Sunset Company: Risk Analysis For Capital Budgeting Using Simulation And Binary Linear Programming

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ABSTRACT

The Sunset Company case illustrates how the study of linear programming and risk analysis are facilitated with popular spreadsheets and their simulation add-ins. From new products having a singular expected value for NPV, binary linear programming (BLP) optimally selects the combination of products that maximizes total NPV given capital constraints. Yet, when probability distributions are used to model risk of the products, an optimized simulation finds a different set of products for a risk adverse strategy. Advances in spreadsheet technology facilitate accounting educators introducing meaningful modeling and risk analysis into the classroom.

Keywords: capital budgeting, risk analysis, linear programming, simulation

INTRODUCTION

This teaching resource illustrates an increasing scope of risk analysis for investment projects within a capital rationing environment. The NPV of five products are first calculated and then used to determine the BLP optimal mix of products that will maximize Total NPV given capital constraints. A simulation for this optimal mix of products is performed and the Total NPV output distribution is used to discuss risk. Still, a strategy for reducing risk by maximizing the 25th percentile for Total NPV leads to a different mix of products. The Total NPV distributions for the two simulations are compared to identify tradeoffs between them.

This case is suited for use in cost accounting or graduate managerial accounting courses that introduce capital budgeting and linear programming. Risk analysis for capital budgeting commonly employs scenarios to determine a skeletal distribution for NPV. With simulation, students quickly grasp how modeling key input variables with probability distributions provides much more output data that fills in the skeletal NPV distribution. Working MBA students are especially intrigued with other applications of simulation, such as cash flow budgets, ratio analysis, and budgeted financial statements.

Linear programming with add-ins included with popular spreadsheets will easily optimize product mix decisions in manufacturing environments. Dichotomous BLP decisions can also be easily modeled by students for tasks such as make-or-buy, sell-or-process further, and optimizing investment portfolios. The capital rationing problem of Sunset Company highlights binary linear programming and optimized simulation as another tool for risk analysis.

Risk Analysis Using Simulation

Traditional risk analyses rely on single point estimates of input variables in determining expected output values of modeled relationships. One clear reason for differences between the expected values of models and actual outcomes is the uncertainty for input estimates. Common techniques to address this uncertainty for input estimates of accounting relationships include sensitivity analysis and scenario analysis; yet, recent add-ins to spreadsheets (e.g., @Risk and Crystal Ball) are better suited for risk analysis (Kelliher *et al.*, 1996; Togo, 2004). These spreadsheet add-ins model input variables with probability distributions, perform a simulation, and then generate output distributions for targeted cells. Distributions for key outputs allow users to better understand risk within the modeled relationship. For a cash budget example, an output distribution for borrowings will identify an amount sufficient to meet the 90^{th} percentile of outcomes, which is in contrast to an expected value when no input probabilities are used.

Linear Programming

A constrained optimization model represents the problem of allocating scarce resources such that an objective function is optimized (Moore and Weatherford, 2001). Constrained linear models have two features in common: an objective function and constraints. An objective function consists of a single performance measure to be maximized or minimized, such as contribution margin or costs. Constraints are restrictions on the set of allowable decisions and they are often in the form of physical, economic and policy limitations or requirements.

In cost/managerial accounting courses, product-mix decisions are often examined with the use of a spreadsheet. While included add-ins to popular spreadsheets will easily optimize constrained linear models (Hilton *et al*., 2003), the topic of linear programming is commonly introduced within an appendix using the graphical approach for just two products (e.g., Horngren *et al.*, 2003).

Linear programming add-ins also have the capability to perform integer and binary linear programming. However, integer linear programming (ILP) is seldom taught as students are commonly told to round to the nearest integer. Binary linear programming (BLP) models utilize integer variables (usually 1 and 0) to indicate logical or dichotomous decisions (e.g., on/off, true/false, or accept/reject). While BLP models are useful for scheduling, financial portfolios, capital rationing environments, and production planning (Moore and Weatherford, 2001), this topic is seldom taught in cost/managerial accounting courses.

Optimized Simulations

A drawback to linear programming is its inability to optimize models that have probability distributions as input variables. Yet, **when a statistic is specified for an output variable,** there are add-ins to spreadsheets (e.g., RiskOptimizer and OptQuest) that will optimize models having probabilistic input. Concurrently, simulation addresses the uncertainty present in the model while optimizing algorithms generate dichotomous values for the decision. The result of this optimized simulation is a set of values (decision) that maximizes or minimizes the objective function while meeting its desired simulation statistic and the constraints of the linear programming model (Palisade Corporation; 2000). With this added capability, managers can examine various risk adjusting strategies by specifying a statistic for the output variable.

SUNSET COMPANY: AN EXAMPLE FOR SIMULATION AND BLP

Background

Sunset Company is a manufacturer of popular water sports equipment. Started as a family business in 1983, Sunset has gradually gained market share and is now the largest supplier of water sports equipment in California. Sunset is recognized for successfully introducing innovative products at a low cost to the customer. Each year Sunset introduces about three to five new products and drops one to two poor performing products. By working closely with their manufacturing equipment suppliers, Sunset has been able to beat competitors to the market using modern manufacturing techniques. The major customers of Sunset are large discount retailers that are willing to purchase new products based on past successes.

The dominance of Sunset Company in the California market is being challenged by a large international sports equipment company based in Southeast Asia. Sunset's management team led by President Jaslyn Mahealani knows that continued success is dependent on the development and manufacturing of novel products at a low cost. In the past, it was not uncommon for products to be selected with little cost analysis and just the support of certain creative individuals. Bad investments were quickly dropped or covered by better than expected sales from other products. However, with an economic downturn expected to tighten the market and significantly increased competition, Sunset Company is faced with less capital available for investment and with more uncertainty in the demand for its water sports equipment.

New Products Decision

The management team is reviewing five new products for the upcoming year found in Panel A of Table 1: Air Mattress, Life Vest, Fins, Boogie Board and Goggles. The controller Deron Kekua distributes cost-benefit analyses for each product, which highlights annual lease payments for new machinery and the first-year annual net cash inflow. The lease payments are disbursed at the beginning of each year and separated from other annual operating cash expenditures. The annual net cash inflow for each product is the difference between expected operating cash receipts and operating cash expenditures other than lease payments. From past experience, the first-year net cash inflow can be used to estimate subsequent net cash inflows over the projected five-year life as presented in Panel B of Table 1. The yearly net cash inflow adjustment factor for year 2 is expected to be 1.15 of the first-year estimate, year 3 is 1.05, year 4 is 0.95, and year 5 is 0.85.

Table 1: Product Information

Panel A: Cash Flows (in thousands)

Product	Annual Lease	First-Year Net Cash Inflow: Mean	First-Year Net Cash Inflow: Probability Distribution
Air Mattress	$ 155	$ 190	Normal, with $15 standard deviation
Life Vest	$ 275	$ 340	Triangular, with $320 minimum, $330 most likely, and $370 maximum
Fins	$ 200	$ 250	Normal, with $50 standard deviation
Boogie Board	$ 135	$ 180	Triangular, with $160 minimum, $180 most likely, and $200 maximum
Goggles	$ 60	$ 75	Normal, with $15 standard deviation
	$ 825		

Panel B: Yearly Net Cash Inflow Adjustment and Capital Constraint

	Year 1	Year 2	Year 3	Year 4	Year 5
Net Cash Inflow Adjustment Factor	1.00	1.15	1.05	.95	.85
Capital Constraint (in thousands)	$ 610	$ 590	$ 560	$ 530	$ 500

The managers submitted to Deron Kekua their estimate for each product's first-year net cash inflow. The mean of the managers' estimates are reported in Panel A. However, there was disagreement among the managers on the sole use of the average for the first-year net cash inflow. Deron then developed a probability distribution of the net cash inflow for each product with the expected value equal to the managers' mean. While most managers found a probability distribution to be more acceptable than just the one expected mean value, they wondered how the distribution could be used in cost-benefit analysis.

Deron Kekua estimates investment capital available for the next five years to steadily decrease from $610,000 to $500,000 as shown in Panel B of Table 1. The management team agrees to an 8% cost of capital for evaluating products and to discount the capital available in future years. Products to be dropped and their released cash are reflected in the available capital for investment. Jaslyn Mahealani knows that consistent investment in new products is critical to the success of Sunset Company. Hence, she seeks to find a combination of products that returns a reasonable NPV while limiting risk during these challenging times. Deron Kekua is asked to present to the management team an approach that improves capital investment decisions and adopts risk adverse positions.

Requirements

Requirement 1: Compute the NPV of each product

Compute the NPV for each product on a spreadsheet using the managers' average for annual net cash inflows. Multiply the net cash inflow of Year 1 with the yearly adjustment factors to obtain the net cash inflows for each of the five years. Assume an 8% discount rate to compute the PVs for lease payments and annual net cash inflows.

Requirement 2: Obtain the BLP solution

Refer to the NPVs computed in Requirement 1 and use binary linear programming to solve for a mix of products which maximizes Total NPV subject to the capital constraints. When using the add-in SOLVER for EXCEL, the decision variables (cells containing "1" or "0") are to be included as a multiplicative factor for both NPV and PV of lease payments. Adjust the yearly capital available for the time-value of money.

Requirement 3: Perform NPV simulation for each product

Obtain a NPV distribution for each product by substituting the mean of Requirement 1 with the probability distribution of the first-year net cash inflow (see Table 1) and then performing a simulation. Use the student version of the EXCEL spreadsheet add-in @RISK to perform the simulation. Present the graphs of the NPV output distributions for just the two products Air Mattress and Life Vest.

Requirement 4: Perform Simulation for BLP Solution

Obtain a probability distribution for Total NPV by performing a simulation for the BLP optimal mix of products found in Requirement 2. Use the product's NPV output distribution of Requirement 3 as inputs to this simulation performed on the BLP of Requirement 2. Generate the Total NPV output distribution for this mix of products and identify the expected value, values at the 5th and 95th percentiles, and the value at the 25th percentile.

Requirement 5: Perform risk adverse simulation

Determine a risk adverse solution to the BLP problem by requiring that Total NPV be maximized at the 25th percentile. Use the NPV output distributions of Requirement 3 as inputs and the RISKOptimizer (2000) add-in to perform the optimized simulation on the BLP of Requirement 2. Identify the mix of products and the expected value when Total NPV is maximized at the 25th percentile.

Requirement 6: Compare results

Compare the simulation results for BLP (Requirement 4) and 25th Percentile (Requirement 5). Identify the different mix of products for BLP and 25th Percentile simulations. Compare their NPV expected value and at the 25th percentile. Discuss the advantages and disadvantages of both simulations.

Solution To Requirements (spreadsheets available on request)

Requirement 1solution: Table 2 calculates the NPV for each product. The present values for lease payments and net cash inflows are computed for each year. Then, the NPV for each product is calculated. Air Mattress has a NPV of $95,000, which is $764,000 PV of net cash inflow less $669,000 PV of lease payment. Life Vest has a $181,000 NPV, Fins has a $143,000 NPV, Boogie Board has a $142,000 NPV, and Goggles has a $42,000 NPV.

Table 2: NPV of Products

Product	PV	Year 1	Year 2	Year 3	Year 4	Year 5
Air Mattress						
Lease payment		155	155	155	155	155
PV of lease payment	669	155	144	133	123	114
First year net cash inflow		190	190	190	190	190
Yearly adjustment factor		1.00	1.15	1.05	0.95	0.85
Net cash inflow		190	219	200	181	162
PV of net cash inflow	764	176	187	158	133	110
NPV of Air Mattress	**95**	21	43	25	10	-4
Life Vest						
Lease payment		275	275	275	275	275
PV of lease payment	1186	275	255	236	218	202
First year net cash inflow		340	340	340	340	340
Yearly adjustment factor		1.00	1.15	1.05	0.95	0.85
Net cash inflow		340	391	357	323	289
PV of net cash inflow	1367	315	335	283	237	197
NPV of Life Vest	**181**	40	80	47	19	-5
Fins						
Lease payment		200	200	200	200	200
PV of lease payment	862	200	185	171	159	147
First year net cash inflow		250	250	250	250	250
Yearly adjustment factor		1.00	1.15	1.05	0.95	0.85
Net cash inflow		250	288	263	238	213
PV of net cash inflow	1005	231	246	208	175	145
NPV of Fins	**143**	31	61	37	16	-2
Boogie Board						
Lease payment		135	135	135	135	135
PV of lease payment	582	135	125	116	107	99
First year net cash inflow		180	180	180	180	180
Yearly adjustment factor		1.00	1.15	1.05	0.95	0.85
Net cash inflow		180	207	189	171	153
PV of net cash inflow	724	167	177	150	126	104
NPV of Boogie Board	**142**	32	52	34	19	5
Goggles						
Lease payment		60	60	60	60	60
PV of lease payment	259	60	56	51	48	44
First year net cash inflow		75	75	75	75	75
Yearly adjustment factor		1.00	1.15	1.05	0.95	0.85
Net cash inflow		75	86	79	71	64
PV of net cash inflow	301	69	74	63	52	43
NPV of Goggles	**42**	9	18	12	4	-1

Requirement 2 solution: Table 3 presents the binary linear programming models **Setup** and **BLP Solution. SETUP** is the base model from which **BLP Solution** is generated. Based on expected values, the **BLP Solution** has the optimal mix (decision = 1) of Air Mattress, Fins and Boogie Board which maximizes total NPV = $380,000. While meeting yearly capital constraints over the five-year period, these three products require PV lease payments of $2,113,000, which is less than the $2,425,000 PV capital available over the same period.

Table 3: Binary Linear Programming

Input Data	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Total**
Capital available	610	590	560	530	500	2790
PV of capital available	610	546	480	421	368	2425

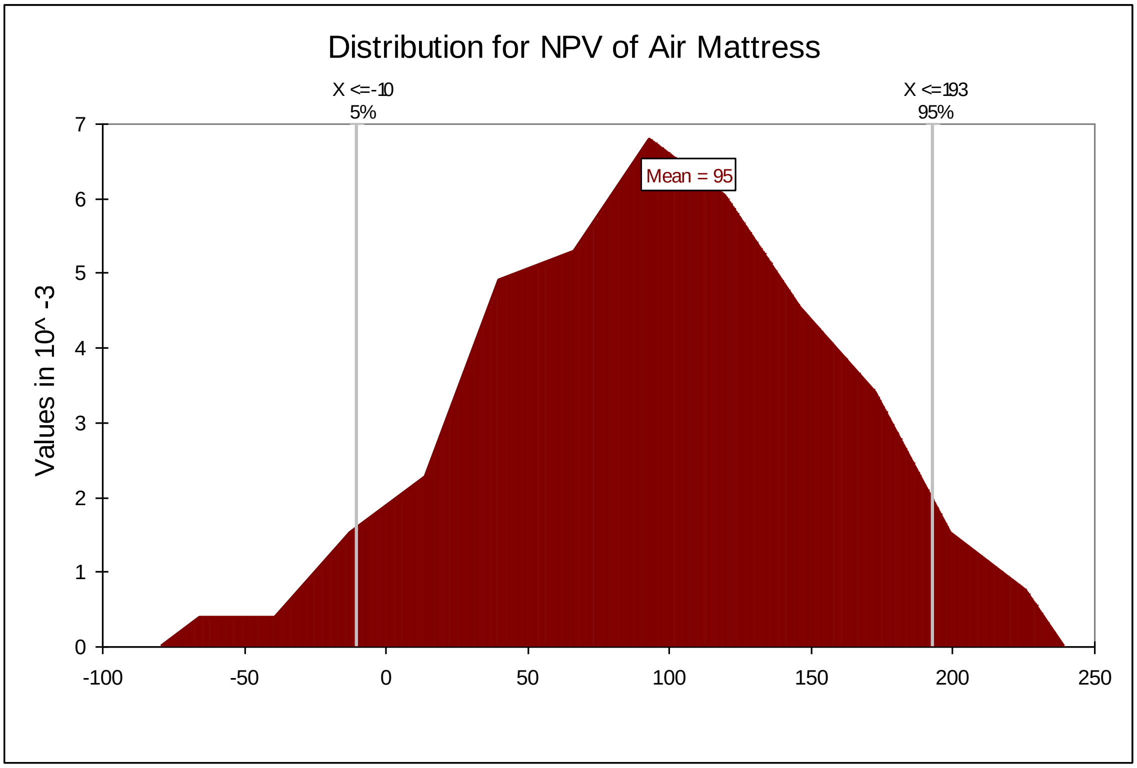
Setup	**Air Mattress**	**Life Vest**	**Fins**	**Boogie Board**	**Goggles**	**Total**		**PV Capital Available**
Decision	**1**		**1**	**1**	**1**	**5**		
NPV	95	181	143	142	42	603		
PV of lease payments		*1*						
Year 1		275	190	135	65	820	<=	610
Year 2	144	255	176	125	60	760	<=	546
Year 3	133	236	163	116	56	704	<=	480
Year 4	123	218	151	107	52	651	<=	421
Year 5	114	202	140	99	48	603	<=	368
Total	669	1186	820	582	281	3538		2425

BLP Solution	**Air Mattress**	**Life Vest**	**Fins**	**Boogie Board**	**Goggles**	**Total**		**PV Capital Available**
Decision	**1**	**0**	**1**	**1**	**0**	**3**		
NPV	95	0	143	142	0	**380**		
PV of lease payments								
Year 1	155	0	200	135	0	490	<=	610
Year 2	144	0	185	125	0	454	<=	546
Year 3	133	0	171	116	0	420	<=	480
Year 4	123	0	159	107	0	389	<=	421
Year 5	114	0	147	99	0	360	<=	368
Total	669	0	862	582	0	2113		2425

25th Percentile Solution	**Air Mattress**	**Life Vest**	**Fins**	**Boogie Board**	**Goggles**	**Total**		**PV Capital Available**
Decision	**0**	**1**	**0**	**1**	**1**	**3**		
NPV	0	181	0	142	42	**365**		
PV of lease payments				**Percentile**	**(0.25)**	**= 311**		
Year 1	0	275	0	135	60	470	<=	610
Year 2	0	255	0	125	56	436	<=	546
Year 3	0	236	0	116	51	403	<=	480
Year 4	0	218	0	107	48	373	<=	421
Year 5	0	202	0	99	44	345	<=	368
Total	0	1186	0	582	259	2027		2425

Requirement 3 solution: By substituting the probability distribution for the first-year net cash inflows (Table 1) for its mean value, the spreadsheet model of Requirement 1 performs a simulation for each product. Using the inexpensive student version of @RISK, Figure 1 displays useful probabilistic information in a graphical format for Air Mattress and Life Vest. The symmetrical NPV graph for Air Mattress supports a mean value of \$95,000, with 5% $\leq$ -\$10,000 and 95% $\leq$ \$193,000. The triangular NPV graph for Life Vest has a mean of \$181,000, with 5% $\leq$ \$121,000 and 95% $\leq$ \$261,000. The graphical output distributions provide NPV estimates for a range of possible outcomes and their likelihood of occurrence.

Figure 1: NPV Distributions of Air Mattress and Life Vest



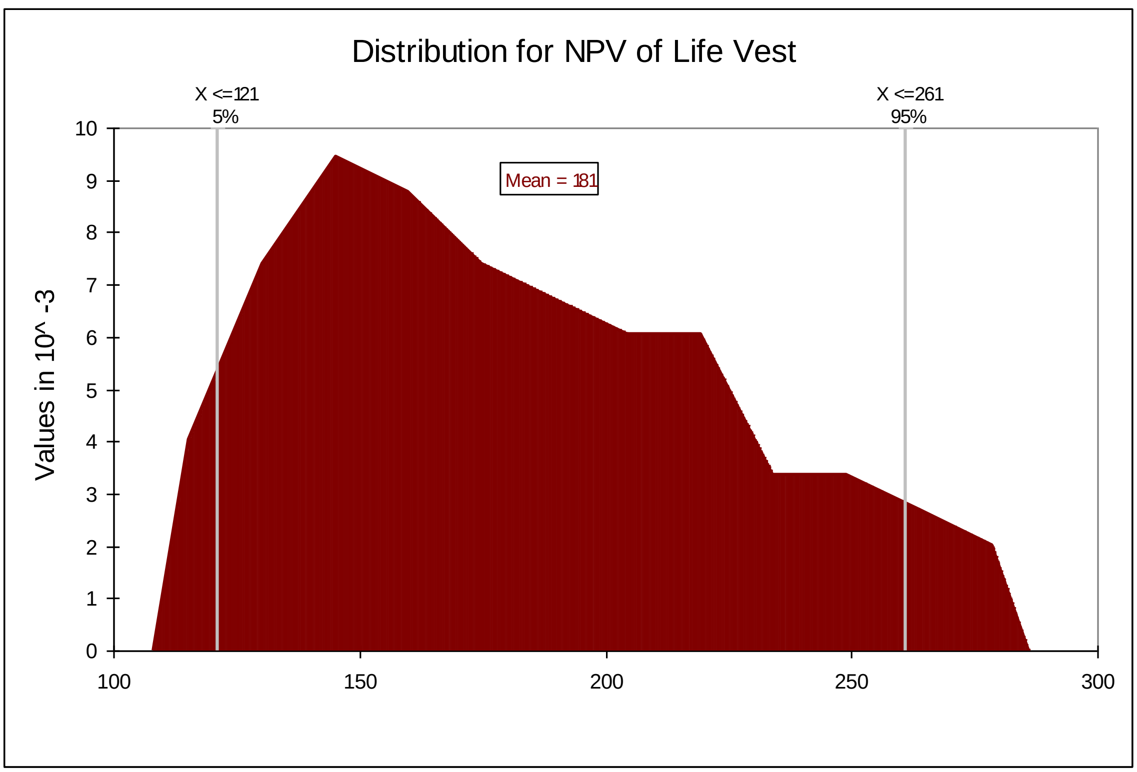
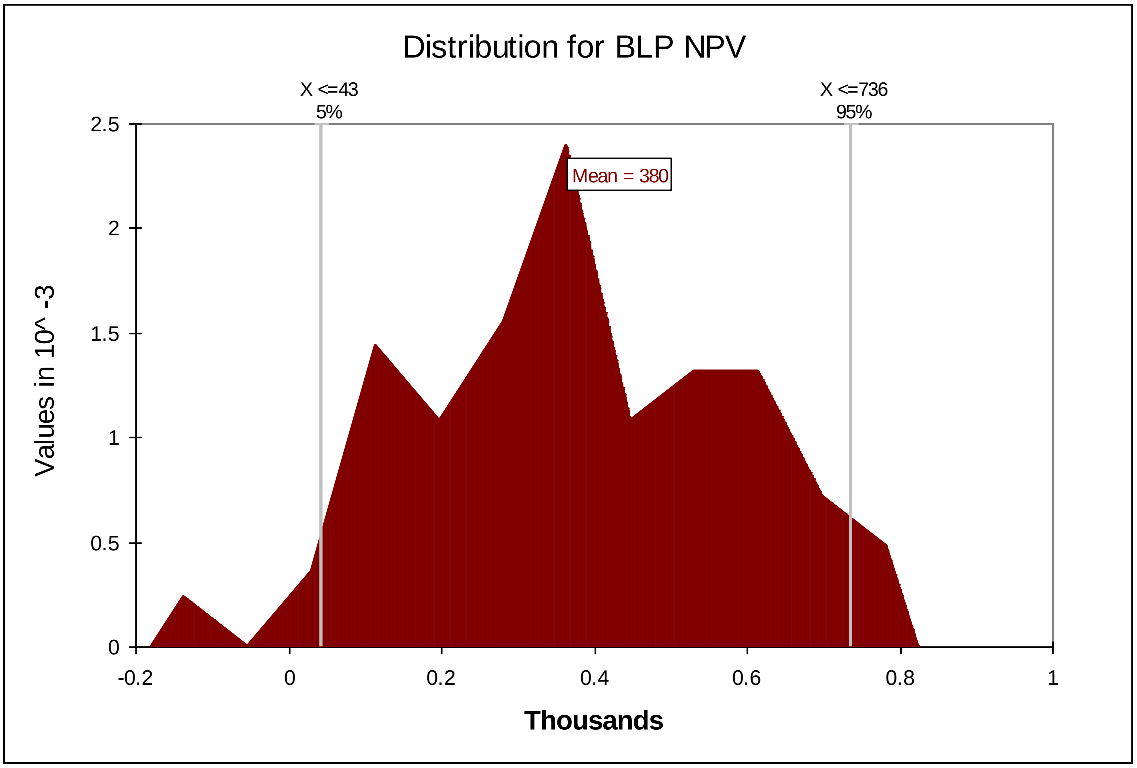
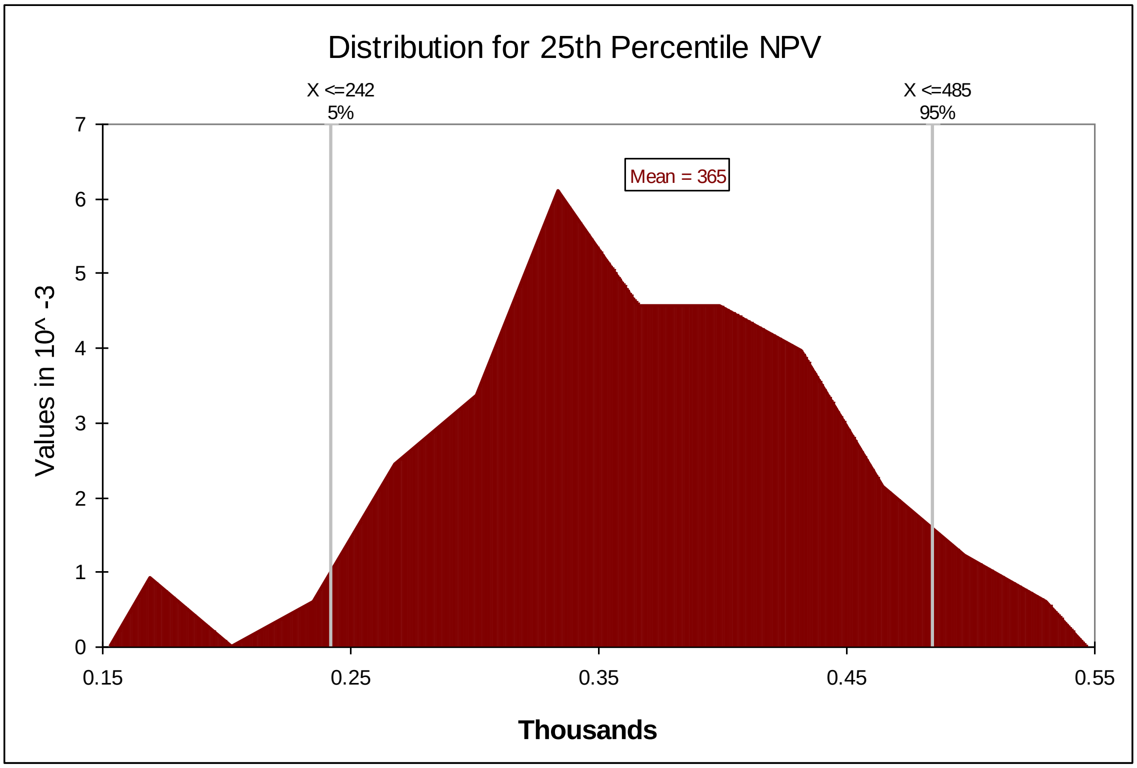


Figure 2: Total NPV Distributions





Requirement 4 solution: Figure 2 displays the distribution for total NPV when a simulation is performed for the BLP optimal mix of Air Mattress, Fins and Boogie Board. As expected it has a mean of \$380,000, with 5th and 95th percentile values of \$43,000 and \$736,000. In addition, the 25th percentile value is \$221,000.

Requirement 5 solution: Table 3 displays the **25th Percentile Solution,** which is a risk adverse solution to the capital constraint problem. The solution (decision=1) consists of Life Vest, Boogie Board and Goggles, which has an expected Total NPV mean of \$365,000, with 5th and 95th percentile values of \$242,000 and \$485,000. The 25th percentile value is \$311,000 and the product mix requires \$2,027,000 of PV lease payments over the five-year period.

Requirement 6 solution: The results of the **BLP Solution** and the **25th Percentile Solution** are displayed in Figure 2. At the 25th percentile of interest, the **25th Percentile Solution** is greater by \$90,000 (\$311,000 - \$221,000). The **25th Percentile Solution** is risk adverse in that its mix of products (a) significantly reduces the likelihood of incurring a loss and (b) requires \$86,000 less PV lease payments (\$2,113,000 - \$2,027,000). At the 5th percentile, the **BLP Solution** has a value of \$43,000, while the **25th Percentile Solution** has a value of \$242,000.

The tradeoffs to this risk adverse strategy are (a) the expected mean of \$365,000 for the **25th Percentile Solution** is \$15,000 less than the expected mean of \$380,000 for the **BLP Solution**, and (b) larger values for NPV are less likely with the **25th Percentile Solution**, as it has a 95th percentile value of \$485,000 in comparison to \$736,000 for the **BLP Solution**.

The management team of Sunset Company is provided with useful information in selecting new products. While only the expected value and the 25th percentile scenarios are presented in this case, the instructor could examine additional scenarios (Requirement 5) by specifying other simulation statistics for the Total NPV distribution.

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NOTES