Estimating Investment Growth Rate and Conditional Internal Rate of Return

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Abstract

Previous analytical and empirical work has shown that the Cash Recovery Rate (CRR) method is a useful technique for estimating a firm's conditional internal rate of return (CIRR). This paper offers a new approach to estimating the investment growth rate used in the CRR method. The new approach reduces bias in estimating a firm's CIRR by the CRR method. This work strengthens the theoretical soundness and research usefulness of the CRR method.

Introduction

A stream of research by Salamon (1982, 1985, 1988) and Gordon and Hamer (1988) develops the Cash Recovery Rate (CRR) method for estimating a conditional internal rate of return (CIRR) of a firm. Compared to the accounting rate of return (ARR) as an estimator of CIRR, the CRR method is justified on theoretical grounds and has been shown to reduce the confounding effects of accrual accounting techniques. Salamon (1988) suggests that the CRR method captures enough of the systematic properties of a firm's underlying CIRR to be useful in research inquiry.

Buijink and Jegers (1989) strengthened the CRR method by showing a new approach to estimating one of the parameters--asset life. They show that the bias in asset life estimates can be reduced by including a growth factor. Responding to the Buijink and Jegers contribution, Salamon (1989) advocated further refinement of the CRR method to strengthen its theoretical soundness and usefulness in empirical research.

We offer a new approach to estimating investment growth rate, for use in the CRR method, that removes potential error in CIRR estimates. Evidence is presented that estimates of investment growth rate by Salamon and others are biased erratically downward. Incorporating the refinement strengthens the theoretical soundness of the CRR method. The modification is important to the soundness of the CRR method both because the growth rate is an important CRR parameter in its own right and because of the interaction demonstrated by Buijink and Jegers (1989) between growth rate and estimated asset life.

Parameters of the CRR Method

Salamon (1982, 1988) showed that a firm's cash recovery rate (CRR) converges over time to:

$$CRR = \left[\frac{g}{(1+g)^{n}-1}\right]\left[\frac{(1+g)^{n}-b^{n}}{(1+g-b)}\right]\left[\frac{(1+r)^{n}(1+r-b)}{(1+r)^{n}-b^{n}}\right] \qquad (1)$$

where:

CRR= cash flow from operations and investments/assets,
b= firm's cash flow profile,
g= annual rate of growth in total assets,
n= weighted average useful life of assets, and
r= implicit estimate of the firm's CIRR.

CRR, g, and n are estimated from external financial statements; b, the cash flow profile parameter, is an assigned value chosen to represent the temporal pattern of cash inflows, such as 1.2 for increasing flows, 1.0 for level flows, or 0.8 for cash flows decreasing over time.

Estimating n

Buijink and Jegers (1989) offered a modification to the way asset life, n, is estimated within the CRR approach of Salamon (1985, 1988). Salamon (e.g., 1988) estimated n for each firm for each year by dividing the average of its gross plant for the year by its depreciation expense for the year. The average of these annual estimates was used as the estimate of the life of each firm's projects. Buijink and Jegers (1989), however, showed that for $g > 0$ and accelerated depreciation methods, Salamon's empirical estimate of n is biased downward. They also showed that underestimation of $\underline{n}$ leads to understatement of CIRR, and that the amount of bias in $\underline{n}$ is positively related to the annual compound rate of growth in gross investment, g. Thus, the higher the asset growth rate, g, the larger the true value of $\underline{n}$, and the greater the difference between Salamon's CIRR estimate and a theoretically sound estimate of CIRR.

Estimating g

The annual compound rate of asset growth, g, is also estimated from financial statement data. Salamon (e.g., 1988) estimated g by continuous discounting of the ratio of the gross investment in the last year to the gross investment in the first year, where the gross investment in a year is the mean of the beginning and ending asset balances in that year. The asset growth rate from 1975 to 1980, for example, would be estimated as the natural log of (gross investment 1980/gross investment 1975) divided by five; gross investment in, e.g., 1975 is computed as the mean of the firm's total assets at the end of 1975 and the end of 1974. Note that four end-of-year asset balances, 1974, 1975, 1979, and 1980, would be employed.

Alternate Method of Growth Rate Estimation

We suggest a different way of estimating the asset growth rate, g, in the CRR method to overcome two problems. The problems, discussed separately below, are his inappropriate use of continuous discounting and his averaging of asset balances.

Sub-annual Discounting vs. Sub-annual Cash Flows

Salamon's use of continuous discounting appears--he does not say--to be motivated by the assumption that cash flows are received continuously by the firm. While that assumption is reasonable, it does not follow that continuous discounting of the annual gross investment data is either necessary or appropriate. Continuous flows and continuous discounting (or compounding) are distinctly different concepts. One does not require or imply the other.

Sub-annual compounding or discounting is well-known to arise in connection with (what U.S. authors usually term) a NOMINAL annual rate. For example, a bank certificate of deposit (CD) with a nominal annual interest rate of 9.531% per year compounded continuously actually bears interest at a true annual rate of $\underline{e}^{.09531} - 1$ or ten percent.

Let us suppose that the bank offers two $1000 CD plans, each bearing interest at a true annual rate of ten percent. Under Plan A, no sub-annual cash flows occur; both the CD principal and accrued interest are paid after one year. Under Plan B, accrued interest is paid to the CD holder daily but is immediately reinvested in a new CD also bearing a ten percent annual rate.[2] The total amount due the investor after one year is $1100 under both plans. The timing of interim cash flows is irrelevant.

In the context of Salamon's g, the annual rate of asset growth also is independent of cash flow frequency. Suppose a firm invests $1000 for a five year period at a true annual rate of ten percent. If interim cash flows from the investment--annual, monthly, daily, or continuous--are immediately reinvested and continue to grow at the ten percent rate, the asset balance after five years will be $\$1000(1.1^5)$ or $1610.51. The true annual rate of asset growth is equal to ten percent whether flows are received continuously or at less-frequent intervals. While Salamon would calculate the rate as ln(1610.51/1000)/5, a "nominal" but wholly irrelevant rate of 9.531%, the true annual compound rate of asset growth is simply the fifth root (minus one) of 1610.51/1000, or 0.10.

Average Gross Investment Balances

Another noteworthy feature of Salamon's g estimation method is his use of average investment balances as the data whose growth rate is estimated. This procedure is benign (though curious) when the year-to-year growth rate is constant over the measurement period, as he assumes. As the constant growth example in Table 1 demonstrates, the degree of bias in Salamon's g estimate is the same--the bias from continuous discounting--whether end-of-year asset balances or average asset balances are employed.

If growth is irregular, however, the averaging procedure can add another small element of error to the computation of g. In Table 2, two contrasting examples are presented in which the true compound rate of growth of ten percent per year--21 percent every two years--is maintained, but where the growth spurts occur in different years. When Salamon's averaging method is applied to an asset balance series with an "early spurt", as in Example A, the bias in his g estimate is even greater than that due solely to continuous discounting. Conversely, some of the bias from continuous discounting is offset if the asset balances shows a "late spurt," as in Example B.

Because empirical growth rates are rarely if ever constant over time, and because there is no evidence that asset balances of all firms "spurt together" year-to-year, we suggest that the averaging procedure adds an unstable and unnecessary element of error to an already biased method of growth rate estimation. A better and simpler procedure would be to estimate the compound growth rate directly from year-end data, as is demonstrated at the bottom of Table 2.

Table 1
Comparison of True Rate of Asset Growth with Salamon's Estimate: A Constant Growth Rate Example

Year	Asset Balance at Year-end	Salamon's "Average" Assets
0	100.00	
1	110.00	105.00
2	121.00	115.50
3	133.10	127.05
4	146.41	139.76
5	161.05	153.73
6	177.16	169.10

Salamon's $\underline{g} = \ln(177.16/100)/6 = \ln(169.10/105.00)/5 = 9.53\%$

True $\underline{g} = (177.16/100)^{1/6} - 1 = (169.10/105.00)^{1/5} - 1 = 10.00\%$

Table 2
Comparison of True Rate of Asset Growth with Salamon's Estimate: Irregular Growth Examples

	Irregular Growth Example A		Irregular Growth Example B	
Year	Ending Asset Balance	Salamon's "Average" Assets	Ending Asset Balance	Salamon's "Average" Assets
0	100.00		100.00	
1	121.00	110.50	100.00	100.00
2	121.00	121.00	121.00	110.50
3	146.41	133.71	121.00	121.00
4	146.41	146.41	146.41	133.71
5	177.16	161.78	146.41	146.41
6	177.16	177.16	177.16	161.78

Salamon's $\underline{g}$:

	Example A	Example B
	$\ln(177.16/110.50)/5$	$\ln(161.78/100.00)/5$
	$= 9.44\%$	$= 9.62\%$

True $\underline{g}$ in both examples: $(177.16/100.00)^{1/6} - 1 = 10.00\%$

Implications

We explore in this section the effect on CIRR estimates from unbiased g estimates.

Bias in g

Bias in the asset growth rate--the difference between the true compound rate, g, and the nominal rate from continuous discounting, ln(1+g)--increases with the absolute value of the true rate. As the first three columns of Table 3 demonstrate, the more the growth rate in a firm's assets deviates from zero, the greater the bias from continuous discounting.

As Table 3 shows, continuous discounting fails to bias an estimate of g if and only if the rate is zero. Examination of Equation 1, however, reveals that r, the estimate of CIRR, is undefined for g = 0. Thus, for all circumstances in which r is defined, estimates of g from continuous discounting are biased to some degree.

Bias in CIRR Estimates

The degree of bias in estimating a firm's internal rate of return, r, associated with a given level of bias in estimating g depends on other parameters in the Salamon model.

For example, the rate of asset growth affects the estimated CIRR only if cash flows are increasing or decreasing over time. Examination of Equation 1 shows

Table 3
Bias Due to Continuous Discounting in Estimates of Asset Growth Rate (g) and Conditional IRR (r)

Asset Growth Rate			Conditional IRR*		
(1) True Rate (g)	(2) Estim. by Continuous Discount	(3) Diff. under est(-) (2)-(1)	(4) True CIRR (r)	(5) Estim. by Continuous Discount	(6) Diff. under est. (5)-(4)
0%	0.00%	-0.00%	**	**	**
5%	4.88%	-0.12%	8.36%	8.30%	-0.06%
10%	9.53%	-0.47%	10.52%	10.31%	-0.21%
15%	13.98%	-1.02%	12.71%	12.27%	-0.44%
20%	18.23%	-1.77%	**	14.09%	**
21%	19.06%	-1.94%	15.21%	14.43%	-0.78%
25%	22.31%	-2.69%	16.72%	15.62%	-1.00%
30%	26.24%	-3.76%	18.39%	17.16%	-1.24%
35%	30.01%	-4.99%	19.82%	18.40%	-1.42%
40%	33.65%	-6.35%	21.01%	19.45%	-1.55%
45%	37.16%	-7.84%	21.99%	20.36%	-1.64%
50%	40.55%	-9.45%	22.81%	21.12%	-1.69%

* CIRR computations are based on straight-line depreciation, cash recovery rates (CRRs) of 0.125, cash flow profiles (b) of 1.2, and average asset lives (n) of 20 years.

** In Equation 1, r is undefined if g=0 or if b=1+g that g is irrelevant to the estimate of r when cash flows are level (b = 1). When b is equal to one, the product of the first two bracketed terms in Equation 1 is unity regardless of the value assumed for g.

Also, the exact effects on empirical CIRR estimates from bias in g depend on interactions between the depreciation method and other firm-specific circumstances. Buijink and Jegers (1989) showed that, with accelerated depreciation, the magnitude of g affects estimates of n, which both affect estimates of CIRR.

However, some general observations can be made with an assumption of straight-line depreciation. In the last three columns of Table 3 are shown the effects of previously-noted growth rate bias levels on CIRR estimates when illustrative values are assumed for b (1.2), n (20) and CRR[2] (0.125). It can be seen that the greater the absolute growth rate, the greater the bias from continuous discounting on resulting CIRR estimates.

The direction and magnitude of CIRR bias from continuous discounting depends on the cash flow profile. If cash flows are increasing over time (b > 1), as in Table 3, CIRR is understated with the continuous discounting approach. However, CIRR is over-stated by continuous discounting when cash flows are decreasing (b < 1). Moreover, the degree of bias increases as the value of b moves farther from unity. Table 4 shows these relationships for several representative values of b.

Table 4 also demonstrates that, ceteris paribus, CIRR bias from continuous discounting is positively related to the average asset life, n. Thus, the empirical importance of the bias should be greatest for firms with a large proportion of non-depreciable or long-lived assets.

Conclusion

Responding to Salamon's (1988, 1989) call for research aimed at removing extraneous contaminating factors in the CRR approach, we have presented an

Table 4
Illustrative Bias in CIRR Estimates Due to Misestimation of Asset Growth Rate (g) when True Rate is Ten Percent (Growth Rate Estimated by Continuous Discounting 9.531%)

(1) Cash Flow Profile (b)	(2) Asset Life (n)	(3) Estim. CIRR (r) for Estim. g of:	(4)	(5) Difference over est. (+) under est. (-) (4)-(3)
		10.00%	9.53%	
0.6	10	-1.79%	-1.35%	+0.43%
0.6	20	13.21%	14.44%	+1.23%
0.6	30	18.92%	20.73%	+1.81%
0.8	10	1.90%	2.08%	+0.18%
0.8	20	11.94%	12.48%	+0.54%
0.8	30	15.35%	16.21%	+0.86%
1.0	10	4.28%	4.28%	-
1.0	20	10.93%	10.93%	-
1.0	30	12.09%	12.09%	-
1.2	10	5.54%	5.43%	-0.10%
1.2	20	10.52%	10.31%	-0.21%
1.2	30	10.86%	10.58%	-0.27%
1.4	10	6.16%	6.00%	-0.16%
1.4	20	10.42%	10.16%	-0.26%
1.4	30	10.69%	10.38%	-0.31%

* CIRR computations are based on straight-line depreciation and cash recovery rates (CRRs) of 0.125.

improvement to his original method for estimating asset growth rate. As the refinement rests on stronger theoretical underpinnings, it serves to strengthen the overall theoretical basis of the CRR method.

The empirical significance of the modification is that it removes a source of bias in estimating the CIRR of a firm by the CRR method. Moreover, due to the model interaction between asset growth rate and estimated asset life shown by Buijink and Jegers (1989) when accelerated depreciation is used, improved estimates of asset growth rates can have a manifold effect in improving the accuracy of ultimate estimates of CIRR. The result can be greater inferential value of research that may be built upon the CRR method.

Suggestions For Future Research

The findings of this and related work offer two avenues of future research. First, the CRR method of estimating a firm's CIRR offers the research community a powerful tool to investigate and direct public policy, business enterprise activity and firm performance. For example, a firm's CIRR may be useful in research studies that investigate the interaction among firm size, profitability and the political cost of financial reporting. Current research exploring the interactions produced questionable results because of confounding created by variation in accounting methods used to compute reported net income. Employing CIRR as a measure of firm profitability may reduce this problem. Although a firm's CIRR is not free of bias, it certainly does not suffer from the same inherent limitations as reported net income. A second avenue of future research would be to investigate the distributional properties of CIRR, by the CRR method, among specific industry classifications. This type of research is necessary to interpret CIRR in statistical analysis. ❧

✻✻✻Notes✻✻✻

1. Daily interest on the CD would amount to $\$1000\ (1.1^{1/365}-1)$ or about $0.26116. While the inferences apply with equal validity to any interest payment frequency, continuous payments are not universally easy to visualize.
2. Effects on CIRR estimates of other values of $\underline{b}$ and $\underline{n}$ are explored in Table 4. The Cash Recovery Rate (CRR) of 0.125 approximates the median empirical rate found by Zeller [1991] for a sample of 779 manufacturing firms.

✻✻✻References✻✻✻

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