Forecasting Restaurant Sales Using Multiple Regression And Box-Jenkins Analysis

Dr. Frank G. Forst, Management Science, Loyola University of Chicago

Abstract

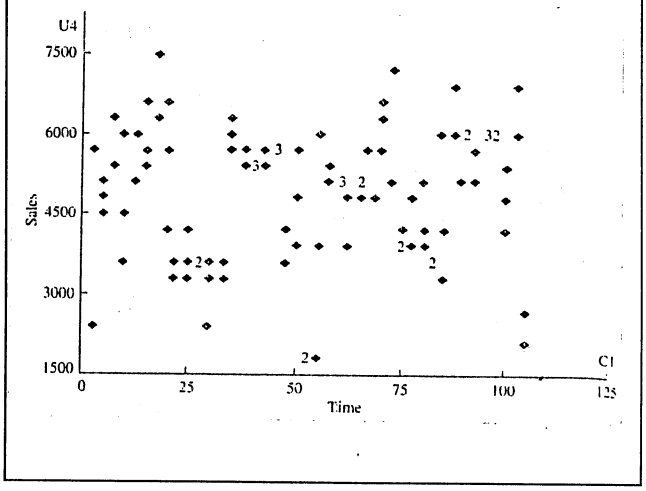
Several regression and Box-Jenkins models were used to forecast weekly sales at a small campus restaurant for Years 1 and 2. Forecasted sales were compared with actual sales to select the three most promising forecasting models. These three models were then used to forecast sales for the first 44 weeks of Year 3, and compared against actual sales. The results indicate that a multiple regression model with two predictors, a dummy variable and sales lagged one week, was the best forecasting model considered.

Introduction

The author and the restaurant manager were interested in forecasting weekly sales of a small restaurant near Marquette University in Milwaukee, Wisconsin. The author decided to employ regression and Box-Jenkins analysis. The restaurant compiled the previous week's sales every Monday morning. We began our analysis by obtaining weekly sales data from the week ending January 4, Year 1 through the week ending December 26, Year 2, a total of 104 observations. (At the restaurant manager's request, the author has disguised the years for which the weekly sales data were obtained). The mean weekly sales for these 104 weeks turned out to be $4862.

Figure 1 is a graph of the weekly sales over time. The graph indicates that weekly sales are quite volatile, ranging from $1870 to $7548, with very little trend.

FIGURE 1



At the time this study was begun, the manager was forecasting the current week's sales as equal to the previous week's sales. The manager was dissatisfied with the accuracy of this "naive" forecasting method. Thus he was quite willing to consider more sophisticated forecasting methods.

Regression Analysis

We tested three predictors. The first predictor was time. The second predictor was a dummy variable indicating whether or not Marquette University was in full session that week (0 means not in full session, 1 means in full session). Examination of the sales data revealed that weekly sales always dropped when Marquette was not in full session; namely, during the Christmas break, the Spring break, and the summer. This is not surprising, since the restaurant is located on Marquette's campus and most of its customers are members of the Marquette community. The third predictor we tried was sales lagged one week, since examination of the data indicated sales for two adjacent weeks were often similar.

Using Minitab, we then computed the simple correlations between the three potential predictors and the dependent variable weekly sales. The results are shown below, in Table 1. As expected, there is almost no trend in the weekly sales. However, the dummy variable is strongly correlated with current sales; that is, whether or not Marquette University is in full session is a good predictor of the current week's sales. The previous week's sales are moderately correlated with the current week's sales. There is also a moderate linear relationship between the dummy variable and the previous week's sales. The other simple correlations are low.

TABLE 1
Correlation Matrix

	Current Sales	Time	Dummy Variable	Sales Lagged One Week
Current Sales	1.000	.049	.772	.580
Time		1.000	-.048	.120
Dummy Variable			1.000	.490
Lagged Sales				1.000

With the aid of Minitab, we experimented with several regression weekly sales. The results of our regression analysis are given in Table 2. Since the sales data manifest almost no trend, the predictor "time" adds very little predictive power to a regression model. Note that model (4) has just a slightly higher coefficient of determination than model (2), and both models possess a significant amount of autocorrelation. Also, models (3) and (5) have the same coefficient of determination, while model (7) has only a slightly higher coefficient of determination than model (6). On the other hand, the predictor "lagged sales" adds a fair amount of predictive power to a regression model. Model (6) has a significantly higher coefficient of determination than model (2), without a significant amount of autocorrelation.

TABLE 2
Autocorrelation

Model Predictor(s)	R^2	Durbin-Watson Statistic	Significant at .05 Level?	*Amount of Collinearity
(1) Time	.0024	0.81	Yes	None
(2) Dummy	.596	1.30	Yes	None
(3) Lagged Sales	.336	1.89	No	None
(4) Time and Dummy	.603	1.32	Yes	Very Little
(5) Time & Lagged Sales	.336	1.89	No	Little
(6) Dummy & Lagged Sales	.649	1.74	No	Some
(7) Time, Dummy, & Lagged Sales	.651	1.73	No	Moderate

*Determined by examination of Table 1, the Correlation Matrix.

We decided to select regression model (6) to forecast weekly sales for the following reasons:

1. Model (6) has the second highest coefficient of determination, only .002 below that of model (7).
2. The three parameters of model (6) are each significantly different from zero at the .001 level. It should be noted that the parameter for the variable "time" was not significant at even the .15 level in any of the regression models which included time as a predictor.
3. Model (6) does not possess a significant amount of autocorrelation.
4. Model (6) is simpler than model (7), and does not have as much collinearity. The regression equation for model (6) is as follows:

$$\hat{Y}_t = 2614.3 + 1610.7X_t + .2605Y_{t-1}, \quad (1)$$

where

$\hat{Y}_t$ = the forecasted sales for week t (in dollars)

X_t = the dummy variable for week t (0 or 1)

Y_{t-1} = the actual sales for week t-1 (in dollars).

R-Square = .649 means 64.9% of the variation in weekly sales can be explained by whether or not Marquette is in full session, the previous week's sales, and the regression plane. R = .806, indicating the regression plane is a very good fit to the weekly sales data. The regression equation implies that weekly sales average about $1611 higher when Marquette is in full session, holding the previous week's sales constant. The MSE for regression model (6) is 517,881.

Box-Jenkins Analysis

Since we had 104 weekly sales observations, we believed a Box-Jenkins analysis would be appropriate to forecast current weekly sales (one-period-ahead forecasts). We were also influenced in our choice of Box-Jenkins analysis by Gardner's results (1979). Gardner was interested in forecasting monthly demand for blood tests at a hospital. He found that the best multiple regression model he tested yielded a smaller mean absolute percentage forecast error than the MAPE for the best Box-Jenkins model tested (5.0% versus 7.7%) in one-period-ahead forecasting. We wanted to compare multiple regression analysis with Box-Jenkins analysis on our weekly sales data.

Using Minitab, we computed the autocorrelations and partial autocorrelations, lagged up to 36 periods. They are shown in Table 3. The 95% confidence limits are +/- .192. The first two autocorrelations are significantly positive, but then the autocorrelations trail off towards zero and negativity until lag 20. The first partial autocorrelation is significantly positive, but then the partial drop off to zero. This pattern suggests the time series is stationary (recall that the regression analysis also indicated an

insignificant trend) and an autoregressive model of order one (AR(1) model) might be an appropriate Box-Jenkins model. Also, the sales data do not appear to be seasonal. We also tested an autoregressive model of order two (AR(2) model), two moving average models of orders one and two (MA(1) and MA(2)), and four autoregressive-moving average models (ARMA(1,1), ARMA(2,1), ARMA(1,2), and ARMA(2,2) models). A constant term was included in each model, since we did not first difference the data.

For each Box-Jenkins model considered, we computed the MSE, tested the significance of the model parameters (not significant means p>.05), and ran a diagnostic check on the residuals. We counted how many autocorrelations of the residuals, lagged up to 36 periods, were significant at the .05 level (the 95% confidence limits are +/- .192). The results are summarized in Table 4.

We decided to select the AR(1) Box-Jenkins model to forecast weekly sales one-week-ahead for the following reasons:

1. Both of the parameters are highly significantly different from zero in the AR(1) model.
2. None of the autocorrelations for the residuals, lagged up to Thus the errors are uncorrelated over time.
3. The MSE of the AR(1) model is only 7.2% higher than the lowest MSE, for the ARMA(2,1) model.
4. The AR(1) model is the simplest Box-Jenkins model.

The AR(1) Box-Jenkins model is:

$$\hat{Y}_t = 1871.46 + .609Y_{t-1} . \quad (2)$$

TABLE 3

Lag	Autocorrelation	Partial Autocorrelation	Lag	ACF	PACF
1	.554	.554	19	-.131	.027
2	.220	-.126	20	.025	.007
3	.138	.103	21	.181	.177
4	.071	-.047	22	.271	.040
5	.065	.064	23	.243	-.007
6	-.006	-.103	24	.194	-.010
7	-.120	-.106	25	.189	-.011
8	-.058	.101	26	.205	.028
9	-.070	-.113	27	.197	.004
10	-.195	-.156	28	.250	.140
11	-.270	-.124	29	.278	.035
12	-.278	-.067	30	.282	.121
13	-.307	-.174	31	.208	-.035
14	-.238	-.004	32	.023	-.072
15	-.257	-.164	33	-.094	-.058
16	-.276	-.101	34	-.160	-.105
17	-.225	-.120	35	-.200	-.023
18	-.230	-.173	36	-.186	-.032

Note that regression model (6) fit the weekly sales for Year 1 and Year 2 better than the AR(1) model, as indicated by the MSEs (517,881 versus 1,011,474). However, this does not imply that the Box-Jenkins model will forecast worse.

Model Performance on More Recent Data

We tested regression model (6) and the AR(1) Box-Jenkins model on 44 weekly sales observations we obtained for Year 3, covering the period from the week ending January 2, through the week ending October 30. We calculated the forecasted weekly sales (one period ahead) and compared them with the actual weekly sales for these 44 weeks. We did not update the forecasts or model parameters. We should point out that Gardner (1979) tested his best multiple regression model on only 24 new observations.

For regression model (6) fitted to the Year 3 weekly sales, the MSE was 1,238,599 and mean absolute percentage forecast error was 13.24%. This MSE is almost 2.4 times as high as the MSE we obtained when fitting regression model (6) to the weekly sales data for the Years 1 and 2. The largest absolute percentage forecast error, 40.73%, was for the week ending May 15, Year 3 (Final Exam

TABLE 4

Model	MSE	Parameters	Parameter Significant?	No. of Significant ACFs of Residuals
AR(1)	1,011,474	Constant	Yes[1]	0
		AR 1	Yes[1]	
AR(2)	1,000,236	Constant	Yes[1]	1
		AR 1	Yes[1]	
		AR 2	No	
MA(1)	1,070,323	Constant	Yes[1]	3
		MA 1	Yes[1]	
MA(2)	1,020,535	Constant	Yes[1]	2
		MA 1	Yes[1]	
		MA 2	No	
ARMA(1,1)	1,000,353	Constant	Yes[1]	1
		AR 1	Yes[2]	
		MA 1	No	
ARMA(2,1)	943,796	Constant	Yes[1]	0
		AR 1	Yes[1]	
		AR 2	Yes[1]	
		MA 1	Yes[1]	
ARMA(1,2)	1,001,167	Constant	Yes[1]	0
		AR 1	No	
		MA 1	No	
		MA 2	No	
ARMA(2,2)	1,000,908	Constant	Yes[1]	1
		AR 1	No	
		AR 2	No	
		MA 1	No	
		MA 2	No	

1 p<.001
2 p<.01

week). We should mention that sales during Finals week of Year 3 were the highest they had ever been at $10,388. The smallest absolute percentage forecast error, 0.57%, was for the week ending July 24, Year 3.

For the AR(1) Box-Jenkins model fitted to the Year 3 weekly sales data, the MSE was 1,843,442 and the mean absolute percentage forecast error was 17.76%. This MSE is over 1.8 times as high as the MSE we obtained when fitting the AR(1) model to the weekly sales data for the Year 1 and 2. The largest absolute percentage forecast error, 65.95%, was for the week ending May 22, Year 3. The smallest absolute percentage forecast error, 0.51%, was for the week ending January 30, Year 3.

As was mentioned in the Introduction, the restaurant manager had been forecasting the current week's sales as equal to the previous week's sales. Applying the manager's approach to the Year 3 weekly sales data, we obtained a MSE of 2,186,701 and a mean absolute percentage forecast error of 17.88%.

For comparison purposes, we also fit the ARMA(2,1) Box-Jenkins model to the Year 3 weekly sales data, since Table 4 indicates this also is a promising model. Well, the MSE turned out to be 2,672,147 and the mean absolute percentage forecast error was 22.06%. We were rather surprised to discover that the ARMA(2,1) model did not forecast the Year 3 weekly sales data as accurately as the AR(1) model, or even the manager's forecasts.

Thus regression model (6) gave more accurate forecasts for the Year 3 weekly sales than either the AR(1) or ARMA(2,1) Box-Jenkins models, or the manager's forecasting method.

In light of these results and the simplicity of the model, the restaurant manager decided to use regression model (6) to forecast sales one-week-ahead.

A Box-Jenkins Transfer Function Model

After the restaurant manager had elected to use regression model (6) for forecasting, the author read an article by Tiao, Box, and Hamming (1975). In this study, the three authors used Box-Jenkins transfer function models to analyze Los Angeles photochemical smog data. These models allow addition of a dummy variable X. Now Marquette University's being in session should produce an immediate, positive response on sales. In a case such as this, the transfer function used should be some constant W, according to Tiao, Box and Hamming (1975, page 266).

We decided to test the following Box-Jenkins transfer function model on our restaurant sales data:

$$\hat{Y}_t = C + WX_t + BY_{t-1}, \quad (3)$$

where

C = a constant

W = the transfer function (a constant)

B = the autoregressive parameter.

A constant term was included in the model, since we did not first difference the data.

We employed SAS to estimate the Box-Jenkins transfer function model parameters, compute the MSE, test the significance of each model parameter, and run a diagnostic check on the residuals. SAS gave us the autocorrelations

$$\hat{Y}_t = 2013.48 + 1159.671X_t + .575545Y_{t-1}.$$

of the residuals, lagged up to 24 periods. The Box-Jenkins transfer function model is:

The MSE turned out to be 974,183. All three model parameters are significant at the .001 level. Only one of the 24 autocorrelations of the residuals is significant at the .05 level (the 95% confidence limits are +/- .192). Reference to Table 4 indicates that only the ARMA (2,1) model has a lower MSE (943,796) than the transfer function model. However, recall that regression model (6) has a MSE of 517,881.

We then tested the Box-Jenkins transfer function model on the 44 weekly sales observations we had obtained for Year 3, as was done in the previous section. For the transfer function model fitted to the Year 3 weekly sales, the MSE was 1,251,350 and the mean absolute percentage forecast error was 17.77%. This MSE is about 28.5% higher than the MSE obtained when fitting the Box-Jenkins transfer function model to the weekly sales data for the Years 1 and 2.

Thus the transfer function model yielded more accurate forecasts (as measured by the MSE) for the Year 3 weekly sales than the AR(1) or ARMA(2,1) models, or the manager's forecasting method. However, it is still the case that regression model (6) gave the most accurate forecasts for the Year 3 weekly sales, as measured by both the MSE and MAPE (1,238,599 and 13.24%).

Discussion

Of the seven regression models and nine Box-Jenkins models considered, regression model (6) appears to be the best choice to forecast the weekly restaurant sales. The fact that the weekly sales for Year 1 and Year 2 displayed almost no trend led us to consider other predictors. The

combination of a dummy variable (denoting whether or not Marquette University was in full session) and the previous week's sales produced a regression model which accurately forecasted Year 1 and Year 2 sales. Unfortunately, this regression model did not forecast nearly as well weekly sales for the first 44 weeks of Year 3 (MSE equals 1,238,559 and mean absolute percentage forecast error equals 13.24%).

To gain some insight as to why this happened, we calculated the correlation matrix for the Year 3 weekly sales data (see Table 5). Again the dummy variable and sales lagged one week appear to be good predictors of Year 3 weekly sales, as was the case with the Year 1 and Year 2 weekly sales. In Table 5 the correlation is very low between time and the dummy variable and moderate between the dummy variable and sales lagged one week, as was the case in Table 1. However, the Year 3 weekly sales have a more substantial, positive trend than the Year 1 and Year 2 weekly sales had (.251 versus .049). In fact, the trend for the Year 3 weekly sales is significantly different from zero at the .10 level (t = 1.6804).

TABLE 5
Correlation Matrix

	Current Sales	Time	Dummy Variable	Sales lagged OneWeek
Current Sales	1.000	.251	.768	.646
Time		1.000	-.064	.288
Dummy Variable			1.000	.464
Lagged Sales				1.000

If this upward trend in weekly sales continues to get stronger in the future, then time should be a good third predictor to add to regression model (6).

*** References ***

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