RISK-ADJUSTMENTS AND REINVESTMENT RATES: NET TERMINAL VALUE

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ABSTRACT

The traditional discounted cash flow techniques of capital investment analysis suffer a variety of problems due to implicit reinvestment assumptions, as well as a general inability to appropriately adjust for differing risk profiles. Net Terminal Value is introduced as a means by which these limitations may be overcome.

INTRODUCTION

Considerable attention has been paid in the literature to the reinvestment rate assumptions inherent in discounted cash flow techniques of analyzing capital expenditures.(1) The problem of reinvestment is a consequence of the mathematics of present value calculations (2) and vanishes when investment analysis is performed with a focus toward maximizing future consumption opportunities in excess of the minimum rate of return required as compensation for undertaking an investment. Net Terminal Value (NTV) utilizes such a future-oriented approach to overcome the limitations imposed by the implicit assumptions of discounted cash flow techniques while allowing for explicit consideration of differing risk profiles of competing investment opportunities. Additionally, NTV accommodates the cash flow patterns of non-normal capital investments.(3)

DISCOUNTED CASH FLOW TECHNIQUES

The objective of capital investment is generally assumed to be the maximization of stockholder wealth. The project that contributes the most to this objective is the appropriate project to select from a set of competing,

mutually exclusive projects. The simplest case usually considered assumes that there is no capital rationing and that the distributions of future cash flows are the same as that of the firm. The relaxation of these assumptions requires modification of the analytical methodology.

The Net Present Value (NPV) criterion of investment selection is considered to be the optimal policy under the assumptions of perfect markets and a single risk class of investments.(4) When a choice between mutually exclusive investments must be made, the project with the highest NPV is preferred. The NPV criterion indicates the value, in today's dollar terms, that accrues to the firm by the acceptance of a specific project without relating the magnitude of the increased value to the size of the initial investment. The concept of an implicit reinvestment of cash flows at the discount rate arises due to attempts to preserve ranking order when measured in terms of future wealth. The consensus that wealth maximization is the assumed objective of the investor is most often presumed to be in present value terms.(5)

The reinvestment issue is usually

how efficiently the firm's limited resources have been invested becomes a primary consideration. Conceptually, capital is rationed to a firm by the marketplace as the cost of obtaining additional capital becomes prohibitively high; nonetheless, many firms often impose strict limits on the capital budget for considerations other than the marginal cost of obtaining additional funds. The Internal Rate of Return (IRR), while reflecting the yield on a per dollar of investment basis, implicitly assumes reinvestment of cash flows at the IRR which must exceed the cost of capital in order for the project to be acceptable. The reinvestment assumption of the IRR thereby contrasts to the NPV assumption of reinvestment at the cost of capital. The conflict in rankings of projects between the two methodologies is a result of the implicit reinvestment assumptions and occurs when either the costs differ (size disparity) or the timing of the cash flows differ.

It has been demonstrated that the application of the cost of capital under the assumption of perfect capital markets should not include reinvestment of cash flows at the firm's average rate on new investment, but rather at the firm's cost of capital as measured by current market rates.⁽⁶⁾ The use of an average rate of return on new investment biases the valuation by attributing returns to the project that are actually due to future opportunities rather than the project itself. For the firm facing capital rationing, on the other hand, the appropriate reinvestment rate would be the marginal rate of return on future investments. This is due to the fact that the timing of internally generated cash flows would influence the firm's ability to take advantage of these future opportunities. Thus, the returns from future investments would accrue to the cash flows generated by a specific project since the availability of funds would determine the extent to which these opportunities could be exploited.⁽⁷⁾

The assumption of certainty about the distributions of future cash flows requires that the risk (variability, or, in terms of modern portfolio theory, covariance) of the projects under consideration be the same as the firm overall in order to justify the use of the firm's cost of capital as a discount rate. When the risks of projects differ from that of the firm as a whole, the approach to project analysis must be significantly altered in order to accommodate the implicit reinvestment assumption. By discounting an above average risk project at a rate that exceeds the cost of capital, the reinvestment assumption implicitly assumes that the future cash flows of the project will be invested in future projects that are also of a high risk nature. Thus, investing in an above average risk project assumes that the new risk profile will be maintained indefinitely. Conversely, discounting at the average cost of capital in order to reflect future investments that are more typical of the firm in terms of risk taken would ignore the higher risk of the specific project.

The conceptually best adjustment for risk is the use of certainty equivalents, where risky expected values are converted into amounts which, if known with certainty, would be considered the equivalent of the risky expected values. Unlike the use of a risk-adjusted discount rate, the certainty equivalent method of adjusting for risk does not assume that the cash flows become riskier over time (although certainty equivalents can accommodate this situation). Additionally, the certainty equivalent method deals explicitly with one's utility function and is therefore more directly related to the individual's objective of utility maximization. The objection most often raised to the certainty equivalent approach is that it requires knowing one's utility function and indifference curves. This becomes even more difficult when applied to a corporation which consists of numerous individuals.

NET TERMINAL VALUE

The use of risk-adjusted required rates of return provides the means by which risk may be properly accounted for in a manner which can be implemented in practice and avoids many of the problems inherent in the use of discounted cash flow techniques.

The Net Terminal Value (NTV) approach to risk adjustment is the following: a cumulative cash account is established which is compounded at the risk-adjusted required rate of return if the value is negative, but compounded at the average cost of capital if the value of the cumulative account is positive.(8) Thus, a negative value at any time represents a net investment to-date in the risky project requiring a risk-adjusted rate of return, while a positive value would be indicative of cumulative funds above and beyond the net investment and the risk-adjusted return. Competing projects would be selected on the basis of the one with the highest NTV.

Consider a project with a life of T periods. Using a forward recursive procedure, the net value of the project at any time t ($0 \leq t \leq T$) is defined as

$$NV_t = NV_{t-1}(1+k_t) + CF_t \quad (1)$$

where

$CF_t \equiv$ Cash flow on date t

$k_t \equiv$ compounding rate for period t

$NV_t \equiv$ the net value of the project at the end of period t

The compounding rate for period t depends upon the sign of the net value for period $t-1$. If the net value for period $t-1$ is negative, the compounding rate equals the risk-adjusted required rate of return since the project is still utilizing net financing costs. If the net value at time $t-1$ is positive, then the project has generated a net surplus,

above and beyond the recovery of principal and required return, which can be invested elsewhere at the corporate opportunity rate, the cost of capital. Thus, the compounding rate, k_t , is a step function where

$$K = \begin{cases} \text{Required rate of return} & \text{if } NV_{t-1} < 0 \\ \text{Cost of capital} & \text{if } NV_{t-1} \geq 0 \end{cases} \quad (2)$$

The boundary conditions to close the problem are

$$NV_0 = CF_0 \text{ initial condition} \quad (3a)$$

$$NTV = NV_T \text{ final condition} \quad (3b)$$

where NTV is the Net Terminal Value.

Figure 1 illustrates the effect such an approach would have on project analysis. The riskier project A, with a required rate of return of 15%, has a risk-adjusted NPV of \$2,126 and an IRR of 22.81%. Average risk project B has a risk-adjusted NPV of \$2,300 when discounted at a 10% cost of capital and an IRR of 17.42%. Based on a comparison of risk-adjusted NPVs, project B is more desirable. Note that the IRR exceeds the risk-adjusted required rate of return in both instances, but gives no indication of which project is preferable.

From a risk-adjusted NTV approach, the rankings are reversed. Project A has an NTV of \$3,644 which assumes that the excess cash inflows are reinvested in average risk projects (10%). Unlike the IRR and risk-adjusted NPV approaches, there is no implicit assumption of reinvestment in above average risk projects (that is, at a high rate of return); the only assumption is that excess returns are reinvested at the corporate opportunity rate, the cost of

Figure 1. Net Terminal Value Illustration

Project A

$$\begin{aligned}
 (15,000)(1.15) &= (17,250) \\
 + \quad 7,000 & \\
 \hline
 (10,250)(1.15) &= (11,787) \\
 + \quad 7,000 & \\
 \hline
 (4,788)(1.15) &= (5,506) \\
 + \quad 7,000 & \\
 \hline
 1,494(1.1) &= 1,644 \\
 + \quad 2,000 & \\
 \hline
 \text{NTV} &= 3,644
 \end{aligned}$$

NPV @ 15% = \$2,126

IRR = 22.8%

PV of NTV @ 10% = \$2,489

BI = .24

Project B

$$\begin{aligned}
 (15,000)(1.1) &= (16,500) \\
 + \quad 6,000 & \\
 \hline
 (10,500)(1.1) &= (11,550) \\
 + \quad 6,000 & \\
 \hline
 (5,550)(1.1) &= (6,105) \\
 + \quad 6,000 & \\
 \hline
 (105)(1.1) &= (116) \\
 + \quad 3,483 & \\
 \hline
 \text{NTV} &= 3,368
 \end{aligned}$$

NPV @ 10% = \$2,300

IRR = 17.4%

PV of NTV @ 10% = \$2,300

BI = .22

capital (or, equivalently, in average risk projects). The risk-adjusted NTV investment criterion has a straight-forward interpretation as well--it is the increase in future wealth above and beyond the risk-adjusted required wealth increase. Since the risk has been adjusted for in the calculation, the present value of the NTV can be determined by discounting at the cost of

capital, leaving the relative rankings unchanged. The present value of Project A's NTV is calculated to be \$2,489 as compared to the \$2,300 present value of Project B's NTV.

A COMPARISON OF METHODOLOGIES

In addition to adjusting for differential risks, the NTV approach overcomes

many of the other problems inherent in the use of discounted cash flow methodologies, as Figure 2 illustrates.

In the event that competing projects have unequal lives, the NTV of the shorter lived project should be compounded at the cost of capital to the

end of the longer lived project for comparative purposes. Project 1, with a useful life of three years, has an NTV of \$361 at the end of the third year. In order to make a valid comparison with the four year life of Project 2, the NTV of Project 1 is compounded at the cost of capital to the

Figure 2. Comparison of Methodologies

Project 1

Project 2

end of the fourth year. This avoids the necessity of calculating replacement chains (which assume that technology, cost relationships, etc., remain constant) or equivalent annual annuities (infinite replacement chains). The NTV approach also reflects the investor's desire to realize a return of principal first since the excess funds do not begin to accumulate until both principal and required return are recovered. Unlike the present value payback method (9) which is designed to reflect this desire for rapid return of principal, however, the NTV continues to consider cash flows beyond the payback period.

Reinforcing the use of the risk-adjusted NTV as a decision criterion is the situation in which there is a future cash outflow. While a higher required rate of return in the calculation of a risk-adjusted NPV would actually result in a lower present value of the cash outflow (increasing the project's desirability), the same would act to make the NTV more negative (less desirable). The NTV approach also circumvents the problem of multiple IRRs. Project 2 illustrates both of these points. In addition to having two IRRs, the NPV of Project 2 increases at higher discount rates, with a maximum occurring at a discount rate of approximately 20%.

In the case of capital rationing, the optimal solution methodology would be to formulate the mixed integer programming problem in order to maximize the total NTV, just as this is the appropriate approach in the case of standard discounted cash flow techniques. In lieu of a formal programming solution, however, the determination of which of two competing projects yielded the largest NTV relative to the initial investment may be accomplished simply by creating a Benefit Index (BI) of

The BI would measure the NTV relative to the size of the investment. (Alternatively, the BI could be defined in terms of the present value of the NTV relative to the initial investment.)

Several authors have recognized the superiority of a terminal value approach to capital investment analysis under conditions that result in the failure of the NPV criterion.(10) Their focus, however, has been on the terminal value of the cash inflows rather than the total cash flows which include the initial outlay. The inclusion of the initial investment in the analysis overcomes the necessity of explicit reinvestment assumptions as well as accommodating the problems of adjusting for different risk profiles.

CONCLUSIONS

The traditional present value approaches to capital investment analysis suffer from a variety of limitations, foremost of which are the implicit reinvestment assumptions and inability to adequately compensate for differences in risk. The employment of Net Terminal Value, which is consistent with the view of investment as the exchange of current consumption for future consumption, centers attention on the accumulation of wealth in excess of the recapture of principal and required return. Net Terminal Value thereby overcomes many of the shortfalls inherent in the use of the discounted cash flow techniques, including unequal lives, future cash outlays, and multiple internal rates of return, while at the same time allowing for explicit risk adjustments. By focusing on the maximization of the surplus wealth of the investor after compensation for risk, the Net Terminal Value criterion results in the unambiguous selection of projects.

$$BI = \frac{NTV}{COST} \quad (4)$$

FOOTNOTES

1. See Beidleman [2] for a summary of differing postures on the subject.
2. See Solomon [13] and Renshaw [12].
3. See Beedles [1].
4. Numerous other assumptions, such as dividend irrelevancy and the separation of financing and investment decisions, bear upon the issue as well but are outside the scope of this article.
5. The Capital Asset Pricing Model explicitly assumes that the investor's objective is to maximize the utility of terminal wealth.
6. See Howe [7] and Meyer [11].
7. An excellent example of the effect that limited access to the capital markets can have on project evaluation is provided in Jean [8].
8. The average cost of capital is a marginal investment opportunity since the firm always has the option of buying back its own stocks and bonds.
9. See Bierman and Smidt [3], p. 41.
10. See Beidleman [2], Brigham [4], and Doenges [5].

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