ESTIMATING A RESIDENCE-EMPLOYMENT DISTRIBUTION: AN APPLICATION OF THE DOUBLE-CONSTRAINED GRAVITY MODEL

by

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A complex problem in socio-economic planning deals with the measurement of the interaction between zones or activity regions. This problem has received considerable attention in transportation, planning, economics and geography, and several types of analytical models have been developed. Several models are inspired by the assumption that the socio-economic forces governing the urban spatial location process behave in analogous ways to the physical interaction of bodies as described by Newton's gravitational law. Newton's law states that the force of attraction between two bodies is directly proportional to the product of their masses and inversely proportional to the distance between them. Gravity-based models contend that the spatial interaction between two zones is directly proportional to the product of zonal socio-economic attraction measures and inversely proportional to the impedance (e.g. time or distance) between them. Masser [2] presents an extensive discussion of these models. In this work, a double constrained gravity model was used to estimate the interaction between geo-political zones due to resident labor population and employment opportunities. The work is part of a larger project which attempts to evaluate different industrial development strategies for the southwestern part of Puerto Rico.

THE DOUBLE-CONSTRAINED GRAVITY MODEL

The double-constrained gravity model can be expressed as:

$$T_{ij} = A_i \, B_j \, P_i \, Q_j \, d_{ij}^{-r}$$

where:

T_{ij} is the interaction (trips) between zones i and j,

A_i and B_j are scaling factors,

P_i is the labor availability of zone i,

Q_j is the employment potential at zone j,

d_{ij} is the distance or travel time between zones i and j,

$-r$ is the exponent of the distance factor.

A_i and B_j are scaling factors ensuring that the interaction of all labor zones with any employment zone equals the employment potential of that zone, and that the interaction of all employment zones with any labor zone equals the labor availability of that zone. The value of these factors is determined via an iterative scheme simultaneously with the determination of the exponent of the distance factor.

Gravity models suffer from several limitations which must be carefully considered before transcending the conceptualization stage. Their principal limitation lies in their lack of a sound theoretical foundation; they are solely analogy models. The determination of the exponent of the distance factor is another problem area. The exponent is unique to the system in question and is determined by comparing distributions generated for different values of r with a true distribution (from the past) of the system being modeled. Like all models that use past data, after calibration it is assumed that although absolute values may change, the distribution properties of the system (as modeled through r) will remain constant over time.

CALIBRATION OF THE MODEL

A search algorithm to calibrate the model was developed and its details are provided in Venta and Riera [4]. The interested reader is referred there for the technical details. The algorithm is a variant of the bisection method, commonly used to solve nonlinear equations. An initial search interval is specified, and the solution mechanism searches for the optimum value in the interval. Optimality is determined by finding the value of r such that the difference between actual and generated interactions is at a minimum. The interval is bisected and the direction in which the search should proceed is determined. This recurring methodology is used to reduce the interval of search until the difference between successive values of r is less than a specified minimum.

RESULTS

The objective of the work was to model the labor residence-employment distribution of ten southwestern municipalities of Puerto Rico. An actual distribution for calibration purposes was obtained from the Commonwealth's Economic Development Administration [3]. A distance matrix for the municipalities was supplied by the University of Puerto Rico [1]. The characteristics of the situation being modeled suggested the treatment of travel time rather than distance since road networks varied dramatically by municipalities. Average travel times, bounded by speed limits, were determined for highways, and for major urban and rural arteries.

The results of the model are shown in Table 1. As can be observed, the model generates a fairly good replica of the actual distribution. Most of the errors obtained are not significant, in light of the nearly 60,000 trips generated. The values obtained for the entries in the diagonal are the most accurate. This is significant in the situation being modeled since the diagonal values in the actual trip matrix are dominant in magnitude with respect to any other row or column value.

The main source of error appears to be the distance to travel time transformation assumptions. Two other important sources of error are the assumption that all within zones travel times were equal, and that the model does not consider different modes of transportation.

Table 1
ACTUAL AND GENERATED TRIP DISTRIBUTIONS

Actual Trip Distribution

Zone	1	2	3	4	5	6	7	8	9	10
1	28332	0	0	0	686	232	1826	34	0	0
2	243	2113	0	0	40	0	21	0	0	0
3	295	0	2826	0	0	135	0	181	39	0
4	147	0	0	1905	259	0	414	0	0	169
5	404	5	0	10	1630	0	793	0	0	113
6	2364	0	54	0	0	2794	0	148	133	0
7	357	0	0	0	32	0	1589	0	0	8
8	460	0	39	0	0	151	11	2175	0	0
9	341	0	7	0	0	120	8	17	1557	0
10	0	0	0	161	427	0	394	0	0	3175

Generated Trip Distribution

Zone	1	2	3	4	5	6	7	8	9	10
1	29008	24	16	11	200	255	1425	64	15	91
2	256	2071	2	2	10	8	50	5	6	6
3	271	3	2772	2	11	141	102	127	39	7
4	96	0	0	1836	173	3	304	1	0	475
5	305	0	0	31	1583	3	500	1	0	524
6	2109	7	70	4	24	2712	292	127	133	13
7	73	0	0	1	17	1	1887	0	0	6
8	375	3	44	2	10	90	85	2214	5	6
9	197	7	30	1	5	212	48	12	1533	3
10	274	0	0	172	1028	4	355	2	0	2316

The Search algorithm has converged at r=2.85 after 10 trials.

CONCLUSIONS

The benefits to be derived in practical applications of gravity models lie mainly in the ability to estimate the interaction between zones at times when detailed data is not available (the periods of time between census constitute a common setting for the application of spatial organization models). The developed models are intended to be planning tools. The model was of value in this application because the cost of a complex study, such as the one from which the calibration data was obtained is enormous.

The use of the model to explain employment allocation among the several municipalities is an important input to the evaluation of industrial location strategies. The indirect jobs generated by a given development plan could also be estimated with the model. Conversely, the model builder could measure the impact of proposed industrial development policies on the required infrastructure. Population-based demands for service facilities such as transportation networks, water and energy distribution, school facilities, commercial centers, and police and fire protection services could be predicted.

REFERENCES

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4. Venta, E. and A. Riera, "An Application of the Double-Constrained Gravity Model to Residence-Employment Distribution," Working Paper, Management Science Department, Loyola University of Chicago (1985).