

Detecting Contagion Effects During The Subprime Crisis Using Different VAR Size Models: Comparison Between OLS And Bayesian Shrinkage

Olfa Kaabia, University of Paris Ouest Nanterre La Defense, France
Ilyes Abid, University of Paris Ouest Nanterre La Defense, France

ABSTRACT

This paper assesses the performance in terms of forecast and structural analysis of different sizes of VAR models while studying contagion effects during the subprime crisis. We compare VAR models of different sizes: a standard VAR estimated by OLS, and a MEDIUM and LARGE VARs estimated by a Bayesian shrinkage procedure.

Using a large database containing national macroeconomic, financial, and trade dynamic variables for 17 OECD countries, we study contagion effects on U.S key variables and OECD housing markets over the period of the subprime crisis.

Our paper unearths that the smallest specification outperforms the largest one, in terms of forecast accuracy and structural analysis. Our results reveal that the SMALL and MEDIUM VARs produce qualitatively similar results; however, the MEDIUM VAR permits us to obtain the responses of a large set of variables and so provides a more comprehensive picture of the effects of the subprime crisis. So, we conclude that the MEDIUM VAR is more successful at extracting pertinent information for a large data set. It appears that dealing with a LARGE VAR provides worst results.

An interesting result that arises is that the standard OLS estimation is interesting; however, the Bayesian shrinkage is a useful tool, in the case of an international contagion study, allowing both a more comprehensive picture of the contagion effects as well as a more complete check of the empirical plausibility of the underlying specification.

Keywords: Contagion; Subprime Crisis; OECD Housing Markets; VAR/ BVAR Models and Bayesian Shrinkage

1. INTRODUCTION

Nowadays, VAR models are standard tools in macroeconomics, and are widely used for structural analysis. They have the advantage of not imposing restrictions on the parameters, and, hence, provide a very general representation allowing the capture of complex data relationships. Other VAR-type models have been proposed as Structural VARs¹; however, we always notice the same criticism of the VAR approach, according to the relatively small amount of information used in VARs². This issue has been especially addressed by Bernanke and *al.* (2005).

¹ Introduced by Sims (1980).

² To conserve degrees of freedom, standard VARs rarely employ more than six to eight variables.

In that sense, many researches propose to add more than eight variables in a VAR model; for instance, the marginal approach proposed by Christiano, Eichenbaum and Evans (1996), or Kim (2001). They define a core set of indicators, and add one variable, or a group of variables, at a time; however, comparison of impulse responses across models is problematic. Moreover, let mention Leeper, Sims, and Zha (1996) who increased the number of variables included by applying Bayesian priors. However, in their study, the VAR systems still contain less than 20 variables. Also, Stock and Watson (2005), and Bernanke and *al.* (2005) introduced the FAVAR models. The idea is that if a small number of estimated factors effectively summarizes large amounts of information about the economy, then a natural solution to the degrees-of-freedom problem in VAR analyses — which have to be of limited dimensions — is to augment the standard VAR with estimated factors. Moreover, Kaabia and Abid (2013) introduced Bayesian techniques in FAVAR model as to better characterize the transmission channels during the Subprime crisis. However, many issues remain unsolved concerning the estimation approach, the number and the nature of the factors.

So our analysis starts from the great criticism of the sparse information set used in the VAR model, which normally does not include more than eight variables to conserve degrees of freedom. In fact, central banks generally use a large information set to analyze the state of the economy before making any decision. In that sense, the VAR approach may exclude important information considered pertinent in the transmission process during turmoil periods. We will study the contagion effects of the subprime crisis which has started in the U.S and spread out to many countries all over the world.

The subprime crisis is not the first turmoil event occurring in the housing markets. Yet, the housing prices boom during the early 2000s has raised many questions, and since then many studies focused on a possible international transmission of housing shocks across countries (Otrok and Terrones, 2004). So, the last subprime crisis has confirmed the fears of possible contagion effects due to liberalization of markets. In fact, since 2006, the decrease of U.S. housing prices, as a result of collapsing residential investments has been followed by a wave of crises, and a sharp decrease in housing prices in other economies. This last crisis, which emerged in the USA in the summer of 2006, was followed by a sharp fall in housing prices in Ireland, New Zealand, Spain, and the United Kingdom. These almost parallel developments provide evidence in favor of a significant correlation across national housing markets.

There appears to be a large degree of co-movements between very different and distant countries. It seems that the U.S. housing boom and bust has spread to other parts of the world, and so confirms that the United States continues to retain its place as the world's principal leading country.

In this paper, we aim at studying contagion effects detected when a local shock affects the propagation mechanism of a large number of OECD variables. In other words, the investigation of contagion effects is to focus on the changes in the transmission mechanisms of shocks, at a critical date. Our analysis is important in terms of policy implications, or even for international investors. The scope of our paper is to contribute to the literature on international transmission.

The idea is to examine the contagion effects of a U.S. housing prices index shock on OECD countries, and to explore which VAR model better reflects the propagation and magnitude of the changes in the transmission processes, so as to reproduce what happened during the subprime crisis. To our knowledge, no paper in the literature has used such comparison to test and model the contagion effects.

Specifically, in our study, we address the following questions:

1. Is it necessary to deal with a large panel of data when studying contagion effects during the subprime crisis?
2. Which VAR model specification does reflect most faithfully what happened in the U.S and OECD countries during the subprime crisis, in terms of forecast and structural analysis?
3. In practice, is Bayesian shrinkage a valid alternative to Ordinary Least Squares (OLS) estimation in the case of international transmission study?

To answer these questions we choose different sizes of VAR models to characterize such dynamics for the OECD countries in our panel. Also, we build on the results obtained by De Mol, Giannone, and Reichlin (2008) and Bańbura, Giannone and Reichlin (2010) by setting the degree of Bayesian shrinkage in relation to the cross-sectional dimension of the model, so as to compare the three models.

Our main contributions to the existing literature are: First, the novelty of dealing with a Bayesian shrinkage procedure allows us to study the impact of U.S. housing prices index shock on the many OECD financial and economic variables included in our dataset in the form of impulse responses. This is particularly relevant given the most recent crisis which has been characterized by sudden shocks of large magnitude. Researchers, investors, and policy analysts are focusing on robust models allowing the reflection of negative effects on many variables in crisis period.

Second, many economic concepts need more than one variable, such as the real activity. So, dealing with a Bayesian shrinkage approach, i.e. the MEDIUM/ LARGE Bayesian VARs may be, a priori, a solution to capture many concepts.

Third, another advantage of using this type of VAR models specifications is that impulse responses can be observed both for variables included only in a small VAR, and for large key variables. If housing markets are contagious, economic policy should focus on structural reforms ensuring a stable domestic market in order to limit the amplification of shocks between housing markets.

Finally, using different VARs specifications allows us to compare our results to previous findings in terms of forecast and structural analysis, in the case of international transmission process.

Dealing with a huge database, we will study the impact of the U.S. housing prices index shock, and analyze the contagion effects on U.S key variables and OECD housing prices. Our database includes the 204 monthly following variables: real GDP, personal consumption, short-term and long-term interest rates, all share prices index, effective exchange rates, housing prices index, consumer price index, unemployment rate, export and import prices, for each of the seventeen considered OECD countries, over the period of 1980: M1 - 2006: M6.³ It is worth emphasizing that this sample is larger and more international than related studies.⁴

We find that the smallest specification outperforms the largest one in terms of forecast accuracy, and structural analysis. Moreover, the SMALL and MEDIUM VARs produce qualitatively similar results suggesting that the MEDIUM VAR is more successful at extracting pertinent information for a large data set. So, we conclude that the MEDIUM VAR is preferable.

Also, the standard OLS estimation is interesting; however, the Bayesian shrinkage is a useful tool, in the case of an international contagion study, providing both a more comprehensive picture of the contagion effects as well as a more complete check of the empirical plausibility of the underlying specification.

The rest of the paper is structured as follows: Section 2 exposes the empirical framework. Section 3 describes the data and presents the results in terms of forecast and structural analysis. Section 4 draws the appropriate conclusions.

2. EMPIRICAL FRAMEWORK

We propose to study contagion effects in the case of simulating the last subprime crisis, using different sizes of VAR models. So, we evaluate forecasting accuracy, and perform a structural exercise on the effect of a U.S. housing shock using different sizes of VARs: a SMALL one, estimated by OLS as well as a MEDIUM VAR, and a LARGE one estimated by Bayesian shrinkage.

³ It is considered as a broad measure of financial and economic co-movements.

⁴ For example, that of Stock and Watson (2005), or even that of De Bandt and Malik (2010).

In this section, first, we follow standard recommendations in the Bayesian literature, and build on the results of De Mol, Giannone, and Reichlin (2008), and Bańbura, Giannone and Reichlin (2010) by coping with the curse of dimensionality using Bayesian shrinkage via the imposition of priors. Second, we evaluate the forecast performance of different VAR model sizes. And finally, we deal with a structural analysis and make the impulse responses.

2.1 Setting Priors

Let a VAR model with p lags, $VAR(p)$, be as follows:

$$Y_t = c + A_1 Y_{t-1} + \dots + A_p Y_{t-p} + u_t \tag{1}$$

where $Y_t = (y_{1,t}, \dots, y_{n,t})'$, $c = (c_1, \dots, c_n)'$ is a vector of constants; A_1, \dots, A_p are the autoregressive $(n \times n)$ matrices and u_t are independent $N(0, \psi)$ errors.

The Bayesian methods combine likelihood function with prior may lead to a valid posterior density even if some parameters are not identified in the likelihood function. However, prior information becomes increasingly important as the number of parameters increases relatively to sample size. In this case, priors on the parameters⁵ A_1, \dots, A_p and the residual covariance matrix, ψ , should be set. In the literature, many priors are suggested. For a complete review of the existing priors, the reader can refer to Koop (2010).

We follow the standard procedure developed by Litterman (1986) for the VAR coefficients priors. Then, we take into account the modifications proposed by Kadiyala and Karlsson (1997), and Sims and Zha (1998) for the residual covariance matrix priors.

First, according to Litterman (1986), a $VAR(p)$ can be considered as “centered” equations around the random walk with drift as follows:

$$Y_t = c + Y_{t-1} + u_t \tag{2}$$

The idea suggested by Litterman is to shrink all VAR coefficients towards zero except for coefficients on own lags of each dependent variable. The latter are either set to one (for variables which exhibit substantial persistence), or zero (for variables which do not).

So Litterman assumes that the Minnesota prior beliefs⁶ are:

$$E \left[(A_k)_{ij} \right] = \begin{cases} \delta_i & \text{if } j = i, k = 1 \\ 0 & \text{otherwise} \end{cases} ; V \left[(A_k)_{ij} \right] = \begin{cases} \frac{\lambda^2}{k^2} & \text{if } j = i \\ \mathcal{G} \frac{\lambda^2}{k^2} \frac{\sigma_i^2}{\sigma_j^2} & \text{otherwise} \end{cases} \tag{3}$$

The coefficients A_1, \dots, A_p are assumed to be a priori independent and normally distributed. Also, the hyper parameters λ and \mathcal{G} control the overall tightness of the prior distribution around the random walk or white noise, and govern the relative importance of the prior beliefs with respect to the information contained in the data.

⁵ The prior on the intercept, c , is diffuse.

⁶ The reader can refer to Litterman (1986) for more details concerning the setting and hypothesis of each parameter.

More precisely, if $\lambda = 0$, the posterior equals the prior and the data do not influence the estimates. If $\lambda = \infty$, the posterior expectations coincide with the OLS estimates. In the rest of the paper, we will choose λ in relation to the size of the VAR model. As the number of variables increases, the parameters should be shrunk some more in order to avoid overfitting.

The parameter k is the lag length and the ratio $1/k^2$ gives the rate at which prior variance decreases with increasing lag length. Also, σ_i^2 / σ_j^2 reflects the different scale and variability of the data.

Second, according to Kadiyala and Karlsson (1997), and Robertson and Tallman (1999), Litterman’s assumption of fixed, and diagonal residual covariance matrix is somewhat unrealistic; that is why they impose a Normal Inverted Wishart prior which retains the principles of the Minnesota prior. Also, this assumption is problematic in the case of the structural analysis, where it is necessary to take into account possible correlation among the residual of different variables.

Consequently, we follow Kadiyala and Karlsson (1997), and Robertson and Tallman (1999) who propose to deal with the matrix form of a VAR(p) model given by:

$$Y = X \beta + U \tag{4}$$

where $Y = (Y_1, \dots, Y_T)'$ is a $(T \times n)$ matrix. $X = (X_1, \dots, X_T)'$ is a $(T \times K)$ matrix, with $X_t = (1, y'_{t-1}, \dots, y'_{t-p})'$ and $K = (1 + np)$ since each row contains p lags for each dependent variable, and an intercept. Also, $\beta = (c, A_1, \dots, A_p)'$ is the $(K \times n)$ matrix of coefficients and $U = (u_1, \dots, u_T)'$ is a $(T \times n)$ residual matrix. The Normal Inverted Wishart prior has the form:

$$\begin{cases} \text{vec}(\beta) | \psi \sim N(\text{vec}(\beta_0), \psi \otimes \Omega_0) \\ \psi \sim iW(S_0, \alpha_0) \end{cases} \tag{5}$$

where the prior parameters β_0 , Ω_0 , S_0 and α_0 are chosen so that prior expectations, and variances of β coincide with those implied by equation (3), and the expectation of ψ is equal to the fixed residual covariance matrix Σ of the Minnesota prior⁷.

In order to match the Minnesota moments in equation (3), and implement the prior of equation (5), it is necessary to add dummy variables as follows:

$$Y^* = X^* \beta + U^* \tag{6}$$

where $Y^* = (Y', Y'_d)'$; $X^* = (X', X'_d)'$; $U^* = (U', U'_d)'$ and the dummy observations Y_d and X_d are:

⁷ See Kadiyala and Karlsson (1997) for more details.

$$Y_d = \begin{pmatrix} \frac{\text{diag}(\delta_1\sigma_1, \dots, \delta_n\sigma_n)}{\lambda} \\ 0_{n(p-1) \times n} \\ \dots \quad \dots \quad \dots \quad \dots \\ \text{diag}(\sigma_1, \dots, \sigma_n) \\ \dots \quad \dots \quad \dots \quad \dots \\ 0_{1 \times n} \end{pmatrix}; \quad X_d = \begin{pmatrix} \text{diag}(1, \dots, p) \otimes \frac{\text{diag}(\sigma_1, \dots, \sigma_n)}{\lambda} & 0_{np \times 1} \\ \dots & \dots \\ 0_{n \times np} & 0_{n \times 1} \\ \dots & \dots \\ 0_{1 \times np} & \eta \end{pmatrix} \quad (7)$$

So we obtain that $\beta_0 = (X_d' X_d)^{-1} X_d' Y_d$, $\Omega_0 = (X_d' X_d)^{-1}$, $S_0 = (Y_d - X_d \beta_0)' (Y_d - X_d \beta_0)$ and $\alpha_0 = T_d - K$.

And in that case, the posterior has the form⁸:

$$\begin{cases} \text{vec}(\beta) | \psi, Y \sim N(\text{vec}(\tilde{\beta}), \psi \otimes (X^* X^*)^{-1}) \\ \psi | Y \sim iW(\tilde{\Sigma}, T_d + 2 + T - k) \end{cases} \quad (8)$$

with $\tilde{\beta} = (X^* X^*)^{-1} X^* Y^*$ and $\tilde{\Sigma} = (Y^* - X^* \tilde{\beta})' (Y^* - X^* \tilde{\beta})$

So we impose a Normal Inverted Wishart prior which retains the principles of the Minnesota prior under the condition that $\vartheta = 1$ ⁹.

The posterior expectation of the coefficients coincides with the OLS estimates of the regression of Y^* on X^* . This expression is common in the Bayesian literature, and coincides with the posterior mean for the Minnesota prior.

After setting the priors, we will move to explain how to make the forecast analysis resulting from different VAR specifications.

2.2 Forecast Analysis

We compute point forecasts using the posterior mean of the parameters. Let $\hat{A}_j^{(\lambda, m)}$ ($\forall j = 1, \dots, p$) and $\hat{c}^{(\lambda, m)}$ be, respectively, the posterior mean of the autoregressive coefficients, and the constant term of a given model ($m = \textit{SMALL}$, \textit{MEDIUM} or \textit{LARGE}) and parameter λ . The point estimate of the one-step-ahead forecast is computed as:

$$Y_{t+1|t}^{(\lambda, m)} = \hat{c}^{(\lambda, m)} + \hat{A}_1^{(\lambda, m)} Y_t + \dots + \hat{A}_p^{(\lambda, m)} Y_{t-p+1} \quad (9)$$

Also, the other forecasts h -steps ahead are computed recursively as follows:

⁸ To insure the existence of the prior expectation of ψ , it is necessary to add an improper prior $\psi \sim |\psi|^{-(n+3)/2}$. See De Mol, Giannone, and Reichlin (2008) and Bańbura, Giannone and Reichlin (2010), for more details.

⁹ See Kadiyala and Karlsson (1997) for more details.

$$\hat{Y}_{t+h|t}^{(\lambda,m)} = \left(\hat{y}_{1,t+h|t}^{(\lambda,m)}, \dots, \hat{y}_{n,t+h|t}^{(\lambda,m)} \right) \tag{10}$$

where n is the number of variables included in the m model and h is the forecast horizon.

Note that in the case of the benchmark model (random walk with drift), the prior restriction is imposed exactly, that is $\lambda = 0$ and the corresponding forecasts are denoted by $\hat{Y}_{t+h|t}^{(0)}$, and are the same for all the specifications.

In our analysis, we compute h -step-ahead forecasts, $\hat{Y}_{T+h|T}^{(\lambda,m)}$, using only the information up to time T . For a given forecast horizon h , in each period $T = T_0 + H - h, \dots, T_1 - h$ where H denotes the longest forecast horizon to be evaluated. T_0 and T_1 are, respectively, the beginning and the end of the evaluation sample.

As for the out-of-sample forecast accuracy, we compute the Mean Squared Forecast Error (*MSFE*) for the variable, i , and a horizon h as:

$$MSFE_{i,h}^{(\lambda,m)} = \frac{1}{T_1 - T_0 - H + 1} \sum_{T=T_0+H-h}^{T_1-h} \left(\hat{y}_{i,T+h|T}^{(\lambda,m)} - y_{i,T+h} \right)^2 \tag{11}$$

And to compare the different specifications, we use the MSFE relative (*RMSFE*) to the benchmark as follows:

$$RMSFE_{i,h}^{(\lambda,m)} = \frac{MSFE_{i,h}^{(\lambda,m)}}{MSFE_{i,h}^{(0)}} \tag{12}$$

We follow De Mol, Giannone, and Reichlin (2008) and Bańbura, Giannone and Reichlin (2010), and set the overall tightness, λ , to yield a desired average "Fit" for the key variables of interest during the pre-evaluation period, and then keep it fixed for the entire evaluation period. So, λ is chosen for a desired "Fit", and is given by:

$$\lambda^{(m)}(Fit) = \arg \min_{\lambda} \left| Fit - \frac{1}{5} \sum_{i \in I} \frac{msfe_i^{(\lambda,m)}}{msfe_i^{(0)}} \right| \tag{13}$$

in with I represents the variables included in the SMALL VAR model, $msfe_i^{(\lambda,m)}$ is an in-sample one-step-ahead mean squared forecast error evaluated using the training sample $t = 1, \dots, T_0 - 1$, and is given for the number of lags p as follows:

$$msfe_i^{(\lambda,m)} = \frac{1}{T_0 - p - 1} \sum_{t=p}^{T_0-2} \left(\hat{y}_{i,t+1|t}^{(\lambda,m)} - y_{i,t+1} \right)^2 \tag{14}$$

More precisely, the desired "Fit" coincides with the one obtained by OLS estimation on the VAR model that is for:

$$Fit = \frac{1}{5} \sum_{i \in I} \frac{msfe_i^{(\lambda,m)}}{msfe_i^{(0)}} \Big|_{\lambda=\infty; m=VAR} \tag{15}$$

In the next section, we will explain how to make the structural analysis resulting from the VAR models.

2.3 Structural Analysis

We follow Stock and Watson (2005) and Bernanke and *al.* (2005), and divide the variables in the data into two categories: slow and fast-moving variables. This distinction is crucial because it implies that slow-moving variables do not respond contemporaneously to an initial shock. This hypothesis is equivalent to ranging the variable in an exogeneity order.

So, if we note S_t as representing the slow variables, r_t is the shocked variable, and Z_t is the fast-moving variable, we can write $Y_t = (S_t, r_t, Z_t)$.

The Structural VAR is written as follows:

$$\tilde{\lambda}_0 Y_t = v + \tilde{\lambda}_1 Y_{t-1} + \dots + \tilde{\lambda}_p Y_{t-p} + e_t \tag{16}$$

with $e_t \sim WN(0, D)$ and where $v = C^{-1}c$, $\tilde{\lambda}_0 = C^{-1}$, $\tilde{\lambda}_j = C^{-1}A_j \quad \forall j = 1, \dots, p$ and e_t is the linear transformation of the VAR residuals: $e_t = (e_{1t}, \dots, e_{nt})' = C^{-1}u_t$.

Let the lower diagonal Cholesky matrix of the covariance of the residuals of the reduced form of a VAR be noted $B = CD^{1/2}$, with $CDC' = E[u_t u_t'] = \psi$ and $D = \text{diag}(\psi)$.

We follow Gordon and Leeper (1994) by generating draws from the posterior of (A_1, \dots, A_p, ψ) . So for each draw, ψ , we compute B, C and even $\tilde{\lambda}_j \quad \forall j = 0, \dots, p$.

3. DATA and RESULTS

3.1 Data

Our large international dataset is drawn from Datastream, Eurostat and the Federal Reserve website, FRED - Saint Louis Fed. The data consists of monthly variables from the period 1981M1-2006M6 for 17 OECD countries, namely United States (U.S.), Canada (CAN), Finland (FIN), France (FRA), Germany (ALL), Ireland (IRL), Italy (ITA), Netherlands (NLD), Spain (ESP), Denmark (DNK), Norway (NOR), Sweden (SWE), Switzerland (SWI), United Kingdom (UK), Australia (AUS), Japan (JPN), and New-Zealand (NZL).

Our dataset includes 204 variables, with 12 variables for each of the 17 countries, encompassing a wide range of financial variables (3-Month Interest Rates, 10-Year Interest Rates, Stock Indices and Housing Price Indices, HPI), variables related to real economy (GDP, Personal Consumption, Industrial Production, Unemployment Rates), aggregate price variables (CPI), trade variables (Import and Export of Goods and Services), and Effective Exchange Rates.

All the data are seasonally adjusted, and the variables are measured at constant national prices. As in the literature, interest rates are differenced, and activity variables are logarithmized. More detailed description is provided in the appendix.

Since the subprime crisis has started in the U.S and spread out to many countries all over the world, we will consider the three following VAR specifications:

- **SMALL:** This is a VAR including three United States key variables (Gross Domestic Product (GDP_USA), Housing Price Index (HPI_USA) and Stock Market Index (SI_USA)). It represents a U.S national VAR model. It will allow us to study the impact of the subprime crisis on the principal U.S key variables.

- MEDIUM: This is the previous VAR augmented by the sixteen housing price index of the OECD considered countries.
- LARGE: This VAR contains all the considered 204 variables in our database: an international model.

3.2 RESULTS

3.2.1 Forecast Analysis

In this section, we evaluate the forecast performance of the three U.S key variables included in the three VAR specifications: GDP_USA, HPI_USA and SI_USA, respectively the U.S Gross Domestic Product, the American Housing Price Index, and the U.S Stock Market Index, over the period going from mid-2006 until the end of 2010.

Note that for the SMALL VAR, we implement information criteria for lag selection and take the optimal lag according to the Bayesian Information Criterion (BIC). The number of lags retained is $p = 11$. This choice confirms the one made by Bernanke and *al.* (2005), indicating that the series are very persistent.

The estimation is based on the sample from 1982:2 to 2006:6, and the results reported are for the same overall shrinkage as given in Table 1.

Table 1: The Value of the Shrinkage Hyper Parameter λ for the Three Considered VARs

	SMALL	MEDIUM	LARGE
λ	∞	0.1064	0.0388

Note: The abbreviations SMALL, MEDIUM and LARGE refer to the VARs with 3, 19 and 204 variables, respectively.

Moreover, to compare models of different sizes, we choose the Bayesian shrinkage hyper parameter in relation with the models dimensions by ensuring that the in-sample "Fit" is constant and equal to 0.7801. As the dimension increases, we set the tightness of the prior so that all models have the same in-sample "Fit" as the smallest VAR estimated by OLS.

Also, we report, in Table 2, the *RMSFE* for the five OECD housing price indices for the three different VARs.

Table 2: The *RMSFE* for Forecast Horizons $h = 1, 3, 6$ and 12 for the Three VARs

Horizons	Variables	SMALL	MEDIUM	LARGE BVAR
h=1	GDP_USA	0.042222	0.060223	0.123261
	HPI_USA	0.052158	0.081442	0.220200
	SI_USA	0.530247	0.717384	0.791179
h=3	GDP_USA	0.907845	0.910629	1.072776
	HPI_USA	1.091275	1.103746	1.967154
	SI_USA	0.535300	0.570965	0.618733
h=6	GDP_USA	1.018012	1.095981	1.685852
	HPI_USA	1.509412	1.588623	1.599839
	SI_USA	0.531530	0.544320	0.720589
h=12	GDP_USA	1.167578	1.400305	2.202496
	HPI_USA	1.730941	1.741873	2.324368
	SI_USA	0.577833	0.580487	0.615186

So, according to Table 2, we notice, that the smallest specification outperforms the largest one in terms of forecast accuracy. It seems that the smaller the model, the better the forecast. This confirms that the USA continues to retain its place as the world's principal leader, and does not depend on the other OECD countries. Moreover, it seems that the SMALL and MEDIUM VARs produce qualitatively and quantitatively similar results. This denotes that dealing with a LARGE VAR does not help to improve the forecast for any of the three U.S key variables considered.

3.2.2 Structural Analysis

In this section, we present the results of the impulse response functions, and the variance decompositions to the U.S. housing price index shock in mid-2006 for the VARs.

We display the impulse response functions for the three models under consideration only for the three U.S. key variables (GDP_USA, HPI_USA, and SI_USA) included in the different VARs.

We divide the variables in the data into slow and fast-moving variables: the slow-moving ones are GDP, consumption, housing price index, CPI, industrial production, export and import of goods and services, and unemployment rate. The rest of the variables e.g. stock prices index, effective exchange rates, short-term and long-term interest rates are the fast-moving ones. Also, r_t is the U.S. housing prices index.

The dotted lines indicate the posterior coverage intervals corresponding to 90% and 68% confidence levels as mentioned in the legend.

At first sight, we notice that the impulse responses maintain the expected sign. We remark that the smaller the VAR is, the tighter the 90% and 68 % confidence intervals are, and vice versa.

For the SMALL VAR, the impulse responses are significant and become nil at about five-month horizon at the exception of the U.S GDP, where the initial shock is persistent for more than one year.

For the MEDIUM VAR, a positive American housing price shock immediately and significantly affects the U.S key variables. We notice that the sharp of the impulse responses are qualitatively the same as for the SMALL but more persistent except for the U.S GDP. The MEDIUM VAR produces a more realistic feature to the contagion effects during the subprime crisis.

For the LARGE VAR, the U.S impulse responses are not significant, for the whole one-year horizon considered, unlike the American housing prices. Unfortunately, the confidence bounds are wide denoting that the impulse responses are unreliable.

The relative tight confidence bounds of the SMALL and the MEDIUM VARs indicate the precision of the response of the U.S. housing prices index positive shock on the other three U.S key variables, which is not the case for the LARGE VAR. In fact, in the case of the last model, the 90 and 68 % confidence bounds are very wide. So, it seems that the SMALL and MEDIUM models provide qualitatively the best results in terms of impulse response functions.

At this stage, combining these conclusions with the previous ones made in the case of forecasting analysis, leads us to say that the SMALL and MEDIUM VARs are preferable to the LARGE one.

Since the SMALL VAR is an interesting model, but, as mentioned by Bernanke and *al.* (2005), it is criticized due to its limited number of variables and to complete our analysis, we report the impulse response functions for the seventeen OECD housing prices in the case of the MEDIUM and LARGE VARs, respectively in figures 2 and 3. In fact, the advantage of the MEDIUM and LARGE VARs, compared to the SMALL one, is that impulse responses can be observed for all the OECD countries.

So, just as a confirmation of our previous findings, it appears, clearly, that the MEDIUM VAR is preferable to the LARGE one. This last model has very wide confidence bounds. For the MEDIUM VAR, we remark that the OECD housing markets have been significantly and immediately impacted by the initial shock but they did not react in the same way. So, the MEDIUM VAR enables us to obtain the responses of a large set of variables and this provides a more comprehensive picture of the effects of the subprime crisis.

Figure 1: Impulse Response Functions for the three U.S. variables (GDP_USA, HPI_USA and SI_USA) for the Three Models (SMALL, MEDIUM and LARGE VARs)

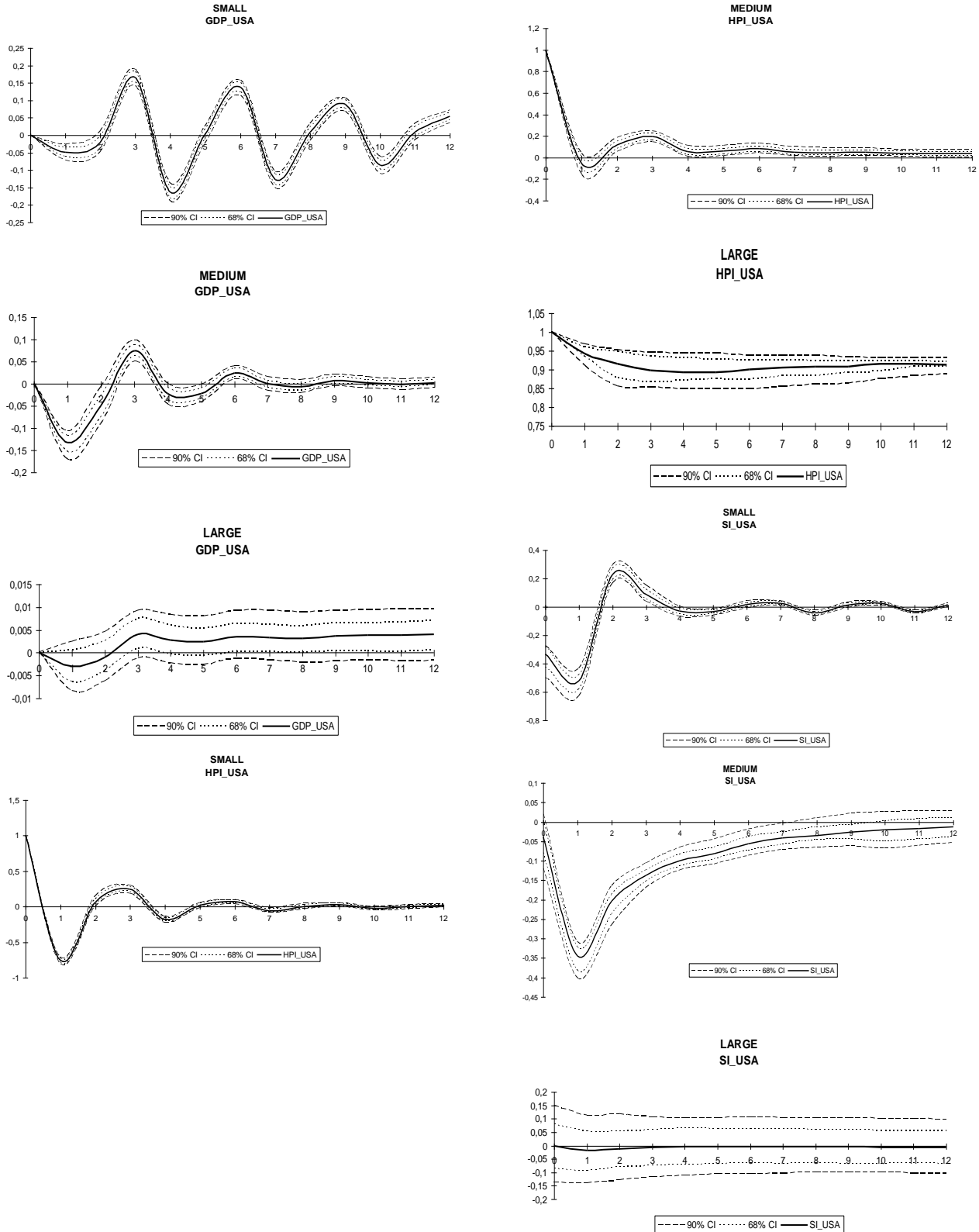
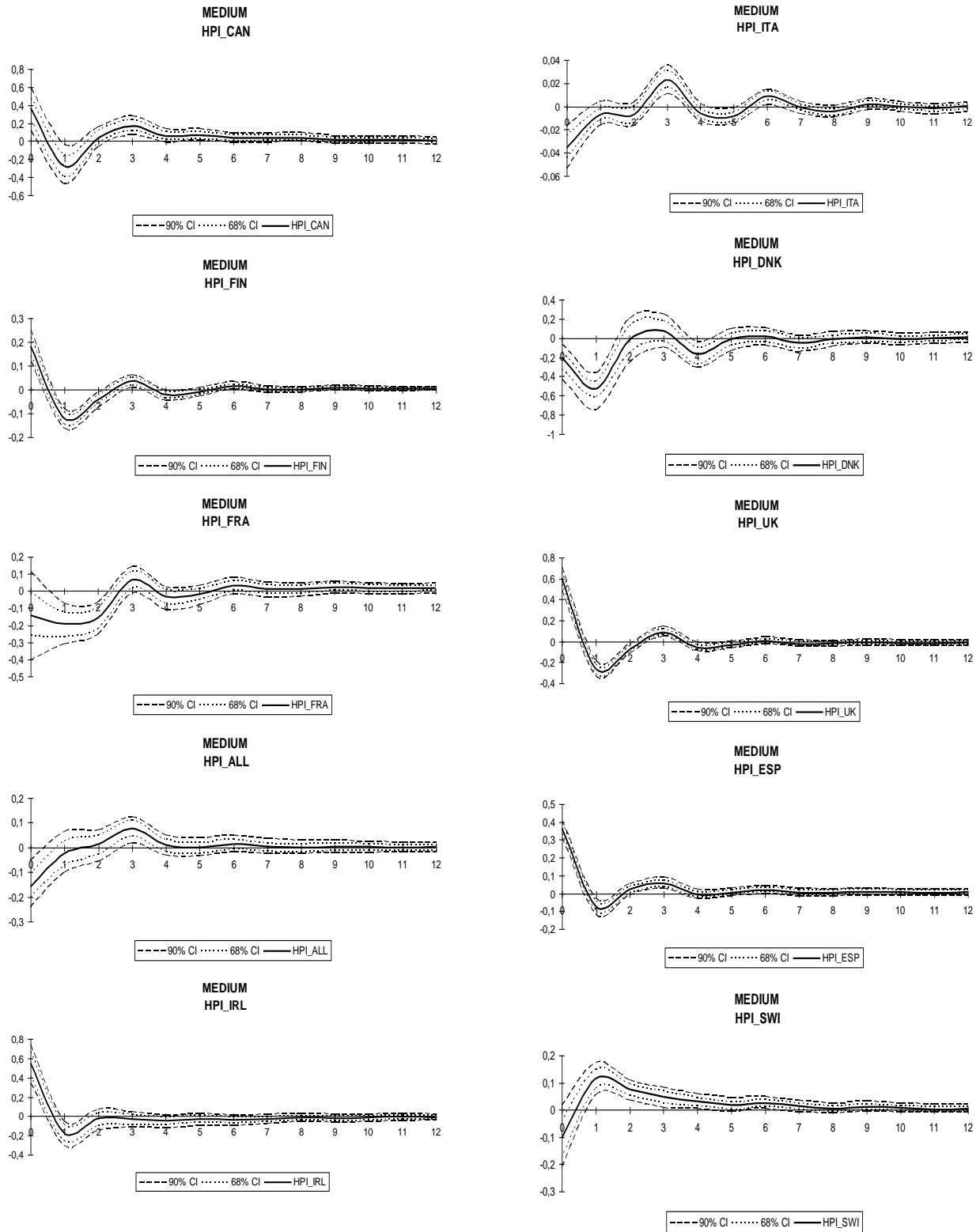


Figure 2: Impulse Response Functions of the Seventeen OECD Housing Price Indices for the MEDIUM VAR



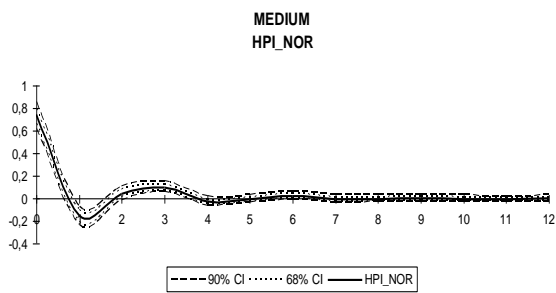
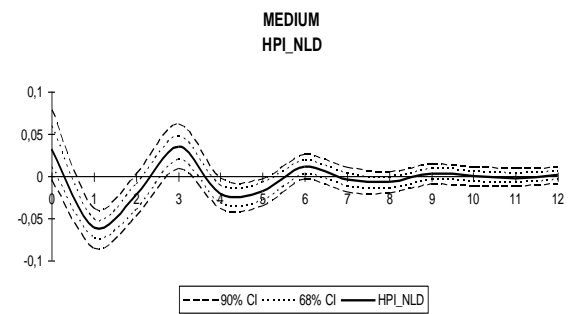
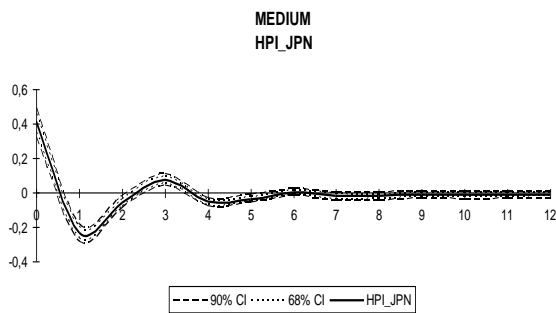
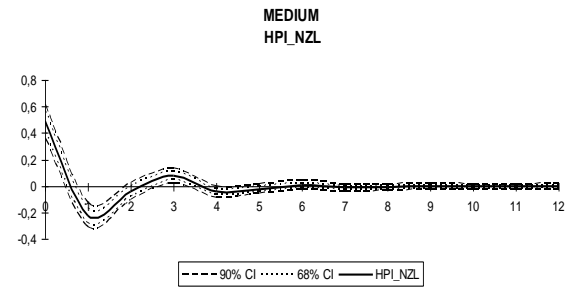
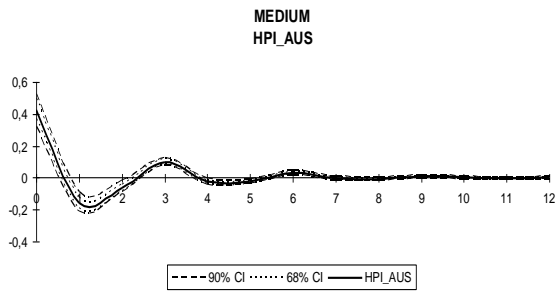
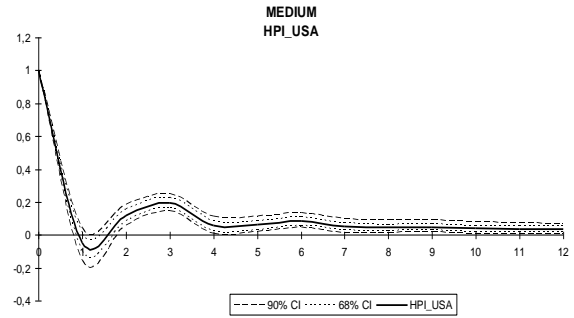
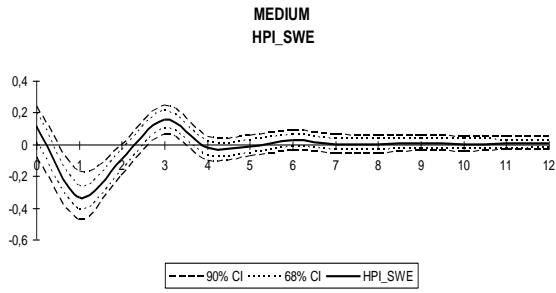
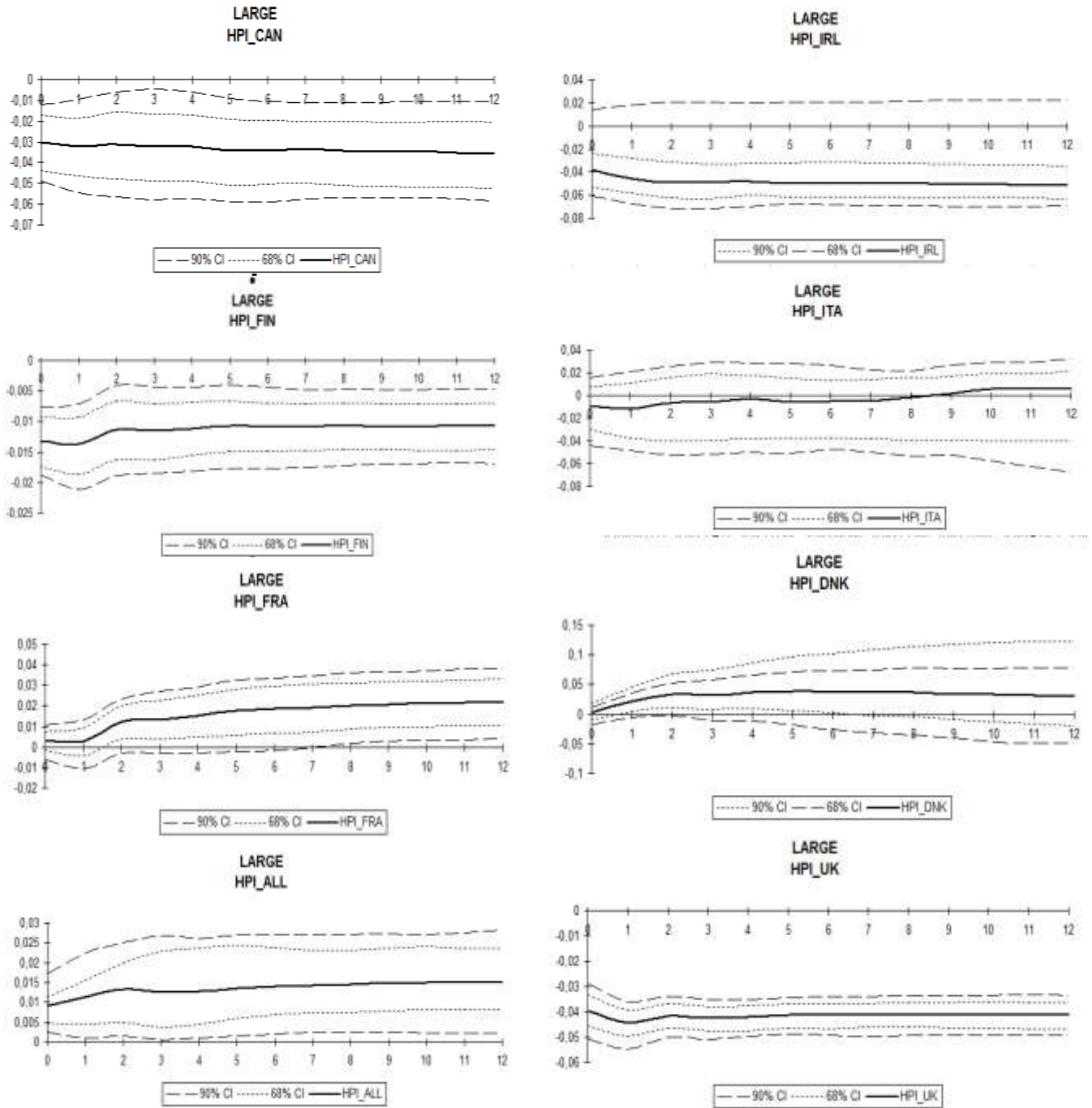
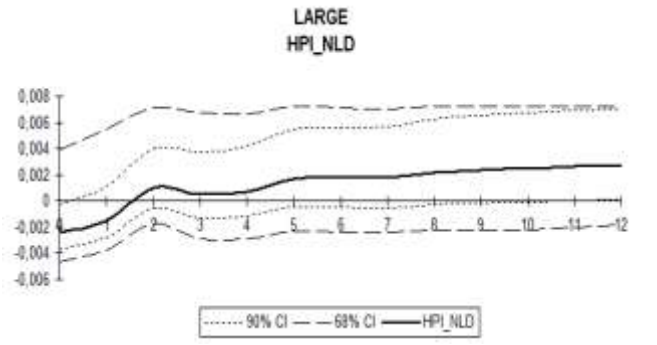
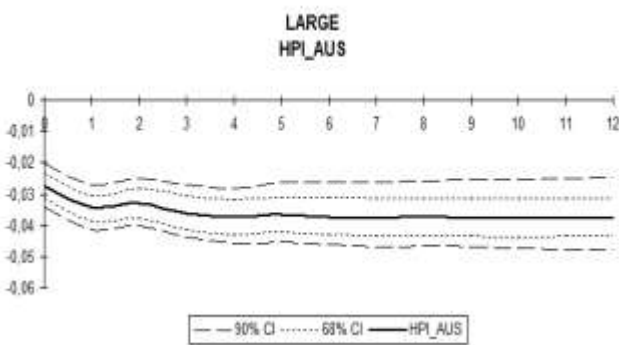
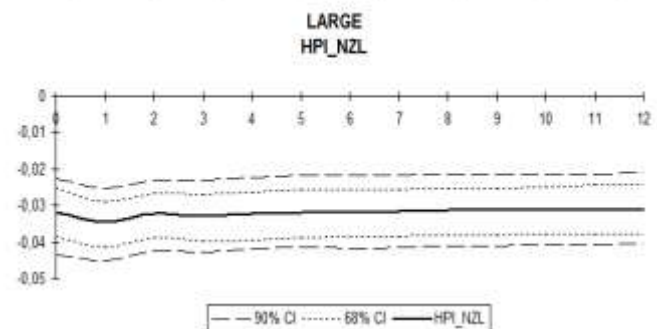
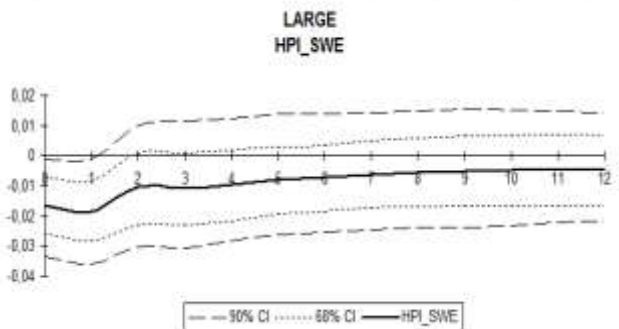
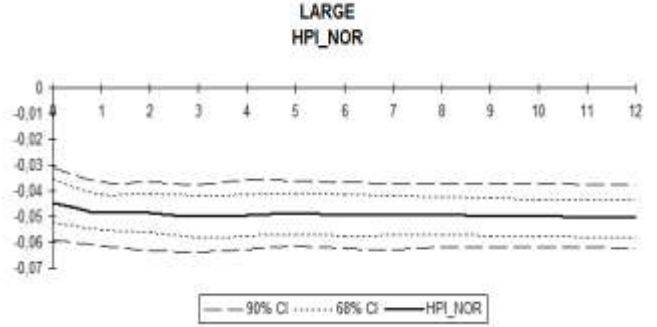
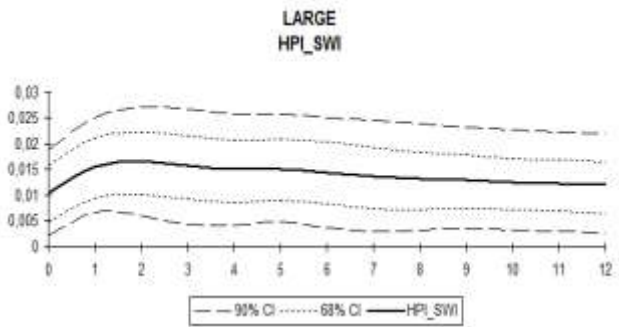
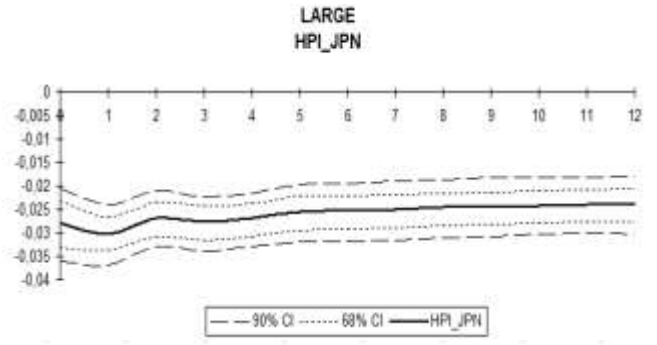
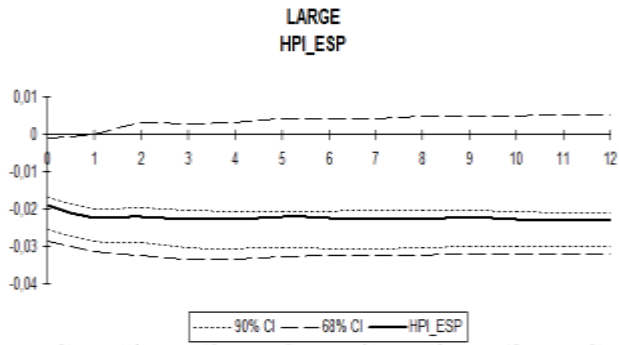
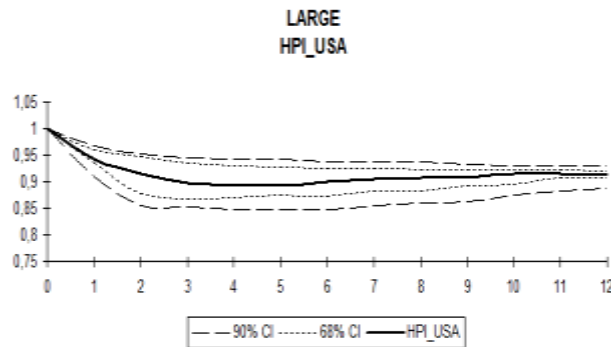


Figure 3: Impulse Response Functions for the Seventeen OECD Housing Price Indices for the LARGE VAR







Consequently, the MEDIUM VAR provides realistic response functions reflecting what happened during the subprime crisis. This result is confirmed also by Kaabia and al. (2013).

Moreover, the same features can be seen from the variance decomposition analysis below:

Table 3: Percentage Share of the US Housing Price Index Shock in the Forecast Error Variance for different Forecast Horizons

Variable	Horizon	SMALL	MEDIUM	LARGE
GDP_USA	1	0	0	0
	3	0.9644	0.34672	0.00128
	6	12.18796	12.77802	0.00358
	12	18.28143	17.50771	0.00696
HPI_USA	1	94.93896	93.18396	8.76813
	3	93.37155	93.29603	19.17080
	6	92.90642	92.88318	30.58735
	12	92.69278	92.35931	45.48698
SI_USA	1	5.457004	4.067679	7.71E-06
	3	15.01895	13.38062	0.00244
	6	15.06545	13.42967	0.00219
	12	15.14763	13.30027	0.00222

The results show that the size of the U.S housing price index shock is clearly more pronounced for the SMALL and MEDIUM models than for the LARGE one.

Besides, for three U.S key variables, and for all horizons, we remark that the size of the initial shock in the SMALL and MEDIUM VARs is bigger than that in the LARGE one.

This suggests that the U.S housing shock is more, and more persistent, even after a horizon of 12 months for the SMALL and MEDIUM VARs. The results show that the American key variables have been quickly affected, and that the size of the positive U.S. housing price index shock is clearly more pronounced for the SMALL and MEDIUM models than for the LARGE one. This is the same feature that we had when making the structural analysis using the MEDIUM VAR.

This finding confirms the previous ones in the case of the structural and forecast analysis denoting that the SMALL and MEDIUM VARs are preferable to the LARGE one. But since the SMALL VAR is criticized due to its limited number of variables, the MEDIUM VAR is a suitable model for international study.

It appears that the standard OLS estimation is interesting; however, the Bayesian shrinkage is a preferable useful tool in the case of an international contagion study allowing providing both a more comprehensive picture of the contagion effects as well as a more complete check of the empirical plausibility of the underlying specification.

4. CONCLUSION

We study the contagion effects of a U.S. housing shock on OECD countries over the period of the subprime crisis. For that, we evaluate forecasting accuracy and perform a structural analysis exercise using VAR models of different sizes: a standard VAR estimated by OLS and a MEDIUM and LARGE VARs estimated by a Bayesian shrinkage procedure.

We took as point of departure the great criticism addressed especially by Bernanke and *al.* (2005) is the sparse information set of the VAR model which can only include a few variables to conserve degrees of freedom. So, we propose to study different-sized VAR models (SMALL, MEDIUM and LARGE). We consider the following specifications: 3, 19 and 204 international macroeconomic and financial variables.

We examine both forecasting accuracy and structural analysis of the effect of a positive U.S. housing price index shock on U.S key variables and OECD housing markets so as to try to replicate what has happened during the subprime crisis.

We find that the smallest specification outperforms the largest one in terms of forecast accuracy, and structural analysis. Moreover, the SMALL and MEDIUM VARs produce qualitatively similar results suggesting that the MEDIUM VAR is more successful at extracting pertinent information for a large data set. So, we conclude that the MEDIUM VAR permits us to obtain the responses of a large set of variables thus providing a more comprehensive picture of the effects of the subprime crisis.

It appears that the standard OLS estimation is interesting however the Bayesian shrinkage is a useful tool in the case of an international contagion study allowing providing both a more comprehensive picture of the contagion effects as well as a more complete check of the empirical plausibility of the underlying specification. These findings are in lines with those of Kaabia and *al.* (2013).

AUTHOR INFORMATION

Olfa Kaabia, EconomiX-CNRS (UMR 7235), University of Paris Ouest Nanterre La Defense, France. E-mail: olfa.kaabia@u-paris10.fr (Corresponding author)

Ilyes Abid, EconomiX-CNRS (UMR 7235), University of Paris Ouest Nanterre La Defense, France. E-mail: ilyes.abid@u-paris10.fr

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APPENDIX: DATA TRANSFORMATIONS DETAILS

As in Stock and Watson (2005a), we use the following transformation codes: 1 - no transformation (levels); 2 - first difference; 3 - second difference; 4 -logarithm; 5- first difference of logarithm and 6- second difference of logarithm.

In the transformation line, ln denotes logarithm, Δln and Δ²ln denote the first and second difference of the logarithm, lv denotes the level of the series, and Δlv denotes the first difference of the series.

Also, following Bernanke and *al.* (2005), we divide the variables into slow moving (denoted by an asterisk: * next to the variable) and fast moving variables.

Short Name	USA GDP*	CAN GDP*	FIN GDP*	FRA GDP*	DEU GDP*	IRL GDP*	ITA GDP*	NLD GDP*	ESP GDP*	DNK GDP*	NOR GDP*	SWE GDP*	SWI GDP*	UK GDP*	AUS GDP*	JPN GDP*	NZL GDP*
Transformation	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln
CODE	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6

Short Name	USA CONS*	CAN CONS*	FIN CONS*	FRA CONS*	DEU CONS*	IRL CONS*	ITA CONS*	NLD CONS*	ESP CONS*	DNK CONS*	NOR CONS*	SWE CONS*	SWI CONS*	UK CONS*	AUS CONS*	JPN CONS*	NZL CONS*
Transformation	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln
CODE	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6

Short Name	USA HPI*	CAN HPI*	FIN HPI*	FRA HPI*	DEU HPI*	IRL HPI*	ITA HPI*	NLD HPI*	ESP HPI*	DNK HPI*	NOR HPI*	SWE HPI*	SWI HPI*	UK HPI*	AUS HPI*	JPN HPI*	NZL HPI*
Transformation	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln
CODE	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6

Short Name	USA CPI*	CAN CPI*	FIN CPI*	FRA CPI*	DEU CPI*	IRL CPI*	ITA CPI*	NLD CPI*	ESP CPI*	DNK CPI*	NOR CPI*	SWE CPI*	SWI CPI*	UK CPI*	AUS CPI*	JPN CPI*	NZL CPI*
Transformation	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln	Δ ² ln
CODE	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6

Short Name	USA PI*	CAN PI*	FIN PI*	FRA PI*	DEU PI*	IRL PI*	ITA PI*	NLD PI*	ESP PI*	DNK PI*	NOR PI*	SWE PI*	SWI PI*	UK PI*	AUS PI*	JPN PI*	NZL PI*
Transformation	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln
CODE	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5

Short Name	USA export*	CAN export*	FIN export*	FRA export*	DEU export*	IRL export*	ITA export*	NLD export*	ESP export*	DNK export*	NOR export*	SWE export*	SWI export*	UK export*	AUS export*	JPN export*	NZL export*
transformation	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln
CODE	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5

Short Name	USA import*	CAN import*	FIN import*	FRA import*	DEU import*	IRL import*	ITA import*	NLD import*	ESP import*	DNK import*	NOR import*	SWE import*	SWI import*	UK import*	AUS import*	JPN import*	NZL import*
transformation	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln
CODE	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5

Short Name	USA iTB_3	CAN iTB_3	FIN iTB_3	FRA iTB_3	DEU iTB_3	IRL iTB_3	ITA iTB_3	NLD iTB_3	ESP iTB_3	DNK iTB_3	NOR iTB_3	SWE iTB_3	SWI iTB_3	UK iTB_3	AUS iTB_3	JPN iTB_3	NZL iTB_3
transformation	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv
CODE	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Short Name	USA iBond 10	CAN iBond 10	FIN iBond 10	FRA iBond 10	DEU iBond 10	IRL iBond 10	ITA iBond 10	NLD iBond 10	ESP iBond 10	DNK iBond 10	NOR iBond 10	SWE iBond 10	SWI iBond 10	UK iBond 10	AUS iBond 10	JPN iBond 10	NZL iBond 10
transformation	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv	Δlv
CODE	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Short Name	USA tx chg	CAN tx chg	FIN tx chg	FRA tx chg	DEU tx chg	IRL tx chg	ITA tx chg	NLD tx chg	ESP tx chg	DNK tx chg	NOR tx chg	SWE tx chg	SWI tx chg	UK tx chg	AUS tx chg	JPN tx chg	NZL tx chg
transformation	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln
CODE	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5

Short Name	USA SI	CAN SI	FIN SI	FRA SI	DEU SI	IRL SI	ITA SI	NLD SI	ESP SI	DNK SI	NOR SI	SWE SI	SWI SI	UK SI	AUS SI	JPN SI	NZL SI
transformation	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln	Δln
CODE	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5

Short Name	USA Unemp*	CAN Unemp*	FIN Unemp*	FRA Unemp*	DEU Unemp*	IRL Unemp*	ITA Unemp*	NLD Unemp*	ESP Unemp*	DNK Unemp*	NOR Unemp*	SWE Unemp*	SWI Unemp*	UK Unemp*	AUS Unemp*	JPN Unemp*	NZL Unemp*
transformation	Δlv	Δlv	Δlv	Δ ² lv	Δ ² lv	Δ ² lv	Δ ² lv	Δ ² lv	Δ ² lv	Δ ² lv	Δlv	Δlv	Δlv	Δ ² lv	Δlv	Δlv	Δlv
CODE	2	2	2	3	3	3	3	3	3	3	2	2	2	3	2	2	2

Notes: The abbreviations GDP (Gross Domestic Product), CONS (Personal Consumption), HPI (Housing Price Index), CPI (Consumer Price Index), PI (Industrial Production), iTB_3 (3-Month Interest Rates), iBond 10 (10-year Government Bond Index), tx change (Effective Exchange rate), SI (Stock Index) and Unemp (Unemployment rate).