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INTERMETROPOLITAN COMPARISONS OF PRODUCTION FUNCTION

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Abstract

A nonhomogeneous production is used to study the features of the production technology across U.S. cities. We compute marginal productivities and scale elasticities for different levels of inputs and outputs. The form of the production function allows variable returns to scale. We can also test the Cobb-Douglas and constant elasticity of substitution forms within the nonhomogeneous specification. Conclusions are drawn concerning returns to scale across cities of different sizes.

1. INTRODUCTION

In his article on the productivity of cities Sveikauskas (1975) suggests that productivity may be systematically higher as city size increases. He argues that a larger city allows more specialization and a greater division of labor which results in an increase in productivity.[1] In a related study Segal (1976) studied the production differences among cities, in an attempt to ascertain whether the larger urban areas have a significant production advantage over the relatively smaller ones. His main concern was to explain variations in labor productivity across urban areas.

The approach taken in these two studies to estimate features of the production technology across metropolitan areas is the Cobb-Douglas (CD) and Constant Elasticity of Substitution (CES) production function forms.[2] The traditional C-D and CES functional forms severely constrain the partial elasticity of substitution to be constant and in the case of the Cobb-Douglas type to be equal to unity. Moreover, the C-D specification assumes constant returns to scale.

In his article on interregional production structures Vinod (1972) suggests a nonhomogeneous production function as a new tool for studying regional economics from empirical studies of production functions. We adopt Vinod's new formulation to study intermetropolitan production structures. The major purpose of this new formulation is to provide empirical computations of marginal productivities, and scale elasticities for different levels of inputs and outputs for the purpose of comparison over the sample observations.[3]

In this study, we use a multiplicative nonhomogeneous production function (hereafter referred to as M-NH) which is presented by Vinod (1972). The nonhomogeneous production function is a generalization of the Cobb-Douglas formulation. The function is linear in its parameters and is amenable to estimation by ordinary least squares techniques (OLS). In addition, the M-NH function removes some of the difficulties in economic interpretation associated with the CES, and it possesses less specification bias than either the C-D and CES forms.[4]

In section 2 we review some of the salient properties of the M-NH production function. In section 3 we discuss the data and present the empirical results of fitting the M-NH formation to aggregate data for 58 metropolitan areas for 1967. The results are compared with the earlier empirical findings from the same body of data. Major differences in implied economic patterns are observed and discussed. In the last section we summarize our findings and draw some implications.

2. THE NONHOMOGENEOUS PRODUCTION FUNCTION

Consider a nonhomogeneous production formulation:

$$Q = a_0 X_1^{a_1 + a_3 \ln x_2} x_2^{a_2}$$

where Q is output, X_1 and X_2 are capital and labor inputs respectively, and the α's denote parameters.[5] Equation (1) can be rewritten in log form and when $\alpha_3 = 0$; the log form of the nonhomogeneous production function collapses to the Cobb-Douglas formulation.[6] From the log form it follows that the marginal elasticities of capital and labor, $\in_{x}1$ and $\in_{X}2$,respectively are obtained by partially differentiating the log of output with respect to the log of the inputs. The scale elasticities $\in$ (defined as the sum of the marginal elasticities of the products) can also be derived. The scale elasticity is defined as the percentage change in output associated with a simultaneous small percentage change equal to all inputs. According to the formula for Scale elasticity $\in$ can assume any value, which imply that returns to scale are variable over the scale of production. The marginal products of capital and labor, MP_X1 and MP_X2, respectively can also be derived.[7]

3. ESTIMATION, DATA AND EMPIRICAL RESULTS

The M-NH function is linear in its parameters, and can readily be estimated by ordinary least squares methods. A unique feature of the specifica tion is that we can obtain the numerical estimates of marginal products, marginal elasticities and scale elasticities for each observed level of Q, X_1 and X_2.[8] For the purpose of estimation, an additive error term U_t is introduced. We assume that its mean and variance are E(U) = 0, Var(U) = ^2_U I,

where I is the identity matrix.

Adding to this production function, we would want to find out whether the larger cities have any advantage such as agglomeration economies and/or increasing return to scale. We would also like to know whether the location of the SMSAs according to region will have any effect on the production process. For that we add on a dummy variable for the size and another dummy variable for the region. For those SMSAs with more than two millions in population we assign a value of 1 and zero otherwise as the dummy variable for size, and those located in the South we assign a value of 1 and zero otherwise as the dummy variable for region.[9] Our final equation on which we fit the data is:

$$\ln Q = \alpha_0 + \alpha_1 \ln X_1 + \alpha_2 \ln X_2 + \alpha_3 \ln X_1 \ln X_2 + \alpha_4 S + \alpha_5 R \qquad (2)$$

A priori, we would expect α_1 through α_4 to be positive. If large cities possess what we call agglomeration economies, size then would make a significant difference in the production process and thus α_4 should be positive. As for α_5, if the contention is correct that the South has been lagging behind the rest of the country in development because of lack of skilled labor, entrepreneurial talent etc.... then we would expect α_5 to be negative.

4. THE DATA AND EMPIRICAL RESULTS

The sample size for this study consists of 58 observations for SMSA's used in Segal's study.[10] The data for capital (X_1) are taken directly from that study. The data for value of output (Q) and total non-agricultural employment (X_2) were taken from the same sources provided by Segal. The value of output (Q) is adjusted for transfer payments and contribution to social insurance. [The sources and details of all the data are given in Segal (1976)].

The ordinary least squares estimates for equation (6) are presented in Table 1. Four regressions are presented. The first is without the measure of SMSA size and regional effects. The second includes a measure of size; the third includes a measure of regional effects and the fourth includes measures of both size and regional effects. The four regressions are almost identical: individual coefficients change little, and the coefficients of determination are quite high and show hardly any variation.

At the 5-percent level of significance all regression coefficients are significant except the product term of capital and labor. That the coefficient of the product term is not significantly different from zero, seems to confirm Segal's findings that SMSAs production function have constant returns to scale. Thus, it seems as though the production function takes the regular Cobb-Douglas form with constant marginal elasticities of inputs and constant returns to scale.

The coefficient of the size variable shows that size of cities do play an important role in the production process. In particular, we find that the production function for large SMSAs is shifted upward by about 11 to 14 percent. This is slightly higher than Segal's findings of an upward shift by about 8 percent due to agglomeration economies. The regional dummy measure indicate that regions in the south lag behind the rest of the country in increases in output.

Now we concentrate on the empirical estimates of marginal productivities of the inputs and scale elasticities generated by equation Model IV (Table 1) for each SMSA in the sample. These results are shown in Table 2 for the entire sample of SMSA's. The values of the marginal product of capital are ranked in decreasing order from the highest value of MPK (.7858) to the lowest value of MPK (.4119). We also show the associated marginal product of labor (MPL) and scale elasticities series ($\in$). A closer look at the scale elasticities series indicates that the values range from .99 to 1.07 which is not significantly different from unity. This means that a one percent change in both inputs would lead to an increase in output of about 1 percent. Also recall that we found that the production function for large SMSAs is shifted upward by about 11 percent to 14 percent. Thus we can say that the production function for each SMSA exhibits constant returns to scale with the largest tones having an advantage in the form of agglomeration economies accounting for between 11 percent to 14 percent of the output.[11]

According to the neoclassical model, if the production function exhibits constant returns to scale, then the marginal product of capital and the marginal product of labor should depend on the capital-labor ratio. We would expect the marginal product of capital to vary inversely with the capital-labor ratio and the marginal product of labor to vary directly with the capital-labor ratio. Thus we would expect an inverse relationship between the marginal product of capital and the marginal product of labor. Looking at the marginal product of capital and the marginal product of labor (Table 2), series, we may want to analyze the results in terms of what we would expect to follow from the neoclassical theory. Our results do not allow us to arrive at such a clear pattern. Several factors may explain this. The variations of the marginal productivities may be due to the highly aggregate data; to differences in production functions for different industries which make up the aggregate set; and to the effect of random variations. For these reasons a comparison of the marginal productivities along neoclassical lines is not straightforward.

5. CONCLUSION

In this paper we took advantage of the available information on capital inputs in various SMSAs and estimated a nonhomogeneous production function from which we generated values of marginal productivities and scale elasticities for each of the SMSAs in the sample. We find that there are constant returns to scale across SMSAs of different sizes. We also find that large cities may increase their output by about eleven to fourteen percent due to agglomeration economies.

Overall our findings using the nonhomogeneous function of production are of special interest in that we obtain measures of marginal productivities and scale elasticities for each SMSA. We provide more information than these obtained from the usual regressions on cross section data (e.g. Segal (1976) which are obtained for the average or typical entity which might not exist. However, we need more disaggregated data at the industry level across cities to carry out more tests of the neoclassical model.

FOOTNOTES

1. Actually Sveikauskas [2] argues that both static and dynamic forces influence productivity. He notes that the most important static advantage is specialization while the dominant dynamic benefit is urban concentration which favors technological progress.

2. Whereas Segal fitted the (CD) production function to metropolitan data at high levels of aggregation, Sveikauskas fitted his (CES) production function to central city data at a more disaggregated level, by using data for two-digit manufacturing industries across cities.

3. These empirical constructs are important if we are interested in practical policy recommendations or implications. Usually when a regression is run to fit cross sectional data, we obtain the result for an average entity which might not even exist. The nonhomogeneous specification allows us to go back to each observation and study the implications of the production function for each observation, in this case for each SMSA.

4. See Vinod [3] for a more detailed justification and development of the multiplicative nonhomogeneous production function.

5. The nonhomogeneous production function can be easily generalized to account for more than two inputs.

6. Note also that if we include the squared terms of the logarithm of x_1 and x_2 respectively, we are expressing the log-quadratic production function. THis point is also expanded on in Vinod [3].

7. Mathematical derivations available from authors.

Table 1

OLS Estimates of the Nonhomogeneous Production Function[a]

Model No.	Constant Term (α_0)	Capital Input (X_1) (α_1)	Labor Input (X_2) (α_2)	Product Term of Capital and Labor Input (α_3)	Size Dummy (α_4)	Regional Dummy (α_5)	R^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
I:	2.058	.079	.796*	.011	---	---	.991
		(1.17)	(5.48)	(0.92)			
II:	1.757	.158*	.801*	.002	.146*	---	.992
		(2.32)	(5.93)	(0.17)	(3.04)		
III:	1.846	.159*	.726*	.009	---	-.134*	.992
IV:	1.625	.214*	.737*	.002	.117*	-.122*	.994
		(3.61)	(6.33)	(0.25)	(2.80)	(-4.54)	

[a]The numbers in parentheses below the estimated coefficients are student t-statistics of the null hypothesis of no association.

*Coefficients are significant at .05 level.

Table 2

Marginal Products and Elasticity Measures

SMSA/State	MPK (1)	MPL (2)	ε (3)
Utica-Rome, N.Y.	.7858	9,554	.99
Reading, PA	.7331	8,393	.99
Lancaster, PA	.6705	8,885	.99
Albany, Schenecdaty, N.Y.	.6630	9,401	1.01
Pittsburgh, PA	.6458	9,916	1.03
Philadelphia, PA	.6405	9,971	1.05
Canton, OH	.6341	9,318	1.00
St. Louis, MO	.6228	9,799	1.04
Kansas City, MO	.6168	9,479	1.02
Peoria, IL	.6160	10,571	1.00
Boston, MA	.6125	10,946	1.04
Allentown, PA	.6078	8,794	1.01
Tulsa, OK	.6039	9,796	1.01
New York, NY	.6033	11,139	1.07
Syracuse, NY	.5898	9,422	1.01
Wilkes-Barre-Hazelton, PA	.5775	7,498	1.00
Davenport-Rock Island, Moline-IA, IL	.5760	10,117	1.01
Springfield, MA	.5722	9,100	1.01
Indianapolis, IN	.5704	9,944	1.03
Grand Rapids, MI	.5695	9,859	1.01
Dayton, OH	.5693	10,097	1.02
Toledo, OH	.5689	10,147	1.02
Chicago, IL	.5654	10,260	1.06
Detroit, MI	.5650	11,366	1.05
Rochester, NY	.5622	10,082	1.02
Erie, PA	.5601	9,189	.99
Wichita, KN	.5546	9,541	1.01
Milwaukee, WI	.5544	10,061	1.03
Cleveland, OH	.5482	10,037	1.04
Cincinnati, OH	.5424	10,122	1.03
Baltimore, MD	.5292	9,433	1.04
Minneapolis, MN	.5244	9,606	1.04
Youngstown, OH	.5221	9,269	1.01
San Francisco, Oakland, CA	.5207	11,684	1.05
Kansas City, MO	.5161	9,415	1.03
Louisville, KY-IN	.5032	9,441	1.02
Akron, OH	.4929	9,813	1.02
Portland, OR	.4878	9,836	1.03
Washington, D.C.	.4834	15,333	1.04
Birmingham, AL	.4734	8,842	1.02
Denver, CO	.4714	9,664	1.03
Columbus, OH	.4693	8,869	1.03
Phoenix, AR	.4680	10,243	1.02
Flint, MI	.4568	10,788	1.01
Los Angeles, Long Beach, CA	.4565	11,047	1.07
Fort Worth, TX	.4530	8,959	1.02
Seattle, WA	.4530	8,959	1.02
Richmond, VA	.4497	8,957	1.02
New Orleans, LA	.4467	9,405	1.03
Chattanooga, TN	.4466	8,281	1.01
Atlanta, GA	.4447	8,979	1.04
Nashville, Davidson, TN	.4421	8,462	1.02
Dallas, TX	.4420	9,413	1.04
Memphis, TN	.4387	8,862	1.02
San Jose, CA	.4360	10,560	1.03
Tampa, St. Petersberg, Fl	.4128	9,479	1.02
San Bernardino, CA	.4149	11,003	1.03
Houston, TX	.4043	9,351	1.04

8. Note that since E_{x1}, E_{x2}, E are all elasticities, they are independent of units of measurement.

9. For the size dummy variable, we follow Segal and separate the SMSAs into two categories: those with a population below 2 million and those above that cut off. The criteria for the cut-off at 2 million are discussed in Segal [1].

10. We find this study to be of independent interest since it is almost impossible to obtain information on capital inputs for cities. THis available data affords us the opportunity to analyze features of the production structure for cities. To the best of our knowledge we know of no study that has attempted an analysis of cities that allows comparisons of key economic constructs across them.

11. By using data that are highly aggregated we have lost some information, since each SMSA has a wide variety of industries. Some of these industries might enjoy increasing returns to scale while others might be operating under conditions of decreasing returns to scale. Thus on the operating under conditions of decreasing returns to scale. Thus on the aggregate, it is difficult to ascertain whether larger cities do enjoy increasing returns to scale. This problem is compounded by the fact that the larger the SMSA is, the more variety it has in the industry mix. The smaller SMSA might have a few dominant industries, but the larger ones probable have a much wider range of industries and thus the aggregate data available does not tell us the whole story. Thus the industry mix is of crucial importance. WHat is needed is data for each industry which would enable us to ascertain whether the industry itself is having increasing or constant or decreasing return to scale. However, with our data the results seem plausible since we find the coefficient a_3 to be insignificant. Furthermore, our results seem to confirm Segal's [1] findings of a constant returns to scale production function for the same sample.

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