization problem is written as follows.

$$\text{Country: } \max_k \; \omega\phi(k)\phi(k')(\lambda+d) + \alpha\phi(k)$$

$$[1-\phi(k')]\lambda + \delta\phi(k')[1-\phi(k)]\lambda - C(k) \tag{5}$$



To simplify the discussion, some parameter values are initially assigned. One can assume there are no foreign shareholders of either firm, $\alpha=1$ and $\delta=0$, and the consumers are equally distributed between the countries, $\omega = 1/2$. Then the maximization problem is the following.

Country:

$$\max_{k} \; .5kk'(\lambda+d) + k(1-k')\lambda - mk^2 \tag{6}$$

The optimal level of investment, from the first order condition is:

$$k = \left[\lambda + k'[.5(d-\lambda)]\right](2m)^{-1} \tag{7}$$

This is the best response function and is downwards sloping.[5]

The Nash equilibrium level of investment is found by setting $k = k'$.

$$k = \lambda\left[2m+.5(\lambda-d)\right]^{-1} \tag{8}$$

Comparative statistics show the level of investment is decreasing increasingly with costs and increasing decreasingly with profits. Additionally, the level of investment increases increasingly as the demand becomes more elastic.

Following is an illustration of the Nash equilibrium. The best response functions differ from those obtained in two firm competition in that they are flatter and truncated although they start at the same points. That is, both the firm and the country invest $\lambda/2m$ when the rival is not investing but the country is not as discouraged as the rival increases investment.

<u>Proposition</u>: The country responds less than a firm in cutting back its investment expenditures in response to rival investments.

This dampened response is due to the country's potential gain of the consumer surplus. This gain occurs when both firms succeed thus investment on the part of the rival imparts some benefit to the competing country. Rival investment causes the country's expected profits to decrease with respect to λ but increase with respect to $\lambda+d$. In contrast, investment on the part of the rival can only be detrimental to the firm. Thus, the firm will retrench more than the country in response to rival investment giving the firm a steeper best response function than the country.

One country, one firm competition

Using the profit maximizing levels of investment for both the country case and the firm case, a Nash equilibrium can be found for an asymmetric scenario where a private firm in one country is competing with a publicly subsidized firm in the other country.

Firm FOC:

$$k = \lambda(1-k')(2m)^{-1} \tag{3}$$

Country FOC:

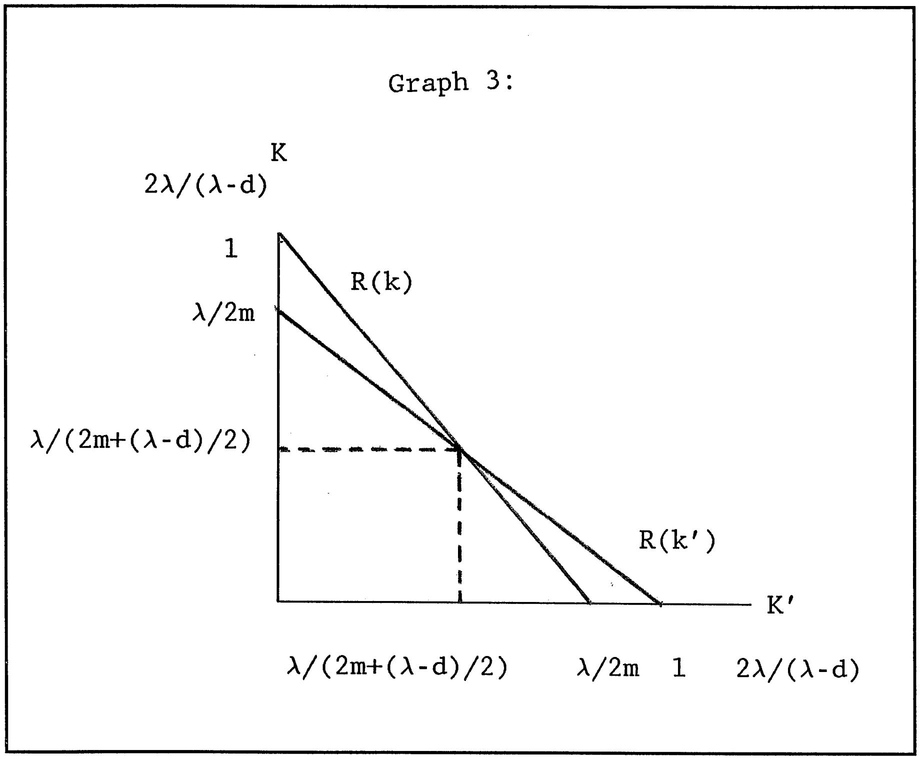
$$k = \left[\lambda + k'[.5(d-\lambda)]\right](2m)^{-1} \tag{7}$$

By substituting the firm's optimal response (from FOC), into the country's optimal choice, the Nash equilibrium investment for the country competing with a firm is obtained. The general solution follows; the first subscript indicates the decision maker, country, and the second indicates the nature of the rival, in this case, a firm.

$$k_{cf} = \lambda(2m)^{-1}[\alpha-(2m)^{-1}(\lambda(\alpha+\delta-\omega) - \omega d)]$$
$$[1-(2m)^{-2}\lambda(\lambda(\alpha+\delta-\omega) - \omega d)] \tag{9}$$

This can be simplified using the parameters specified earlier.

$$k_{cf} = (2m-(\lambda-d)/2)[(2m)^2\lambda^{-1}-(\lambda-d)/2]^{-1} \tag{10}$$



Conversely, one can substitute the country's optimal response into the firm's function to obtain the firm's optimal level of investment when competing with a country.

$$k_{fc} = (2m-\lambda)[(2m)^2\lambda^{-1}-(\lambda-d)/2]^{-1} \qquad (11)$$

The comparative statistics show that the country will increase its level of investment with the elasticity of demand as the greater the elasticity, the greater the consumer surplus to be gained. As expected the country will also increase k as λ increases and decrease k as m increases the cost component. The firm's responses are less clear, increasing investment with elasticity and ambiguous with respect to profits and costs. This ambiguity can be attributed to the fact that the country responds to each parameter change and thus the firm is affected not only by the parameter change but also by the country's investment response. The illustration of the equilibrium and best-response functions follows.

Proposition: Subsidization may not always be welfare-enhancing.

The country's more aggressive stance in deciding levels of investment will cause the rival firms to contract their investments. This reduces the probability of both succeeding and thereby reduces the possibility of gaining the consumer surplus. As a result, the country's subsidy may lead to a greater expectation of gaining λ but less of a chance of gaining $\lambda+d$. Even though the country takes into account the contraction of the rival firm's investment and the reduced probability of gaining the consumer surplus, their optimizing framework precludes the lesser investment undertaken by a firm in the same situation (proof in next proposition). Thus, the country's subsidization of the firm may be detrimental.

Proposition: The country's optimizing framework precludes replicating what a firm would choose.

The firm's best response function begins at $\lambda/2m$ which is the greatest amount the firm invests. At that level, the country invest $\lambda[1-(\lambda-d)/4m]/2m$ which is the least amount the country ever invests, being it's response to the firm's largest level of investment. This minimum investment is greater than the Nash Equilibrium investment attained in a two firm competition, $\lambda/(2m+\lambda)$.

Proof:
Suppose the opposite:

$$\frac{\lambda\left[1-[(\lambda-d)/4m]\right]}{2m} \le \frac{\lambda}{2m+\lambda} \qquad (12)$$

Divide by λ and cross-multiply:

$$8m^2 - 2m(\lambda-d) + 4m\lambda - \lambda(\lambda-d) \le 8m^2 \qquad (13)$$

Reduces to:

$$4m\lambda \le (2m+\lambda)(\lambda-d) \qquad (14)$$

But $4m > 2m+\lambda$ because $\lambda<2m$

and $\lambda > (\lambda-d)$ (15)

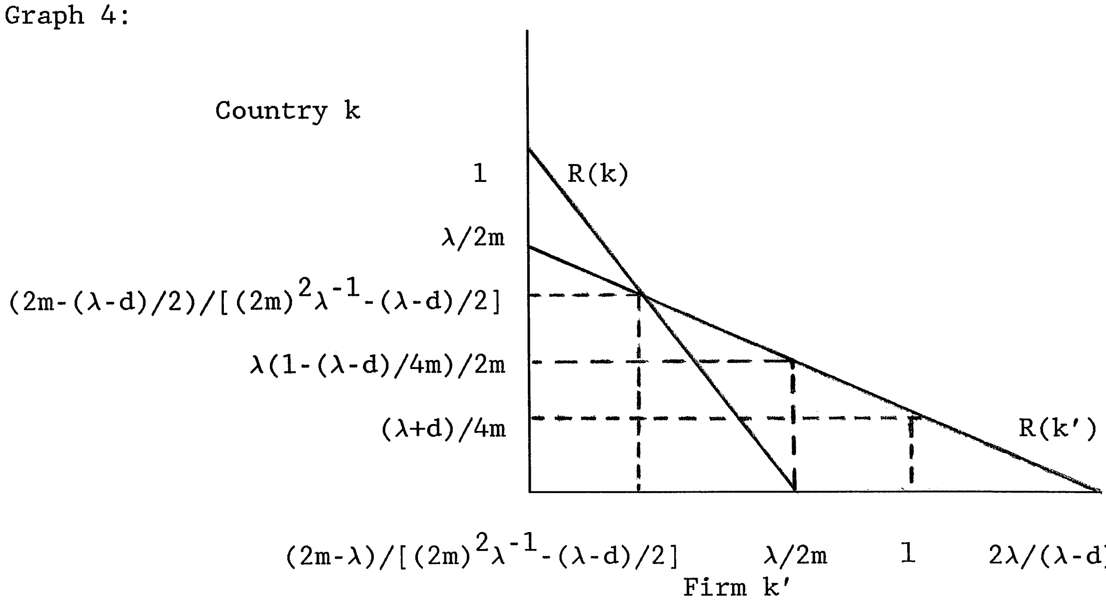
Thus,

$$4m\lambda > (2m+\lambda)(\lambda-d).$$

Proof by contradiction, *Q.E.D.* (16)

Thus while the country is behaving optimally, it will never invest what the firm facing the same rival would invest; the country's best response function precludes the possibility of duplicating the firm's actions.

Graph 4:



Proposition: The benefit to be derived from a subsidy decreases as d increases.

The parameter d is the difference between the profits to a firm and the increased consumer surplus benefit to the country. When this difference is large, the country's greater investment and subsequent withdrawal of the rival result in losses in the expected consumer surplus that are greater than the increase in expected profit. Thus when d is large, indicating relatively elastic demand, non-intervention is superior to subsidization on the part of a country. The proof of this proposition is in the following Welfare Analysis section.

Summary of the Nash Equilibrium levels of Investment

Two firms:

$$k_{ff} = \lambda(2m+\lambda)^{-1} \tag{17}$$

Two gov't subsidized firms:

$$k_{cc} = \lambda\left(2m+(1/2)(\lambda-d)\right)^{-1} \tag{18}$$

A gov't subsidized firm facing a single firm:

$$k_{cf} = (2m-(\lambda-d)/2)\left[(2m)^2\lambda^{-1}-(\lambda-d)/2\right]^{-1} \tag{19}$$

A firm facing a gov't subsidized firm:

$$k_{fc} = (2m-\lambda)\left[(2m)^2\lambda^{-1}-(\lambda-d)/2\right]^{-1} \tag{20}$$

Proposition: A country will invest more than a firm, especially when the rival is not another country.

Proof: The equilibrium levels of investment can be ranked unambiguously:

$$k_{cf} > k_{cc} > k_{ff} > k_{fc}. \tag{21}$$

Optimal Subsidy

From these levels of investment, the optimal subsidy can be found. The subsidy lowers the costs for the firm thus it enters the optimization problem through the cost function: C(k)(1-s). Applying a subsidy here is equivalent to enhancing the efficiency parameter, m. The appropriate level of the subsidy is then the level that will cause the firms to invest what the country would have chosen. The firm's investment, equation (4) will equal the country's investment, equation (8), when the subsidy is as follows.

$$k_{ff} = \lambda[2m(1-s)+\lambda]^{-1} = k_{cc} = \lambda\left(2m+(1/2)(\lambda-d)\right)^{-1} \tag{22}$$

The optimal subsidy, s, is the following:

$$s = (\lambda+d)/4m \qquad s \in (0,1) \tag{23}$$

If both countries set a subsidy equal to $(\lambda+d)/4m$, a Nash equilibrium is attained.

Graph 5:

		Foreign	
		~S'	S'
Home	~S	$W(k_f,k_f')$, $W'(k_f,k_f')$	$W(k_f,k_c')$, $W'(k_f,k_c')$
	S	$W(k_c,k_f')$, $W'(k_c,k_f')$	$W(k_c,k_c')$, $W'(k_c,k_c')$

but the firm does not, this parameter creates the difference in incentives between the two agents. Previously established elasticity constraints, $2d \leq \lambda \leq 2m$, limit the parameters to the following intervals: $\lambda \in [0,2)$ and $d \in [0,1)$.

Benefit from Subsidization

To subsidize or not to subsidize, that is the question.

Proposition: A subsidy is welfare enhancing only when d is small, that is, when there is little difference between profit, λ, and consumer surplus, $\lambda+d$.

Proof: In order to determine if subsidization is welfare enhancing, one must compare the welfare state when two non-subsidized firms are competing to the welfare state when one of these firms is subsidized. The welfare function is graphed for each case, substituting in the relevant levels of investment as follows.

$$W(k,k') = \lambda k - [(\lambda-d)/2]kk' - mk^2 \quad (26)$$

Substitute in

$$k_{ff} = k'_{ff} = \lambda/(2m + \lambda) \quad (27)$$

This gives us the welfare when neither firm is subsidized:

$$W(k_{ff},k'_{ff}) = \frac{\lambda^2}{2(2+\lambda)} + 1/2 \frac{\lambda^2}{(2+\lambda)^2} d \quad (28)$$

This function is linear in d, and has a positive intercept and slope, as shown in the following graph.

The welfare function is graphed again in the same space but using the levels of investment relevant to country/firm competition. The relevant Nash Equilibrium levels of investment are:

A country vs. a firm:

$$k_{cf} = (2m-(\lambda-d)/2)[(2m)^2\lambda^{-1}-(\lambda-d)/2]^{-1} \quad (19)$$

Welfare Implications

The welfare analysis is ironic in that the well-being of a country may be harmed when the government, rather than the firm, decides the level of R&D. We show that the subsidy is detrimental when demand is elastic and the expected consumer surplus is significantly larger than the expected profit region.

The following diagram compares welfare states where the firms are either subsidized, S, or not, ~S. The domestic and foreign welfare are then a function of each agent's Nash equilibrium level of investment.

Welfare is a function of k, given the rival's choice of k', and the subsequent expected gains to consumer surplus and shareholder profit.

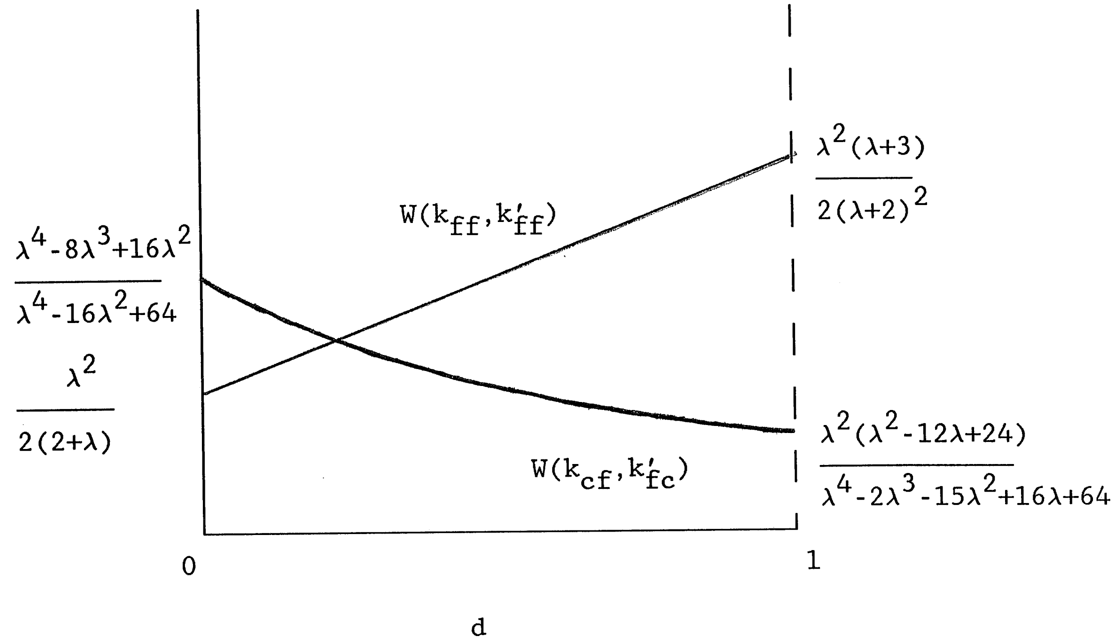
$$W(k,k') = \alpha\lambda k(1-k') + \delta\lambda k'(1-k) + \omega(\lambda+d)kk' - mk^2 \quad (24)$$

For the given parameter specifications, this becomes:

$$W(k,k') = \lambda k - [(\lambda-d)/2]kk' - mk^2 \quad (25)$$

Graphs of this welfare function using specific levels of k and k' illustrate when the subsidy is beneficial. To simplify the graphs, we assume that m = 1; m being the efficiency parameter for each firm's research efforts. Welfare is then analyzed as a function of the demand parameter d, the difference between the profit region and consumer surplus. Because the country vies for d

Graph 6:
Welfare



A firm vs. a country:

$$k_{fc} = (2m-\lambda)[(2m)^2\lambda^{-1} - (\lambda-d)/2]^{-1} \quad (20)$$

The welfare function when one firm is subsidized is then:

$$W(k_{cf}, k'_{fc}) = \frac{\lambda^4 - 8\lambda^3 - 4\lambda^3 d + 8\lambda^2 d + 16\lambda^2}{\lambda^4 - 2\lambda^3 d + \lambda^2 d^2 - 16\lambda^2 + 16\lambda d + 64}$$

The intercept of this function is greater than the intercept for the two firm welfare function. These magnitudes are reversed at d = 1. Thus, the functions cross, as illustrated below. One can conclude that subsidization is welfare enhancing for a country only for small levels of d.

Subsidization is superior to laissez-faire for a country only for small values of d. This parameter, d, is the difference between the profit rectangle and the consumer surplus trapezoid. Welfare is enhanced only if the difference is small; consumer surplus not considerably greater than profits. This is the case when demand is relatively inelastic.

When the country becomes involved in subsidizing trade, the rival firm decreases its investment in response to a decreased probability of winning and thus a decreased expected profit. The lower investment by the rival then leads to a lower probability of both innovating. This decreases the expectation of gaining the consumer surplus for the country although it increases the expectation of the domestic firm's profits. Only if d is small does the increase in the expected profit to the firm outweigh the decreased expected consumer surplus.

Benefit from Retaliation

(29) Proposition: If a rival is subsidized, then it is welfare enhancing to counter-subsidize, that is, to retaliate.

Proof: The welfare function is now written using the Nash equilibrium levels of investment found when a firm is facing a country (subsidization). This is compared to the welfare function using the levels of investment obtained when both countries are subsidizing R&D (retaliation).

To establish the non-subsidizing country's welfare when facing a subsidized rival, substitute in k_{cf} and k_{fc}. See Equation (30). Then, to establish welfare when both countries are subsidizing their firms' R&D, substitute in See Equations (31) and (32).

The retaliation welfare function is everywhere greater than the subsidization case, therefore if the rival is subsi-

$$W(k_{fc},k'_{cf}) = \frac{\lambda^5 - 2\lambda^4 - 8\lambda^3 - 6\lambda^3 d + \lambda^4 d + 2\lambda^2 d^2 + 8\lambda^2 d + 16\lambda^2}{\lambda^4 - 2\lambda^3 d + \lambda^2 d^2 - 16\lambda^2 + 16\lambda d + 64} \quad (30)$$

$$k_{cc} = k'_{cc} = \lambda\left[2m + (1/2)(\lambda - d)\right]^{-1} \quad (31)$$

$$W(k_{cc},k'_{cc}) = \frac{\lambda^2}{2+\lambda} + \frac{\lambda^2 d - 2\lambda^2 - \lambda^3}{8 + 4\lambda - 4d + \lambda^2/2 - \lambda d + d^2/2} \quad (32)$$

dized, it behooves the government to subsidize its firm as well.

This conclusion is strengthened as d gets larger and the welfare functions diverge. A country's welfare is an increasing function of d when both are subsidizing but a decreasing function of d when only the rival is subsidizing. When both countries are subsidizing, there is an increased probability that both will innovate, increasing the expectation of a gain due to the consumer surplus. This augments the welfare of the country the larger d is. If only one firm is subsidized, then the probability of both innovating decreases as the non-subsidized firm retrenches. As d gets large, this expected loss increases, thus the downward sloping welfare function.

Two Equilibria

Two equilibria can now be established. If d is small, it is rational to subsidize and a firm facing a subsidized rival will retaliate. That is, if the consumer surplus is not significantly greater than the profit, then both countries subsidizing their firms is the equilibrium.

If d is large, it does not pay for either country to subsidize; a free-trade equilibrium results. Yet if one government irrationally subsidizes, the trembling hand scenario from game theory, the rival will retaliate thus obtaining the government-government equilibrium.

The illustration of strategies at the beginning of the welfare section is redrawn here. The two equilibria are the diagonals of the box, the northwest and southeast corners, where the actions of each agent are symmetric. Having identified the two equilibria, one can ask which is optimal. In order to answer this question, one compares the welfare function using the laissez-faire levels of investment, k_{ff} and k'_{ff}, and welfare using the subsidized levels of investment, k_{cc} and k'_{cc}. See Equations (28) and (32).

$$W(k_{ff},k'_{ff}) = \frac{\lambda^2}{2(2+\lambda)} + 1/2\,\frac{\lambda^2}{(2+\lambda)^2}d \quad (28)$$

Welfare when both countries subsidize R&D is greater than the welfare in the laissez faire state, when neither subsidize.

Welfare Conclusions

A government subsidy will be detrimental when demand is elastic meaning the expected consumer surplus is significantly larger than the expected profit. Then the government's intervention causes the rival to withdraw R&D, decreasing the probability of a tie in the innovation race, thus decreased expectation of consumer surplus.

When a subsidy is beneficial, the rival, being identical, counter-subsidizes (retaliates). This equilibrium yields higher welfare compared to the non-subsidized equilibrium. The optimality of government intervention thus depends on the probabilities of innovating alone or tieing, and the elasticity of demand which determines the difference between profits and consumer surplus.

Extensions of the Model

Several variations of the model reveal the robustness of the results. These include a multi-period setting, an asymmetric distribution of consumers, entry of n rivals, and an exponential probability function.

Multi-period Decision Making

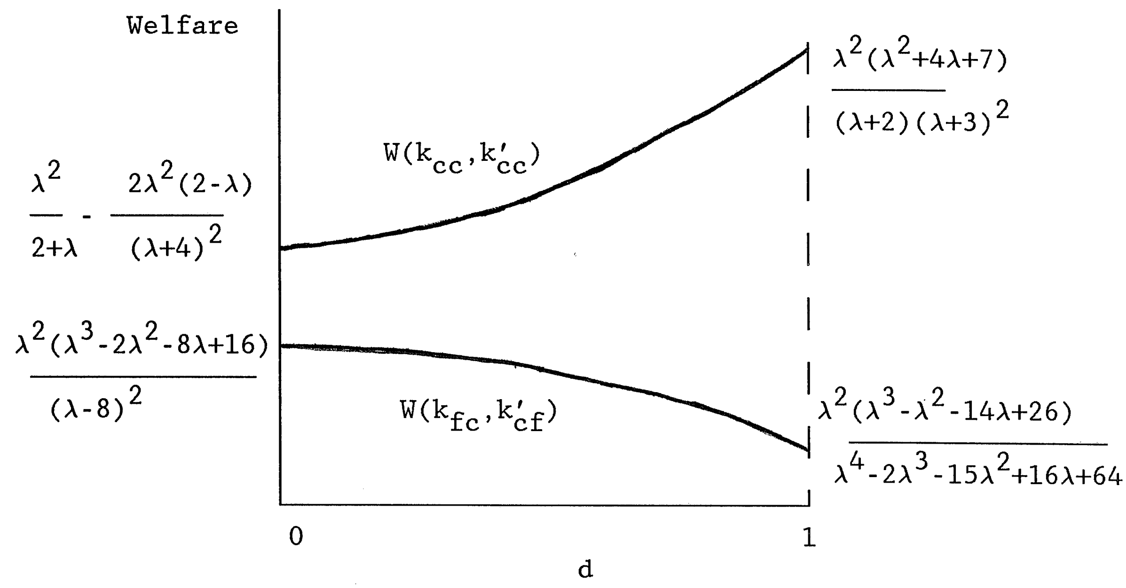
A multi-period game is diagramed in Graph 10.

The first three possible outcomes are the same as in the single period game. If one firm successfully innovates, it reaps all of the profits for the first period but imitation by the rival in the next period drives future profits to zero. When both innovate, there are zero present and future profits. The fourth case is where the multi-period model diverges from the single period. If neither firm innovates, rather than a zero pay-off as in the single period game, the firm faces the expected benefit from playing again.

Proposition: The multi-period Nash equilibrium level of investment is less than the single period Nash equilibrium level of investment.

Proof: By introducing ß as the discount factor for future periods, the value of the game can be expressed algebra-

Graph 7:



ically as follows.

Expected Value =

$$\underbrace{Pr(B)U(B)+Pr(D)U(D)+Pr(R)U(R)}_{V} + \underbrace{(1-Pr(B)-Pr(D)-Pr(R))}_{\alpha}\beta(V + \alpha\beta(V+\alpha\beta(V+.....) \quad (33)$$

$$\text{Expected Value} = V/(1-\alpha\beta) \quad (34)$$

The firm chooses the level of investment to maximize the expected value of this stream of profits. Using the linear probability function, V is written as a function of k as it is the same as the single period maximization function, and α is the probability that neither firm innovates, $(1-\phi(k))(1-\phi(k)')$:

$$\underset{k}{\text{Max}} \quad \frac{k(1-k')\lambda - mk^2}{1-(1-k)(1-k')\beta} \quad (35)$$

Setting the first derivative equal to zero defines the optimal level of investment, k. Then setting k = k' yields the implicit Nash Equilibrium:

$$\frac{\beta mk^3 - \beta k^2(\lambda+3m) + k(2\beta(\lambda+m) - \lambda - 2m) - \lambda(\beta-1)}{(\beta k^2 - 2\beta k + \beta - 1)^2} = 0. \quad (36)$$

In the interval [0,1), this is a monotonically decreasing function of k. Substituting the one period solution, $k_{ff} = \lambda/(2m+\lambda)$, for k in the last equation yields a negative value. As the implicit solution is found when the function equals zero and it is a decreasing function, then a negative value for the function indicates that the value substituted in is greater than the Nash equilibrium value. Q.E.D. This concludes the proof of the proposition that the single period level of investment is greater than the multi-period level of investment.

Although this comparison is incomplete without knowing the extent to which a multi-period setting alters the countries decision-making, it suggests an impatient firm invests more in R&D. There is an inverse relationship between ß and k, thus as the future is discounted

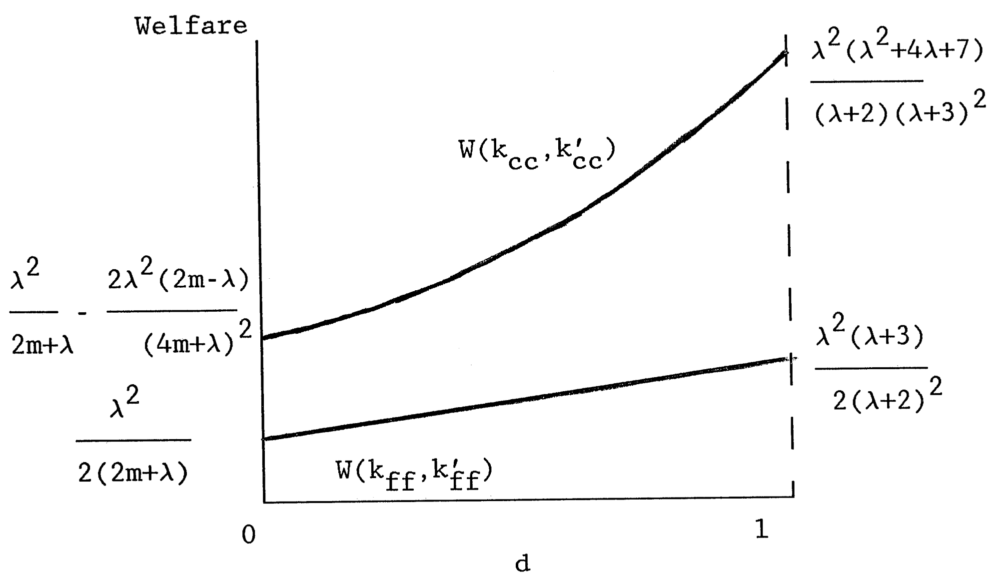
Graph 8:

Foreign

		$\tilde{S}$	S
Home	$\tilde{S}$	Equilibrium if d is large	
	S		Equilibrium if d is small

more, that is, less weight to future profits via a smaller ß, the level of investment, k, increases.

Graph 9:



Asymmetric Distribution of Consumers

When the distribution of consumers is asymmetric, $\omega \neq 1/2$, one can study how the country's incentives change as a function of the location of consumers.

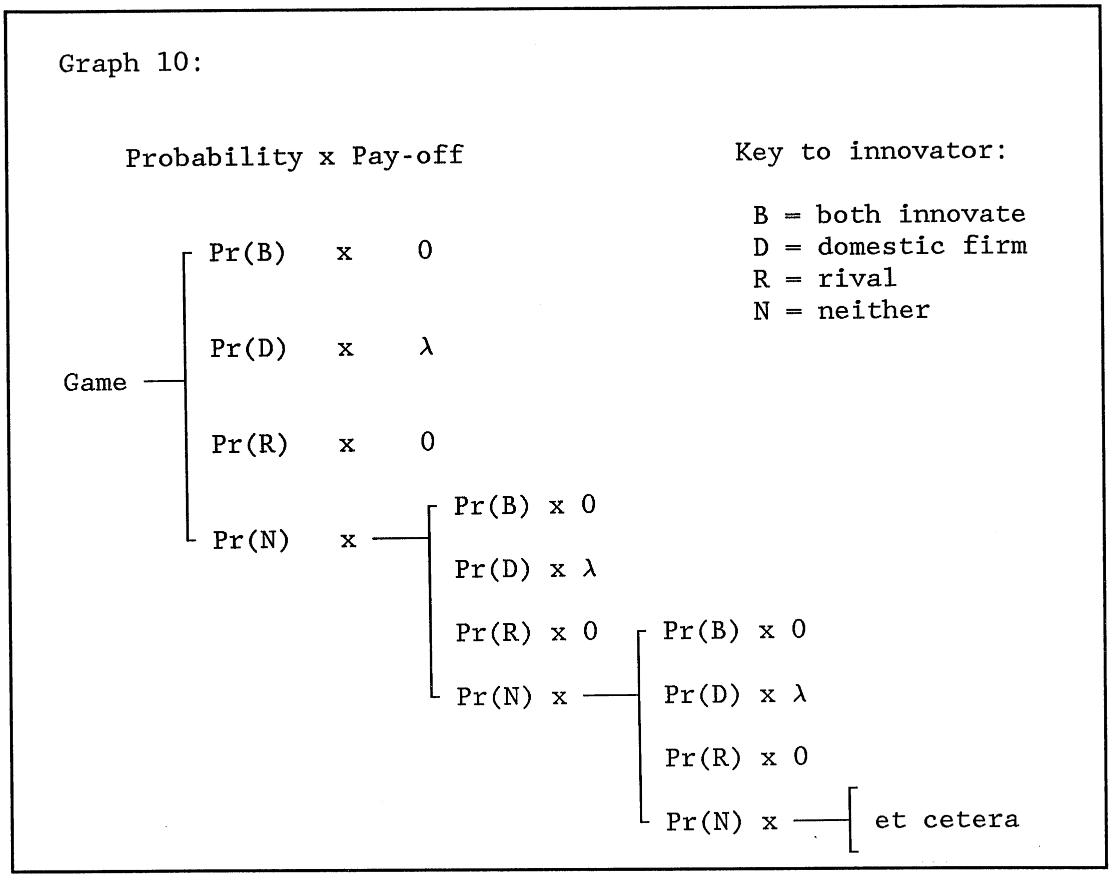
Proposition: The firm is not affected by the location of consumers but the country is, it will invest more as more consumers are nationals.

Proof: The firm maximizes expected profits without regard to the location of consumers or consumer surplus thus there is no change in the firms' decisions.

As the country's optimization includes consumers, the derivative of the country's best response function shows how the country's choice changes with respect to ω. As this is positive, the country will invest more as more consumers are nationals:

$$\partial k_c/\partial\omega = (\lambda+d)k'/2m > 0. \qquad Q.E.D. \qquad (37)$$

The best response function for a country will be upward sloping if $\omega > \lambda/(\lambda+d)$. The elasticity constraint, $\lambda > 2d$, determines that the minimum value for $\lambda/(\lambda+d)$ is 2/3 and it approaches one as demand becomes inelastic. Thus, at a minimum, 2/3 of the consumers would have to be in one country, $\omega > 2/3$, for the best response function to be upward-sloping. This implies an aggressive response to increased rival investment; as the rival invests more, the competitor will also invest more.



Exponential Probability

The same model of international competition can be solved with an exponential probability function, rather than linear, to determine the sensitivity of the results to the probability specification. The likelihood of innovating will depend on the level of investment, k, as well as an efficiency parameter μ: $\phi(k) = 1 - e^{-\mu k}$. The probability of failure is then $e^{-\mu k}$. The cost function is now linear so that $C(k) = k$.

The maximization problem for the firm is then:

$$\max_{k} \lambda(1-e^{-\mu k})(e^{-\mu k'}) - k \qquad (38)$$

This yields the best response function for the firm:

$$k_{ff} = (\ln\mu\lambda)/\mu - k' \qquad (39)$$

In deriving the maximization problem for the country, the parameter specifications are maintained, $\alpha = 1$, $\delta = 0$ and $\omega = 1/2$: See Equation (40).

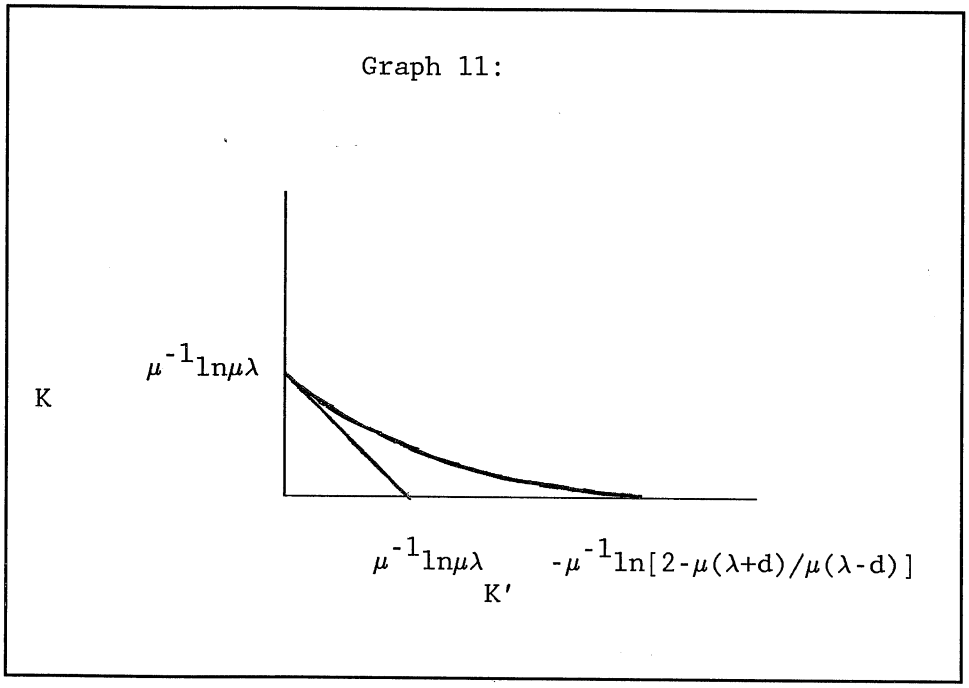
This yields the best response function for the country: See Equation (41) below. The best response functions can be graphed to illustrate the Nash equilibria of the various scenarios, as summarized below. Following is the graph of the quadratic best response function of a country graphed against that of a firm. They intersect on the boundary where $k_{fc} = 0$ and $k_{cf} = (\ln\mu\lambda)/\mu$.

Exponential Summary of Nash Equilibria:

These levels of investment can be ranked in the same order as in the model with a linear probability function and the model with asymmetric distribution of consumers: $k_{cf} > k_{cc} > k_{ff} > k_{fc}$. This supports the idea that the country invests more than the firm, especially when the rival is a (non-subsidized) firm. An interesting note is that the aggregate level of investment is the same whether one firm is subsidized or neither is subsidized: $k_{cf} + k_{fc} = k_{ff} + k_{ff} = \mu^{-1}\ln\mu\lambda$. See Equations (39), (41), (42), and (43).

$$\max_{k} (1-e^{-\mu k})e^{-\mu k'}\lambda + (1-e^{-\mu k})(1-e^{-\mu k'})(\lambda+d)/2 - k \qquad (40)$$

$$k_{cc} = \mu^{-1}\{\ln\mu + \ln[(\lambda+d)/2 + ((\lambda-d)/2)e^{-\mu k'}]\} \qquad (41)$$



$$k_{cf} = (\ln\mu\lambda)/\mu \quad (42)$$

$$k_{fc} = 0 \quad (43)$$

Welfare in the Exponential Case

The corner solution implies that a firm will withdraw all R&D investment rather than compete with a government subsidized firm. This means that the expected gain to the country providing the subsidy is reduced, as there is no possibility of gaining any consumer surplus. If the country foresees that it's subsidy will cause the rival firm to withdraw completely, it will have incentive to stay out of the game. Then the firms may still tie and increase consumer surplus but the government can not intervene to increase this expected benefit. For the firm, on the other hand, a government subsidy will eliminate the competition and thus enhance their expected profit as a sole innovator. To the extent that some proportion of the profits go to foreign shareholders, the government would choose a lower level of investment than the firm. Thus subsidizing research is not welfare enhancing.

N Rival Firms

Rather than limit competition to a duopoly, one can examine competition between n+1 firms where n is the number of rivals. For a firm to succeed and thus obtain the profit, it must be the sole innovator and the n others must fail. This alters the firm's maximization problem, calculated here with both the linear, and later, with the exponential probability function.

$$\max_k \lambda k(1-k')^n - mk^2 \quad (44)$$

The best response function is:

$$k = \lambda(1-k')^n/2m \quad (45)$$

and the implicit Nash equilibrium is:

$$k = \lambda(1-k)^n/2m \quad (46)$$

Comparative statistics show that the firm will reduce it's Nash Equilibrium level of investment in response to the entry of new firms. This is consistent with Loury's conclusions on the impact of market structure on R&D.

The exponential probability function provides more explicit results.

$$\max_k \lambda(1-e^{-\mu k})(e^{-\mu k'})^n - k \quad (47)$$

The best response function is:

$$k = \mu^{-1}(\ln\lambda\mu) - nk' \quad (48)$$

and the Nash equilibrium is:

$$k = (\ln\lambda\mu)/(n+1)\mu > 0 \quad \text{if } \mu\lambda > 1 \quad (49)$$

Comparative statistics show the equilibrium level of investment decreases as the number of rivals increases and increases with the level of profits and efficiency. Furthermore:

Proposition: The aggregate level of investment in the industry is invariant to the number of firms in the industry.

Proof: The industry levels of investment are found by summing the levels of investment for the various numbers of firms. If there are n rivals, then there are n+1 firms. The equilibrium level of investment for one firm:

$$k = (\ln\lambda\mu)/(n+1)\mu \quad (50)$$

If there are n+1 firms, they invest

$$(n+1)k = (n+1)(\ln\lambda\mu)/(n+1)\mu = (\ln\lambda\mu)/\mu \quad (51)$$

The sum of research expenditures is the same regardless of the number of firms. Q.E.D. This sum is also the same whether there are two firms or a firm competing with a country.

The invariance of the industry level of expenditure to the number of firms is the same as found by Sah and Stiglitz (1987). There is a certain level of investment that is optimal, depending on the possible profits to be earned, and that level is unaffected by the number of firms. One firm could pursue many research directions or many firms could each pursue one research project. Here is the same invariance of expenditure to market structure. The only case that does not conform is the non-market scenario of two countries deciding levels of investment through subsidies to the firms whereby their subsidies increase the aggregate level of spending on R&D.

Conclusions

The best response function for the government subsidized firm reveals that the country responds less than the firm in cutting back its investment expenditures in response to the rivals investment. The country's more aggressive stance and the rival's retraction reduce the expectation of the consumer surplus. This results in a welfare reducing subsidy when there is a large difference between profits, λ, and consumer surplus, $\lambda+d$. This difference, d, is large when demand is relatively elastic; then a government subsidy to a firm's R&D will be detrimental. When d is small, the reduction of expected consumer surplus is outweighed by the increase in expected profits.

A subsidy enacted to counter a subsidized rival, retaliation, is always welfare enhancing. Thus, if one country subsidizes its firm, it can expect the other country to subsidize, too. If d is small, both countries subsidize their firms, resulting in a higher welfare equilibrium compared to laissez-faire.

Extensions of the model reveal the following. A multi-period setting reduces the optimal level of investment for a firm. Allowing entry reveals invariance of industry investment to the number of firms although this optimal industry level is not the level obtained when the government intervenes. Asymmetrically distributed consumers may cause the country's best-response function to slope upward, indicating an aggressive response to an aggressive rival rather than withdrawal. An exponential rather than linear probability function strengthens the results: the firm facing a subsidized rival does not simply reduce its investment but withdraws from the race, causing welfare to decline. Thus, an opportunity to enhance firm competitiveness through a subsidy may end up harming the country through its effects on consumers.

Suggestions for Future Research

Future research might build on any of the extensions of the model. For example, expansion of the section on different locations of consumers or consideration of the cross-ownership of firms which is usually neglected. The strategic decisions of the actors could also be modeled as a three stage game where the government decides optimal R&D policy, the firms then decide the level of investment and then prices.

I am very grateful to Boyan Jovanovic for many insightful conversations. Additional comments from Elizabeth Granitz were most helpful. All remaining errors are my own.

*****Footnotes*****

1. This is known as a process innovation which is in contrast to a product innovation that differentiates product characteristics.
2. Demand is restricted so that $\lambda>2d$. Otherwise inelasticity would have the innovator lower price by more than epsilon.
3. Second order conditions for a maximum are satisfied: -2m , 0.
4. K can not exceed one as it is a probability. The maximum level of k is $\lambda/2m$, thus $\lambda/2m \leq 1$ and $\lambda \leq 2m$. Where $\lambda > 2m$, the best response functions would be truncated at one and the Nash equilibria is unaffected. Multiplicity of equilibria eliminated in the case $\lambda = 2m$ by the symmetry assumption. If $\lambda = 2m$, the functions both start and end at one. Despite this coincidence, only one equilibrium is possible, at the midpoint, as $\lambda/(2m+\lambda)$ is then equal to 1/2 and the assumption of symmetry is satisfied.
5. The innovator lowers the price by epsilon only, thus demand must be inelastic which implies $\lambda>2d$. The best response function is downward sloping if the derivative, $\partial k/\partial k' = -(\lambda-d)/4m$, is negative, which it is if $\lambda>d$. The importance of λ is that it is the profit for which the firms vie, while d is the consumer surplus in excess of profit for which the government vie.

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