The Rule of 78's: Practical Applications and Theoretical Limitations

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INTRODUCTION

There are various techniques used for recording income and expense which require additional clarification from that currently found in the literature. One such technique is the rule of seventy-eights. This technique is widely used by lending institutions as a means to account for realized interest income on certain types of loan transactions. It may also be employed by retail merchants who transact sales on an installment basis. The rule of seventy-eights is a computational procedure that is used to determine the portion of precomputed interest charges which have been earned as of a particular date over the course (or life) of an installment loan transaction. It is also used, in a reciprocal manner, to compute interest rebates on installment loans that are paid in full prior to maturity.

The rule of seventy-eights is applied to installment loan transactions wherein interest is computed on the basis of an Add-On rate--where such a rate is equivalent to the stated rate per the dollar amount of the loan per the number of years that the loan will be outstanding (e.g., 12% per 100 per year). When using Add-On, the interest is charged as if the loan will be repaid in one lump sum on the maturity date, even though it will actually be systematically amortized over the life of the loan. Clearly, a deficiency exists with regard to recognizing revenue since an Add-On rate, per se, gives no consideration to the principal amount which will be outstanding at various points over the course of the loan.[1] To overcome this deficiency, the rule of seventy-eights is employed in order to systematically account for interest earnings.

The purpose of this article is to provide an in-depth explication of the basic concepts and operational precepts of the rule of seventy-eights so that the accountant may better understand and, therefore, better advise his client in regard to interest income recognition. This income recognition technique does have wide application and as will be shown, is a theoretically valid approach to recording interest revenue so long as certain constraints are not violated.

REVENUE RECOGNITION

For accounting purposes, revenue is generally recognized as a flow process, but there is no standardized definition for the term. The Committee on Accounting Concepts and Standards of the American Accounting Association proposes the traditional definition where revenue is considered a flow of assets or net assets into the firm as a result of sales of goods or services.[2] This approach is supported by Sprouse and Moonitz in their definition of revenue as "the increase in the net assets of an enterprise as a result of the production or delivery of goods and the rendering of services".[3] Both of the preceding definitions indicate an exchange requirement; however, revenue can

also be realized when permitting others to use enterprise resources--such as the interest which accrues over time in a lending transaction. In the case of banks and other lending institutions, as well as with those retail outlets that customarily deal in extended credit, interest on installment loans may constitute an important, or the main source of current income; obviously, such income should be properly accounted for in terms of generally accepted accounting principles.[4]

These definitions, however, lend confusion to the problems of measurement and timing of interest income since management does have the option of reporting such revenue by use of the cash- or accrual-basis of accounting.[5] This paper will attempt to isolate the interest income area of revenue recognition and delve into the interpretation and application of the rule of seventy-eights as related to certain types of lending activities.

<div align="center">BASIC CONCEPTS OF THE RULE</div>

Application of the rule of seventy-eights involves the following two steps: (1) Determination or computation of the appropriate income factor; and then, (2) Multiplication of this factor by the loan's total precomputed in-interest charge to yield the dollar amount of realized income. The factor takes the form of a fraction and ranges between some value greater than zero but no more than one. In order to compute the correct value, it is necessary to separately determine the denominator and the numerator of the fraction. The denominator is equal to the sum of the values of the number of months in the original term of the loan, where the first month carries a value of one through the last month which carries a value equal to the number of months in the original term of the loan.[6] For example, with an 18 month loan, the denominator would equal:

$$1 + 2 + 3 + \ldots + 16 + 17 + 18 = 171$$

Moving to the numerator of the factor, it is found by determining the sum of the values of the number of months that the loan has been outstanding beginning with the highest value for the series (where this value equals the number of months in the original term of the loan) and adding to this the next highest value for each additional month that the loan has been outstanding.[7] For example, with an 18 month loan that has been outstanding for 12 months, the numerator equals:

$$18 + 17 + 16 + \ldots + 9 + 8 + 7 = 150$$

and the realized income factor equals:

$$\frac{18 + 17 + 16 + \ldots + 9 + 8 + 7}{1 + 2 + 3 \ldots + 16 + 17 + 18} = \frac{150}{171} = 87.72\%$$

The figure of 87.72 represents the percent of total precomputed interest charges that have been earned by the twelfth month of an 18 month loan. Obviously, the percentage figure will change according to the term of the loan and the number of months that have expired. However, the factor itself is not affected by the size of the loan or the Add-On rate being used. These variables are considered in the amount of total precomputed interest charges. In Add-On loans, the total amount of interest charges are computed on the basis of: (1) the interest rate employed; (2) the total amount loaned; and

(3) the length of the loan. For example, the precomputed interest charges for a twelve percent, $2,000, 18 month loan would be: (.12) (2,000) (1.5) = $360. And it is to this, or some comparable figure, that the interest earned <u>factor</u> is applied in order to determine the <u>dollar</u> amount of interest income earned; for the above series of illustrations, the amount of recognized income would equal: (.8772) (360) = $315.79.

The comments and examples above demonstrate the computational procedures associated with two of the basic applications of the rule of seventy-eights.[8] The interest earned over a period of time is actually equal to the sum of individual monthly realized income; where the amount of monthly income declines over time. This principle is illustrated below:

For an 18 month loan that has been outstanding for 12 months, the interest earned factor equals:

$$\frac{18 + 17 + 16 + \ldots + 9 + 8 + 7}{1 + 2 + 3 + \ldots + 16 + 17 + 18} = \frac{150}{171} = 87.82\%$$

This, of course, is equivalent to:

$$\frac{18}{171} + \frac{17}{171} + \frac{16}{171} + \ldots + \frac{9}{171} + \frac{8}{171} + \frac{7}{171} = \frac{150}{171}$$

In turn, this equivalency can be restated as:

$$10.53\% + 9.94\% + 9.36\% + \ldots + 5.26\% + 4.68\% + 4.09\% = 87.72\%$$

where 18/171 or 10.53% equals realized income in the first month, 17/171 or 9.94% equals realized income in the second month, and so on down to 7/171 or 4.09% as realized income in the twelfth month of this 18 month loan.

Clearly, the rule of seventy-eights provides the amount of interest that has been earned at interim stages over the course of a loan. More importantly, it does so by recognizing the greatest monthly income in the earlier stages of the loan; i.e., when the greatest principal balance is outstanding.

The foregoing discussion and computational procedures have been couched in terms of installment loans that are amortized on a monthly basis. While this is the most common type of installment loan, with slight adjustments, the rule of seventy-eights may also be used to compute realized income in situations where the credit is repaid in a manner other than on a monthly basis. For there are really only three conditions which need exist in order to justify the use of the rule of seventy-eights. First, interest charges must be computed on an Add-On basis; second, the loan must be amortized in equal dollar installments; third, the installment payments must be made in equal intervals.[9] So long as each of these conditions is met, the rule of seventy-eights may <u>technically</u> be applied to installment credits which are amortized in any manner; for example, on a quarterly basis. The factor computational procedures associated with non-monthly repayment schedules is very similar to those described above. The basic difference is that rather than working with the values of the number of months in the loan, the procedure depends on the user working with the values of the number of <u>payment periods</u> in the loan.[10]

ACCURACY AND OPERATIONAL PRECEPTS OF THE RULE

Because Add-On interest rates do not recognize the fact that the principal balance of an installment loan declines over time, it is necessary to use

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the rule of seventy-eights in order to determine realized income. It has been shown previously that the rule does attempt to relate the amount of monthly interest income to the outstanding principal balance of the loan. Yet, one important question remains unresolved; i.e., just how well does the rule of seventy-eights accomplish this function? In order to judge the validity of the system, it is necessary to go beyond basic mathematics and consider instead the underlying principles upon which the rule is based. In effect, what the rule accomplishes is the recognition of interest earnings in a manner which approximates that which would accrue on an equivalent simple interest basis. However, certain conditions or constraints must be fulfilled in order to obtain a tolerable accuracy level.[11]

Although there is no simple formula for conversion, any Add-On loan can be converted to a comparable installment loan where interest is computed on the basis of an equivalent simple interest rate. Exhibit I provides an illustrative list of equivalent simple interest rates for various Add-On rates with different loan maturities.

EXHIBIT I

RELATIONSHIP BETWEEN ADD-ON AND SIMPLE INTEREST RATES

Add-On Interest Rate	Simple Interest Rates for Selected Loan Maturities (given equal monthly amortization)				
	12 Mos.	18 Mos.	24 Mos.	36 Mos.	60 Mos.
8%	14.45	14.65	14.68	14.55	14.13
9%	16.22	16.42	16.43	16.24	15.71
10%	17.97	18.17	18.16	17.92	17.27
11%	19.72	19.91	19.87	19.57	18.80
12%	21.46	21.64	21.57	21.20	20.31
13%	23.19	23.36	23.26	22.81	21.79
14%	24.91	25.06	24.92	24.40	23.25

In order to find the exact equivalent simple interest for a given Add-On loan transaction, it is necessary to use the present value concept. Specifically, one must determine the discount rate which equates the payment annuity to the principal amount of the loan--the discount rate providing the desired simple interest rate. This procedure can be put in the form of the equation:

$$L = \sum_{t=1}^{n} P_t (1+i)^{-t}$$

where: L = principal amount of the loan;
P = the amount of the loan payment;
n = the number of payments necessary to amortize the loan (the number of months in the term of the loan); and
i = the interest or discount rate per payment period (for a loan amortized on a monthly basis, i equals the annual rate divided by 12).

Admittedly, when the calculations are done manually,[12] the procedure is cumbersome; yet, if such conversion is desired, there is no alternative to present value.

The rule of seventy-eights accounts for interest income in the mechanical

manner as set out in the previous portion of this paper; but this accounting is such that, given certain conditions, it actually results in earnings which are comparable or close to those which would accrue with use of an equivalent simple interest rate. Exhibit II provides an illustration of the interest income that is recognized through application of the rule of seventy-eights vs. the earnings realized on the basis of simple interest. In this illustration, the borrower is assumed to have taken out a $1,000.00, one-year, twelve percent Add-On loan with equal monthly payments of $93.33. This transaction would be equivalent to a similar 21.45% simple interest loan; i.e., such a loan would also require equal monthly payments of $93.33 and would involve total precomputed interest charges of $120.00. Observe in Exhibit II the similarity of all values throughout the life of the loan--note especially the near equality of comparative interest earnings. Note, however, that because there are differences in income resulting in an accelerated recognition of earnings (albeit small), the true rate of interest being charged on an _interim_ basis through use of the rule of seventy-eights is actually somewhat greater than that of the equivalent simple interest rate.

It is clear from the above illustration that use of the rule of seventy-eights can result in interest earnings that are similar to those recorded on a simple interest basis; i.e., where the interest income is relative to the unrepaid principal balance of the loan. Of course, as evidenced by the difference in earnings, the methods are not perfect substitutes for one another nor can they ever be, as long as use of the rule results in accelerated recognition of earnings--as was the case in Exhibit II. But since the absolute ($) and relative (%) earnings differentials are so small, the above illustration suggests a high degree of accuracy and considerable justification for using the rule of seventy-eights.

EXHIBIT II

Monthly earnings from (Hypothetical) $1,000, One-Year Installment
Loan Transaction: Earnings As Realized from
Application of Rule of Seventy-Eights vs. Simple Interest

	12% Add-On; 93.33 Mon Pmts		21.45% Simple 93.33 Mon Pmts		Monthly Diff. in Earnings		Cum. Diff. in Earnings	
Pmts	Prin.	Int.*	Prin.	Int**	$	%***	$	%****
1	$74.87	$18.46	$75.46	$17.88	0.58	3.28%	0.58	3.28%
2	$76.41	$16.92	$76.81	$16.53	0.40	2.42%	0.99	2.87%
3	$77.95	$15.38	$78.18	$15.15	0.23	1.55%	1.22	2.46%
4	$79.48	$13.85	$79.58	$13.76	0.09	0.68%	1.31	2.07%
5	$81.02	$12.31	$81.00	$12.33	-0.02	-0.19%	1.29	1.71%
6	$82.56	$10.77	$82.45	$10.89	-0.11	-1.04%	1.18	1.36%
7	$84.10	$ 9.23	$83.92	$ 9.41	-0.18	-1.89%	1.00	1.04%
8	$85.64	$ 7.69	$85.42	$ 7.91	-0.22	-2.74%	0.78	0.75%
9	$87.18	$ 6.15	$86.95	$ 6.38	-0.23	-3.57%	0.55	0.50%
10	$88.71	$ 4.62	$88.50	$ 4.83	-0.21	-4.39%	0.34	0.30%
11	$90.25	$ 3.08	$90.08	$ 3.25	-0.17	-5.19%	0.17	0.15%
12	$91.79	$ 1.54	$91.69	$ 1.64	-0.10	-5.91%	0.08	0.06%
Totals	$999.96	$120.00	$1,000.04	$119.96	0.08			

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Situation: Borrower repays loan in 12 equal ($93.33) monthly installments;
payments being made on regularly scheduled payment dates: the
first payment one month from date of transaction; the 21.45% simple
interest rate is equivalent to the 12% Add-On.

*	Interest earnings as computed on rule of seventy-eights.
**	Interest earnings as computed on principle of simple interest.
***	Relative (%) diff. = dollar amount of diff. relative to amount charged on simple interest basis.
****	Cum. % diff. = cum. dollar amount of diff. relative to cum. amount of interest as charged on simple basis

The example discussed above is, of course, but one of a wide range of
possible loan transactions; as a result, the effects of interest rate, size,
and maturity have not been considered up to this point. The effect of the
interest rate feature was examined through a series of tests wherein the Add-
On rate was changed as the size of the loan and the maturity were held
constant. As expected, these tests revealed that both the absolute and
relative earnings differentials are directly related to the Add-On rate. In
other words, a higher Add-On rate results in more acceleration of earnings,
and, therefore, provides monthly income figures that are less comparable to
those as generated from (equivalent) simple interest. This principle holds
regardless of the original amount of the cash advance or the term-to-maturity
of the loan (see Exhibit III for selected examples).

Moving to the effects of the size feature, it was determined that the
size of the loan affects only the absolute dollar amount of the earnings
differential. Specifically, it was found that the spread between the earnings
recorded on the basis of simple vs. Add-On is a multiple of the relative
difference in the size of the loan. For example, for any given month, the
dollar amount of the earnings differential would be three times greater for a
$3,000 loan than for a $1,000 loan. Conversely, the size factor was found to
have no effect when the earnings difference is measured on a relative basis--
ie., the difference is 3.25% (in the first month) for any one year, twelve
percent Add-On loan, regardless of the amount of the cash advance (again, see
Exhibit III for selected examples.

In contrast to the size factor, it was found that the maturity of the
loan does affect both the absolute and relative amounts of the earnings
difference. That is, as the term-to-maturity lengthens, the absolute and
relative differences become greater. In effect, the spread in earnings, in
both dollar and percentage terms, is considerably more in the early stages of
longer lived loans and considerably less in the final stages[13] --Exhibit III
also provides selected examples of this feature. Thus, the accuracy of the
rule deteriorates as maturity lengthens and the rate of acceleration increas-
es.[14]

Exhibit III lists the maximum dollar and percentage earnings differen-
tials on a monthly and total basis--the latter being the greatest cumulative
difference. (Note that the greatest cumulative dollar difference occurs in
the last month that the earnings from the rule of seventy-eights are greater
than those derived from simple interest, while the greatest monthly percentage
difference occurs in the first month of the loan.) Thus, to the extent that
the relative differential is the most (or only) important criteria, examina-
tion of the exhibit reveals that the precision of the rule of seventy-eights
is unaffected by the size of the loan, but is adversely affected with longer
loans and/or with loans carrying higher Add-On interest rates.

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EXHIBIT III

EARNINGS DIFFERENTIALS FROM SELECTED INSTALLMENT LOAN
TRANSACTIONS: EARNINGS AS REALIZED FROM RULE OF
SEVENTY-EIGHTS VS. SIMPLE INTEREST

Term-to-Mat /Size of Loan	Int Charged @ 10% Add-On				Int Charged @ 12% Add-On				Int Charged @ 14% Add-On			
	Max Mon Diff		Max Cum Diff		Max Mon Diff		Max Cum Diff		Max Mon Diff		Max Cum Diff	
	$	%	$	%	$	%	$	%	$	%	$	%
12 Mos/1000	$0.41	2.73%	$0.90	2.73%	$0.58	3.25%	$1.28	3.25%	$0.78	3.76%	$1.73	3.76%
12 Mos/3000	$1.23	2.73%	$2.71	2.73%	$1.74	3.25%	$3.85	3.25%	$2.34	3.76%	$5.18	3.76%
12 Mos/5000	$2.05	2.73%	$4.52	2.73%	$2.90	3.25%	$6.42	3.25%	$3.91	3.76%	$8.65	3.76%
36 Mos/1000	$1.28	8.59%	$7.61	8.59%	$1.79	10.15%	$10.73	10.15%	$2.37	11.65%	$14.30	11.65%
36 Mos/3000	$3.85	8.59%	$22.83	8.59%	$5.38	10.15%	$32.20	10.15%	$7.11	11.65%	$42.91	11.65%
36 Mos/5000	$6.41	8.59%	$38.07	8.59%	$8.96	10.15%	$53.67	10.15%	$11.85	11.65%	$71.51	11.65%

Note: Both the monthly (Mon) and cumulative (Cum) differences are computed relative to the amount that would be charged on the basis of an equivalent simple interest; the dollar amount lists the added interest that would be charged with use of the rule of seventy-eights (and Add-On interest), while the percentage value shows the added interest amount relative to simple interest charge.

ACCOUNTING CONSIDERATIONS

For accounting use in interest income recognition, three conditions must be met before applying the rule of seventy-eights: 1) interest must be computed on an Add-On basis; 2) the loan must be amortized in equal dollar installments; and 3) installment payments must be made in equal intervals. When these conditions are met and so long as the maturity is not too long nor the interest rate too high, the use of the rule of seventy-eights will recognize interest income in a manner which approximates simple interest recognition. Should the accountant wish to determine the exact simple interest when an Add-On rate was originally used, it is necessary to use present value in order to execute the conversion.

Since the inflow of revenue from installment loans using Add-On interest rates may be an important or major source of revenue, the necessity of a valid technique for revenue recognition becomes readily apparent. The rule of seventy-eights has been long used and is today a widely accepted method of simple interest recognition since it is based on a declining balance. The actual application is very similar to the use of sum-of-the-years-digits approach to depreciation which, however, is applied to revenue recognition. This approach to revenue recognition results in a larger recognized earnings during early stages of a loan and a lower recognized earnings during latter stages than that which would result when using simple interest, but the dollar or percentage variations are not significant given previously identified constraints.

For the accountant, a clear understanding of the rule of seventy-eights is essential if lending institutions or retail credit outlets are on the work horizon, whether in conjunction with consulting, auditing, or other related service areas. This will enable the accountant to thoroughly review procedures relating to the installment loan area, to recommend to management appropriate conditions under which Add-On rates are a viable charge procedure (although in the final analysis, state statutes will determine the legal limits), and to properly evaluate the process of interest income recognition for statement presentation purposes.

Exhibit IV
Differential Earnings Percentages Using Simple Interest
Computations as Base. (Refer to Exhibit II and III for background.)

Maturity					
3 year	5.09%	6.93%	8.59%	10.15%	11.65%
2 year	3.54%	4.58%	5.73%	6.82%	7.87%
1 year	1.54%	2.16%	2.73%	3.25%	3.76%
	6%	8%	10%	12%	14%

Add-On Interest Rates

Note

Table provides comparisons - Example: use 8% ADD-ON as compared to Simple Interest Equivalent of 14.45% for a 1 year loan. Maximum difference between 8% ADD-ON of $12.30 and 14.45% Simple Interest of $12.04 equals .26, therefore $.26 divided by $12.04 = 2.16% (see table above).

CONCLUSION

Given acceptable maturities and interest rates, the use of the rule of seventy-eights with Add-On installment loans provides a valid method for computing interest income. Granted there is some acceleration in recognizing earnings and the method is not precisely accurate in relation to simple interest, but it does account for interest income in a manner which is similar to, though not identical with, that which accrues from use of an equivalent simple interest. Therefore, given that the rule of 78's is in essence a shortcut in computing interest earned, in all but the most atypical situations its deviation is too slight to affect the validity of its results. Exhibit IV provides a quick reference of interest comparability.

FOOTNOTES

1. While Add-On rates and the rule of seventy-eights can be used with install-ment loans whose amortization schedules are on other than a monthly basis (quarterly, for example), the greatest majority of such loans do, in fact, carry terms which require monthly repayment. As a result, most of the comments in this paper will be in terms of monthly amortized loans.

2. AAA Committee on Accounting Concepts and Standards, Accounting and Report-ing Standards for Corporate Financial Statements and Preceding Statements and Supplements. (Columbus, Ohio: AAA, 1957), p. 15.

3. Robert T. Sprouse and Maurice Moonitz, "A Tentative Set of Broad Accounting Principles for Business Enterprises, Accounting Research Study No. 3. (New York: AICPA, 1962), P. 46.

4. Accounting Principles Board, "Statement of Accounting Principles Board No. 4," (New York, New York: AICPA, 1970), P. 59.

5. Prentice-Hall, Inc., Federal Regulations, Vol. I (Englewood Cliffs, New Jersey: By Author, June 1972), Reg. 1.451-(a).

6. Note that the rule's name is derived from the factor associated with a one year (12 month) loan transaction--i.e., the denominator of the income factor in this situation is equal to 78.

7. Refer to sum-of-years depreciation method for similar computational techniques,e.g., Harry Simons, Intermediate Accounting, Comprehensive Volume. (Cincinnati, Ohio, 1972), p. 482.

8. Other applications of the rule of seventy-eights include determination of monthly interest earned and the determination of interest earned over a period of time within the term of the loan. These applications are based on the same computational principles as set out in the text. For example, the percent of interest earned during the third month of an 18 month loan would be: $16/171 = 9.36\%$; whereas the interest earned over a 12 month period between, say, the third and fourteenth months of an 18 month loan would be:

$$\frac{16 + 15 + 14 + \ldots + 6 + 5}{171} = \frac{126}{171} = 73.68\%$$

9. Technically, this third condition implies that there are 12-30 day months

in a 360 day year; this inference, in turn, accrues directly from the mathematical procedures used to compute the income factors. Regardless of whether the payment interval is one month, two months, or three months, it is obviously impossible to obtain exactly equal intervals unless the payment date changes every month. However, the effects of this practical feature are very slight and certainly are not sufficient to negate use of the rule.

10. For example, with a two year loan that is amortized on a quarterly basis, the computation of the denominator would be dependent upon the number of payment periods over the original term of the loan. Since there are eight such intervals in this loan, the denominator would be: $1 + 2 + \ldots + 7 + 8 = 36$. Again, working in payment periods, the numerator is computed along the same lines as set out earlier in this paper. The numerator of the realized income factor (after the fourth payment) would equal: $8 + 7 + 6 + 5 = 26$; and the realized income factor would be: $26/36 = 72.22\%$. However, it should be pointed out that while the rule technically could be utilized with loans that carry, say, semi-annual or annual amortization schedules, from a practical point of view, the wisdom of using an Add-On rate in these circumstances is subject to question. For it has been found that when the interval between payment dates becomes too long (say beyond three or four months), the validity of the rule also becomes suspect.

11. In this paper, note that the accuracy of the interim earnings figures obtained with the use of the rule of seventy-eights is judged in relation to those which would be derived from application of (equivalent) simple interest-this approach follows logically from the proven theoretical qualities of simple interest.

12. Fortunately, manual calculation can be avoided. Computers can be easily programmed and readily accessed to perform the necessary present value computations; in addition, there are published charts available which provide simple interest conversions for a wide range of Add-On rates and loan maturities. Also, if precise accuracy is not desired, there is simple formula available which provides approximate conversion; it is as follows:

$$(2) \quad \frac{\text{(number of payments in one year) (interest charge in \$)}}{\text{(principal amount of loan) (total number of payments +1)}}$$

The user should be cautioned, however, that this formula tends to overstate the simple interest rate and, therefore, does not provide a true equivalent rate.

13. As a matter of interest, the results of a large series of tests revealed that the cross-over point (i.e., the point in the life of the loan that interest from the simple basis first exceeds that accruing from application of the rule of seventy-eights) is equal to approximately one-third the maturity of the loan, regardless of the amount of the loan or the Add-On rate being used. That is, the monthly interest income recognized with the rule vs. that recognized on the basis of simple interest is greater during the first third of the life of the loan and less during the last two thirds.

14. While the discussion in this portion of the paper is devoted exclusively to loans that are paid off on a monthly basis, tests were also run on quarterly amortization schedules. The results of these tests showed behavioral patterns with regard to accuracy and to the effects of interest rate, size and maturity that are identical to those which appear with monthly schedules.

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