Commodity Price Correlation And Time Varying Hedge Ratios

Amine Lahiani, Université d'Orléans & ESC Rennes Business School, France
Khaled Guesmi, IPAG Lab, IPAG Business School & EconomiX, University of Paris West Nanterre La Défense, France

ABSTRACT

This paper examines the price volatility and hedging behavior of commodity futures indices and stock market indices. We investigate the weekly hedging strategies generated by return-based and range-based asymmetric dynamic conditional correlation (DCC) processes. The hedging performances of short and long hedgers are estimated with a semi-variance, low partial moment and conditional value-at-risk. The empirical results show that range-based DCC model outperforms return-based DCC model for most cases.

Keywords: Range-Based Dynamic Conditional Correlation; Downside Risk; Transaction Costs

1. INTRODUCTION

Hedging strategies are the cornerstone of portfolio theory and portfolio risk management. This requires minimizing the risk of the portfolio for a given return. This principle is called the minimum-variance criterion. It is now common to hedge a spot position with futures (Johnson, 1960; Stein, 1961) but also to hedge one asset with another (Bos & Gould, 2007). This requires a prior understanding of the dynamics of the considered assets, the link between them; i.e., correlation and cointegration. Some studies show that neglecting these features would lead to an inaccurate hedging strategy underperforming alternative strategies.

The recent literature on hedging performance employs bivariate models and mainly the bivariate GARCH-based models. Baillie and Myers (1991) used bivariate GARCH models to estimate the optimal hedge ratio to hedge spot prices of six commodity prices with futures contracts; they conclude that the dynamic hedge ratios outperform the static ones. Kroner and Sultan (1993) consider a model that accounts for both the long-run cointegration between assets and the heteroscedasticity in the error dynamics. Results show that their model outperforms the naïve and conventional hedging, that include time-invariant unconditional moments, both within-sample and out-of-sample. Lien (2004) assesses the effects of omitted cointegration relationship between spot and futures prices on optimal hedge ratio and hedging effectiveness and finds that omitting cointegration leads to a smaller hedge ratio but the loss of hedging effectiveness is minimal. More recently, in contrast to the previous findings, Lien and Tse (2002) fail to reject the null of constant-correlation based optimal hedging ratio computation. They further argue that the constant correlation optimal hedge ratio outperforms the time-varying optimal hedge ratio.

Because of the mixed results found in the literature, the research question on the optimal hedge ratio is still of paramount importance. Numerous papers attempt to derive the optimal of hedging ratio by considering different extensions. Chen et al. (2014) compute the optimal hedge ratio by minimizing the riskiness of hedged portfolio returns.[1] Results show that the riskiness-minimizing hedge ratio is effective in reducing the riskiness of the spot as compared to the variance-minimizing hedge ratio. Choudhry (2003) shows that the time-varying hedge ratio based on bivariate GARCH and bivariate GARCH-X models outperforms the constant minimum-variance hedge ratio.

Chen et al. (2006) estimate a time-varying optimal hedge ratio by using a range-based multivariate volatility. Out-of-sample forecasting shows that there is a gain in hedging effectiveness in terms of percentage

[1] The riskiness measure is defined in Aumann and Serrano (2008) as the reciprocal of the absolute risk aversion of an individual with constant absolute risk aversion who is indifferent between taking and not taking that gamble.

variance reduction over when comparing its performance to that of static methods and alternative time-varying methods.

The major contributions of this paper are twofold. First, we investigate the weekly hedging strategies relying on both return-based and range-based asymmetric dynamic conditional correlation dynamic conditional correlation processes. Over the existing papers our approach allows not only to account for the variability of both assets but also the co-movement between pairs of the considered assets. Second, the hedging performances of short and long hedgers are estimated with semi-variance, low partial moment and conditional value-at-risk. These measures are common in the finance literature as risk measurements. The semivariance produces better portfolios than those based on variance. The low partial moment has theoretically favorable features and is empirically preferred to the mean-variance optimization strategy. Value-at-Risk is a widely accepted measure of financial risk and is proved to outperform the mean-variance portfolio optimization strategy.

The remainder of the paper is organized as follows. Section 2 introduces the econometric models used in the paper and discusses their advantages relative to alternative modeling techniques used in the related literature. Section 3 explains each of the above optimal hedging ratio measures. Section 4 presents the data used and discusses the main findings. Section 5 concludes.

2. METHODOLOGY

2.1 The GARCH Model

Conditional volatility models of ARCH type are concerned with the modeling of the second moments of the shocks to returns. Although such shocks are typically assumed to be independent, they are likely to be dependent in practice. Indeed, the generalization of univariate GARCH model to a multivariate setting is written as follows:

$$\varepsilon_t / \mathrm{I}_{t-1} \to N(0, H_t)$$

where the ε_t is the vector of the error terms of the conditional mean. $\eta_t = vec(\varepsilon_t \varepsilon_t^{'})$ refers to a sequence of independently and identically distributed (i.i.d) random errors; and $H_t = \begin{pmatrix} h_t^1 & h_t^{12} & \cdots & h_t^{1m} \\ h_t^{21} & h_t^2 & \cdots & h_t^{1m} \\ \vdots & & \ddots & \vdots \\ h_t^{m1} & h_t^{m2} & \cdots & h_t^m \end{pmatrix}$ is the conditional variance-covariance matrix of assets returns with m is the number of assets.

Unless η_t is a sequence of independently and identically distributed random vectors, or alternatively a martingale difference process, the assumption of constant conditional correlation - $\rho_{ij} = \frac{h^{ij}}{\sqrt{h^i}\sqrt{h^j}}$ - will not be valid, so that it would be more likely that the condition correlations $\Gamma_t = \{\rho_{ijt}\}$ are time-varying $\Gamma_t = \{\rho_{ijt}\}$ for i,j = 1,…,m and t = 1,…,T. Engle and Kroner (1995) developed the BEKK[2] model to capture the time-varying behaviour of conditional covariances. The BEKK(1,1) model has the following representation:

$$Q_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'Q_{t-1}B \tag{1}$$

where the second term in (1) is singular. In addition to including a large number of parameters, which leads to serious computational difficulties, BEKK models the dynamic conditional covariances rather than what is typically

[2] BEKK stands for Baba, Engle, Kraft, & Kroner.

of primary interest to practitioners in finance, namely the dynamic conditional correlations. In the specific model given in (1), BEKK can be interpreted as accommodating serial correlation of unknown form in the standardized residuals. Although it was not considered explicitly in Engle and Kroner (1995), the dynamic conditional correlations associated with BEKK can be derived from $Q_t = D_t \Gamma_t D_t$ as $\Gamma_t = D_t^{-1} Q_t D_t^{-1}$, where $D_t = (diag Q_t)^{1/2}$. The element of D_t are defined as univariate GARCH models as follows:

$$H_{ii,t} = c_i + \sum_{q=1}^{q_i} \alpha_{iq} \varepsilon_{i,t-q}^2 + \sum_{p=1}^{p_i} \beta_{ip} H_{ii,t-p} \tag{2}$$

The following usual restrictions for non-negativity and stationarity hold in Equation (2) $\forall\, i = 1,2,\ldots,m.$

(i) $c_i > 0,$

(ii) $\forall p = 1,\ldots, p_i; \forall q = 1,\ldots,q_i : \alpha_{ip}\ and\ \beta_{iq}$ are such that $P(H_{ii,t} > 0) = 1,$

(iii) $\sum_{q=1}^{q_i} \alpha_{iq} + \sum_{p=1}^{p_i} \beta_{ip} < 1$

In order to capture the dynamics of time-varying conditional correlation, Γ_t, Engle (2002) and Tse and Tsui (2002) proposed the closely related Dynamic Conditional Correlation (DCC) model and the Variable Conditional Correlation (VCC) model, respectively, as extensions of the CCC model. No explanation was given as to how the shocks to returns in either the VCC or DCC model would have to be modified to yield the dynamic structure of the conditional correlations. The DCC model is given by:

$$Z_t = (1 - \theta_1 - \theta_2)\overline{Z} + \theta_1 \eta_{t-1} \eta_{t-1}' + \theta_2 Z_{t-1} \tag{3}$$

where the second term in (3) is singular, and θ_1 and θ_2 are scalar parameters. When $\theta_1 = \theta_2 = 0$, $\overline{Z}$ in (3) is equivalent to the CCC model. As Z_t in (3) is conditional on the vector of standardized residuals, (3) would be the conditional covariance matrix, and hence also the conditional correlation matrix, if η_t were a vector of independently and identically distributed random variables. However, there is no discussion of the properties of η_t within the DCC model framework[3] although Engle (2002, p. 342) does state that "the errors are a Martingale difference." As (3) does not satisfy the definition of a conditional correlation matrix, Engle (2002) calculates the appropriate dynamic conditional correlation matrix as follows:

$$\Gamma_t^* = \left\{(diag Z_t)^{-1/2}\right\} Z_t \left\{(diag Z_t)^{-1/2}\right\} \tag{4}$$

The VCC model uses a transformation of the standardized shocks to estimate the time-varying conditional correlations, written as follows:

$$\begin{cases} \Gamma_t = (1 - \theta_1 - \theta_2)\Gamma + \theta_1 \Psi_{t-1} + \theta_2 \Gamma_{t-1} \\ 1 - \theta_1 - \theta_2 \ge 0 \\ \theta_1 \quad and \quad \theta_2 \ge 0 \end{cases} \tag{5}$$

[3] Engle (2002) states that "the errors are a Martingale difference by construction" in suggesting how to estimate the model.

where the typical element in the non-singular second term, which is a lagged recursive conditional correlation matrix given by:

$$\Psi_{ijt-1} = \sum_{l=1}^{M} \eta_{it-l}\eta_{jt-l} \bigg/ \left\{ \sum_{l=1}^{M} \eta_{it-l}^{2} \sum_{l=1}^{M} \eta_{jt-l}^{2} \right\}^{1/2} \tag{6}$$

where M ≥ m. When $\theta_1 = \theta_2 = 0$, Γ in (5) is equivalent to the CCC model. No standardization of (5) is required because it satisfies the definition of a conditional correlation matrix, albeit of standardized shocks, η_{it}, that are not independently distributed, although they are explicitly (and incorrectly) assumed to be serially independently distributed in Tse and Tsui (2002).

The primary structural difference between DCC and VCC is that (5) standardizes Z_t to obtain the dynamic conditional correlation matrix, whereas VCC assumes that the time-varying conditional correlation matrix can be calculated recursively using (5).

Chan et al. (2003) proposed the Generalized Autoregressive Conditional Correlation (GARCC) model which, unlike the DCC and VCC models, motivates the dynamic structure of the conditional correlations explicitly through serial correlation in the vector of standardized shocks. They showed that, if η_{it} follows an autoregressive process rather than being a sequence of independently and identically distributed random vectors, that is:

$$\begin{cases} \eta_t = \nu_t(\eta_{1t,}\ldots,\eta_{mt})', \nu_t \sim iid(0,1) \\ \eta_{it} = \sum_{l=1}^{L} \phi_{il}\eta_{it-l} + \xi_{it}, \xi_{it} \sim iid\left(0,\sigma_i^2\right), i = 1,\ldots,m \end{cases} \tag{7}$$

Amore general dynamic model than DCC and VCC can be obtained when $L \to \infty$, as follows:

$$W_t = \overline{W} + \Phi_1 \otimes \eta_{t-1}\eta_{t-1}^{'} + \Phi_2 \otimes W_{t-1} \tag{8}$$

where Φ_1 and Φ_2 are ($m \times m$) matrices and $\otimes$ is the Hadamard (or element by element) product. The first equation in (8) was introduced in the univariate ARCH literature (that is, for m = 1) by Tsay (1987) as a random coefficient autoregressive approach to deriving ARCH models, with a straightforward extension to univariate GARCH models. Chan et al. (2003) showed that, when $\phi_{il} = \phi_{1i}\delta_{il}$ and $\delta_{il} \sim iid(0, \phi_{2i}^{l-1})$, (8) is the dynamic conditional correlation matrix of the standardized residuals, η_{it}, which are not independently distributed because of the presence of serial correlation in (7). The GARCC conditional correlation matrix can be obtained formally as:

$$\Gamma_t = \left\{ \left(diag W_t\right)^{-1/2} \right\} W_t \left\{ \left(diag W_t\right)^{-1/2} \right\} \tag{9}$$

which makes clear the importance of recognizing the serial correlation in the vector of standardized residuals. Chan et al. (2003) show that the standardization in (8) is not required, in practice, so that (8) is effectively the conditional correlation matrix. However, the standardization in (4) for the DCC model is required as (3) does not satisfy the definition of a conditional correlation matrix of the standardized shocks.

As an extension of the above model, asymmetric effects can be accommodated in GARCC by modifying the conditional covariance matrix in (8) with the indicator function in (5) to produce the asymmetric GARCC (AGARCC) model.

2.2 The Measure of Hedging Performance

The out-of-sample hedging performances of return-based and range-based DCC-GARCH models for both short and long hedgers are evaluated not only with traditional variance-based and utility-based measurements but also with approaches based on downside risk, such as semi-variance, low partial moment and conditional value-at-risk. These evaluation measures are briefly described as follows:

$$\begin{cases} E_1 = [1 - \dfrac{\text{variance}(r_{h,t})}{\text{variance}(r_{s,t})}] \times 100\% \\ E_2 = [1 - \dfrac{\text{utility}(r_{h,t})}{\text{utility}(r_{s,t})}] \times 100\% \quad ;\text{utility}(x) = E(x) - \gamma \text{var}(x) \\ E_3 = [1 - \dfrac{\text{semivariance}(r_{h,t})}{\text{semivariance}(r_{s,t})}] \times 100\% \quad ;\text{semi variance} = E\{(\max[0, \tau - x])^2\} \\ E_4 = [1 - \dfrac{\text{LPM}(r_{h,t})}{\text{LPM}(r_{s,t})}] \times 100\% \quad ;\text{LPM} = E\{(\max[0, \tau - x])^n\} \\ E_5 = [1 - \dfrac{\text{CVaR}(r_{h,t})}{\text{CVaR}(r_{s,t})}] \times 100\% \quad ;\text{CVaR} = E\{(\max[0, \tau - x])^n\} \end{cases}$$

where $r_{h,t}$ define as the return of hedged portfolio and calculated as:

$$r_{h,t} = r_{s,t} - \hat{\delta}_t r_{f,t}$$

γ is the degree of risk aversion and τ is the target return and $\tau = 0,\ n=1$.

3. EMPIRICAL RESULTS

The weekly data used for our empirical study is composed of futures prices of three major stock indices (S&P 500, DAX, and Nikkei 225), two precious metal commodities prices (gold and silver), and one crude oil price (WTI) spanning the period from January 1, 1993 to December 25, 2009. These data (including high, low, and close prices) for the entire period are collected from DataStream.

The out-of-sample hedging performances of return-based (DCC-GARCH) and range-based (DCC-CARR) models are reported in Tables 1 and 2.

Table 1: Out-of-Sample Evaluation of Hedging Performance

	Variance	Utility	Semi-Variance	LPM	CVaR
Panel 1: Stock Markets					
S&P 500-short					
DCC-GARCH	98.362*	98.348*	98.657*	99.896*	87.209*
DCC-CARR	*98.511**	*98.515**	*98.839**	*99.918**	*88.343**
S&P 500-long					
DCC-GARCH	98.362*	98.375*	97.928*	99.655*	86.415*
DCC-CARR	*98.511**	*98.507**	*98.030**	*99.685**	*86.912**
DAX-short					
DCC-GARCH	95.897*	95.820*	93.718*	97.864*	79.436*
DCC-CARR	96.092*	96.020*	93.983*	97.948*	79.998*
DAX-long					
DCC-GARCH	95.897*	95.970*	97.638*	99.460*	87.234*
DCC-CARR	*96.092**	*96.161**	*97.773**	*99.496**	*87.586**
Nikkei225-short					
DCC-GARCH	97.915*	98.028*	98.295*	99.821*	84.702*
DCC-CARR	*98.360**	*98.364**	*98.556**	*99.874*	*85.340*
Nikkei225-long					
DCC-GARCH	97.915*	97.803*	97.516*	99.626*	84.676*
DCC-CARR	*98.360**	*98.356**	*98.178**	*99.745**	*87.719**
Panel 2: Commodities					
Gold-short					
DCC-GARCH	97.125*	97.214*	97.614*	99.567*	84.989*
DCC-CARR	*98.198**	*98.220**	*97.922**	*99.626**	*85.940**
Gold-long					
DCC-GARCH	97.125*	97.037*	96.728*	99.228*	85.405*
DCC-CARR	*98.198**	*98.176**	*98.472**	*99.813**	*88.335**
Silver-short					
DCC-GARCH	97.677*	97.690*	98.254*	99.699*	86.919*
DCC-CARR	*97.738**	*97.708**	*98.010**	*99.668**	*85.062**
Silver-long					
DCC-GARCH	97.677*	97.664*	96.865*	98.584*	87.188*
DCC-CARR	*97.738**	*97.767**	*97.352**	*99.063**	*87.823**
WTI-short					
DCC-GARCH	*92.290**	*92.282**	93.002*	95.510*	*85.586**
DCC-CARR	91.985*	91.952*	93.441*	96.309*	83.095*
WTI-long					
DCC-GARCH	*92.290**	*92.298**	*91.445**	*93.599**	*86.615**
DCC-CARR	91.985*	92.017*	90.233*	91.658*	86.349*

Note: The numbers in the table indicate hedging performance as described in Section 2.2. Variance is for E1, Utility is for E2, Semi-variance is for E3, LPM is for E4, and CVaR is for E5. * indicates that the coefficients are significant at the 1% level.

To investigate the possible influence on hedging performance while considering transaction cost, following the procedure given in Kroner and Sultan (1993), we assume that the hedging positions are rebalanced if and only if there is benefit to do so, that is, a mean-variance expected utility-maximizing investor will rebalance at time t if and only if increased utility from rebalancing is great enough to offset transaction costs. Accounting for these transaction costs, a hedger's expected daily utility is calculated as $E(R_h) - \gamma\, var(R_h) - TC$, where TC is transaction costs. The results are presented in Table 2.

Table 2: Out-of-Sample Evaluation of Hedging Performance with Utility Metric under Transaction Cost

	DCC-GARCH	DCC-CARR
Panel 1: Stock Markets		
S&P 500-short		
TC = 0.01%	98.387 (94)	*98.498 (79)*
TC = 0.02%	98.387 (94)	*98.498 (77)*
TC = 0.03%	98.387 (94)	*98.494 (74)*
S&P 500-long		
TC = 0.01%	98.418 (94)	*98.492 (79)*
TC = 0.02%	98.418 (94)	*98.491 (79)*
TC = 0.03%	98.418 (94)	*98.488 (74)*
DAX-short		
TC = 0.01%	95.812 (110)	*95.978 (83)*
TC = 0.02%	95.812 (110)	*95.978 (83)*
TC = 0.03%	95.812 (110)	*95.978 (81)*
DAX-long		
TC = 0.01%	95.937 (110)	*96.133 (83)*
TC = 0.02%	95.937 (110)	*96.133 (83)*
TC = 0.03%	95.937 (110)	*96.132 (81)*
Nikkei225-short		
TC = 0.01%	97.998 (98)	*98.392 (50)*
TC = 0.02%	97.998 (97)	*98.390 (45)*
TC = 0.03%	97.998 (97)	*98.391 (42)*
Nikkei225-long		
TC = 0.01%	97.745 (98)	*98.368 (50)*
TC = 0.02%	97.744 (97)	*98.367 (45)*
TC = 0.03%	97.744 (97)	*98.366 (42)*
Panel 2: Commodities		
	DCC-GARCH	**DCC-CARR**
Gold-short		
TC = 0.01%	97.329 (64)	*98.237 (56)*
TC = 0.02%	97.329 (64)	*98.237 (54)*
TC = 0.03%	97.328 (64)	*98.238 (53)*
Gold-long		
TC = 0.01%	97.171 (64)	*98.216 (56)*
TC = 0.02%	97.171 (64)	*98.216 (54)*
TC = 0.03%	97.171 (64)	*98.217 (53)*
Silver-short		
TC = 0.01%	97.728 (61)	*97.668 (106)*
TC = 0.02%	97.728 (61)	*97.668 (105)*
TC = 0.03%	97.728 (60)	*97.668 (105)*
Silver-long		
TC = 0.01%	97.699 (61)	*97.726 (106)*
TC = 0.02%	97.699 (61)	*97.725 (105)*
TC = 0.03%	97.699 (60)	*97.726 (105)*
WTI-short		
TC = 0.01%	*92.284 (119)*	91.883 (106)
TC = 0.02%	*92.284 (119)*	91.883 (104)
TC = 0.03%	*92.284 (119)*	91.883 (103)
WTI-long		
TC = 0.01%	*92.300 (119)*	91.948 (106)
TC = 0.02%	*92.300 (119)*	91.948 (104)
TC = 0.03%	*92.284 (119)*	91.948 (103)

Note: The numbers in the table indicate hedging performance as described in Section 2.2. Variance is for E1, Utility is for E2, Semi-variance is for E3, LPM is for E4, and CVaR is for E5. The number of portfolio rebalancings is given in parentheses. * indicates that the coefficients are significant at the 1% level.

4. CONCLUSION

This article reinvestigates the weekly hedging strategies generated by return-based and ranged-based dynamic conditional correlation (DCC) models for three stock indices, two metal commodities, and one crude oil. Following the work of Cotter and Hanly (2006), the hedging performances of short and long hedgers are evaluated not only with traditional variance-based and utility-based measurements but also with approaches based on downside risk, such as semi-variance, low partial moment, and conditional value-at-risk. The empirical results indicate that range-based DCC model outperforms return-based DCC model for most cases, which is also supported by Chou et al. (2009). The DCC-CARR Process incorporates both the superiority of range in forecasting volatility and the elasticity of the GARCH model. The estimation results may provide an alternative to risk management and asset allocation. At last, a range-based dynamic hedging strategy sustains its predominance even if the transaction cost is considered.

AUTHOR INFORMATION

A. Lahiani is an Associate Professor of Economics at Université d'Orléans and ESC Rennes Business School in France. He earned his Ph.D. in Economics from the University Paris X Nanterre in France and the University of Geneva in Switzerland. His research interests include the study of long memory models and threshold models. Professor Lahiani's research papers are published in the *International Journal of Forecasting, Economic Modelling, Quarterly Review of Economics and Finance, Research in International Business and Finance, Energy Economics and Energy Policy* among others. E-mail: amine.lahiani@univ-orleans.fr

Dr. Khaled Guesmi is a Professor of Finance in IPAG Business School. He holds an MSc and a PhD in Economic from the University of Paris Ouest Nanterre la Défense (France). He is also an Associate Researcher at EconomiX, University of Paris Ouest Nanterre la Défense. His principal research areas concern emerging markets finance, international cost of capital, energy finance, and stock market integration. His most recent articles are forthcoming and published in refereed journals such as *Economic Modelling, Journal of Banking and Finance, Energy Economics, Economics Bulletin, Macroeconomic Dynamics,* and *Journal of Applied Business and Research*. E-mail: khaled.guesmi@ipag.fr (Corresponding author)

REFERENCES

1. Aumann, R. J., & Serrano, R. (2008). An economic index of riskiness. *Journal of Political Economy, 116*, 810-836.
2. Baillie, R. T., & Myers, R. J. (1991). Bivariate GARCH estimation of the optimal commodity futures hedge. *Journal of Applied Econometrics, 6*, 109-124.
3. Billio, M., & Caporin, M. (2004). *A generalized dynamic conditional correlation model for portfolio risk evaluation*. (Unpublished paper). Department of Economics, University of Venice Ca' Foscari.
4. Billio, M., Caporin, M., & Gobbo, M. (2004). *Flexible dynamic conditional correlation multivariate GARCH for asset allocation*. (Unpublished paper). Departement of Economics, Univariaty of Venice Ca' Foscari.
5. Bos, C. S., & Gould, P. (2007). *Dynamic correlations and optima hedge ratios*. (Tinbergen Institute Discussion Paper 07-025/4).
6. Capiello, L., Engle, R. F., & Sheppard, K. (2003). *Asymmetric dynamics in the correlations of global equity and bond returns*. (ECB Working paper No. 204).
7. Chan, F., Hoti, S., & McAleer, M. (2003). *Generalized autoregressive conditional correlation*. (Unpublished paper). School of Economics and Commerce, University of Western Australia.
8. Chen, Y. T., Ho, K. Y., & Tzeng, L. Y. (2014). Riskiness-minimizing spot-futures hedge ratio. *Journal of Banking & Finance, 40*, 154-174.
9. Chen, Y. T., Chou, R. Y., Liu, N., & Shyy, G. (2006). *Estimating time-varying hedge ratios with a range based multivariate volatility model*. 2007 AEA Conference Papers.
10. Chou, R. Y., Wu, C. C., & Liu, N. (2009). Forecasting time-varying covariance with a range-based dynamic conditional correlation model. *Review of Quantitative Finance and Accounting, 33*, 327-345.

11. Choudhry, T. (2003). Short-run deviations and optimal hedge ratio: Evidence from stock futures. *Journal of Multinational Financial Management, 13*, 171-192.
12. Cotter, J., & Hanly, J. (2006). Reevaluate hedging performance. *Journal of Futures Markets, 26*(7), 677-702.
13. Engle, R. F. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics, 20*, 339-350.
14. Engle, R. F., & Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH. *Econometric Theory, 20*, 339-350.
15. Hafner, C., & Franses, P. H. (2003). *A generalized dynamic conditional correlation model for many asset returns*. Econometric Institute Report EI 2003-18, Erasmus University Rotterdam.
16. Johnson, L. L. (1960). The theory of hedging and speculation in commodity futures. Review *of Economic Studies, 27*(3), 139-151.
17. Kroner, K. F., & Sultan, J. (1993). Time-varying distributions and dynamic hedging with foreign currency futures. *Journal of Financial and Quantitative Analysis, 4*, 535-551.
18. Lien, D. (2004). Cointegration and the optimal hedge ratio: the general case. *The Quarterly Review of Economics and Finance, 44*, 654-658.
19. Lien, D., & Tse, Y. (2002). Some recent development in futures hedging. *Journal of Economic Surveys, 16*, 357-396.
20. Stein, J. L. (1961). The simultaneous determination of spot and futures prices. *The American Economic Review, 51*, 1012-1025.
21. Tsay, R. Conditional heteroskedastic time series models. *Journal of the American Statistical Association, 82*, 590-604.
22. Tse, Y. K., & Tsui, A. K. C. (2002). A multivariate GARCH model with time-varying correlations. *Journal of Business and Economic Statistics, 20*, 351-362.

NOTES