Demonstrating The Use Of Vector Error Correction Models Using Simulated Data

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ABSTRACT

In this paper, we demonstrate the use of time series analysis, including unit roots tests, Granger causality tests, cointergation tests and vector error correction models. We generate four time series using simulation such that the data has both a random component and a growth trend. The data are analyzed to demonstrate the use of time series analysis procedures.

Keywords: Time Series Analysis; Simulation; Unit Root Tests; Granger Causality; Cointegration; Vector Error Correction Models

INTRODUCTION

Regressions between levels of variables may have high covariation because of persistence in the base levels of the variables rather than persistence in the changes in the values of the variables. Taking the first differences of the variables may eliminate, or at least reduce, the dependence between the variables. Gross national income from period to period is an integrated process, but the changes in GNI are not an integrated process. The first differences of GNI are an independent, identically distributed process which are only weakly dependent. An alternative transformation to differencing is to take the natural logarithm of the ratio of the two levels to generate the percentage rate of change which generates a continuously compounded rate of change.

STATIONARITY AND DICKEY-FULLER

Ordinary Least Squares regression requires that the time series being evaluated be stationary. Otherwise, OLS is no longer efficient, the standard errors are understated, and the OLS estimates are biased and inconsistent. Stationarity requires that the time series values for the mean, the standard deviation, and the covariance, be invariate over time[1].

$E(\mu_{t-1}) = E(\mu_t)$, i. e., μ_t is constant over time,
$E(\sigma_{t-1}) = E(\sigma_t)$, i. e., σ_t is constant over time, and
$E(cov_{t-1}) = E(cov_t)$, i. e. the covariance of (x_t, x_{t-1}) is constant over time.

That is, the mean for any time (t-1) will equal the mean for any time (t), the standard deviation for any time (t-1) will equal the standard deviation for any time (t), and the covariance for any time (t-1) will equal the covariance for any time (t).

One method to test for stationarity is the unit root test of Dickey-Fuller (1979). To test for a unit root of a stochastic time series, the value of the random variable is regressed against lagged values of the same random variable

$$x_t = \alpha + \beta x_{t-1} + \varepsilon_t \qquad [1]$$

[1] See Wooldridge (2003) for a more detailed discussion of the theoretical models discussed in this paper.

where, x_t is the value of the time series at time (t), α is the intercept term, β is the regression coefficient, x_{t-1} is the lagged value of the time series, and ε_t is the residual. If β is equal to one, then the process generating the time series is non-stationary. The null hypothesis is that H_0: $\beta=1$ and the alternative hypothesis is that β is less than one, H_1: $\beta<1$. The actual test is run after subtracting x_{t-1} from both sides of Equation [1]. The regression is

$$\Delta x_t = \alpha^* + \beta^* x_{t-1} + \varepsilon^*_t \quad [2]$$

where the (*) indicates the parameters from the regression adjusted by subtracting x_{t-1}. The null hypothesis is that H_0: $\beta^*=0$ and the alternative hypothesis is that β is less than zero, H_1: $\beta^*<0$.

This model is only valid for AR(1) processes. If the underlying return generating process exhibits serial correlation of order greater than one, Augmented Dickey-Fuller tests must be used. Higher order terms are included in the regression

$$\Delta x_t = \alpha^* + \beta^* x_{t-1} + \beta_1 \Delta x_{t-1} + \beta_2 \Delta x_{t-2} + \ldots + \beta_n \Delta x_{t-n} + \varepsilon^*_t \quad [3]$$

where, the additional terms are derived from the higher order AR() terms. The null hypothesis is that H_0: $\beta^*=0$ and the alternative hypothesis is that β^* is less than one, H_1: $\beta^*<0$.

CO-INTEGRATION AND ENGLE-GRANGER

Co-integrated processes are random in the short-term but tend to move together in the long-term. Wooldridge (2003) shows that six-month Treasury bill rates and three-month Treasury bill rates are both unit root processes that are independent in the short-term but do not drift too far apart in the long-term. If either rate moves too far from equilibrium, either too high or too low, investors move money from the low (high) rate alternative to the high (low) rate alternative. This process will raise (lower) the rate in the low (high) rate market.

Engle and Granger (1987) show that if a linear combination of non-stationary time series is stationary, the time series is co-integrated. If two time series are integrated of order one, the time series resulting from adding the two is integrated of order one. If $y_t \sim I(1)$ and $x_t \sim I(1)$, then $(y_t + x_t) \sim I(1)$. However, if a beta, β, exits such that $(y_t - \beta x_t) \sim I(0)$, then y_t and x_t are said to be co-integrated. This co-integration equation reflects the long-term relationship between y_t and x_t.

If we can construct a linear combination of y_t and x_t such that the difference of the two variables has a unit root, the two variables are co-integrated and the regression coefficient is the co-integration parameter.

$$y_t = \beta_0 + \beta_1 x_t + u_t$$

If u_t is I(0), then y_t and x_t are co-integrated. The model for testing for co-integration with a time trend includes a time variable.

$$y_t = \beta_0 + \beta_2(t) + \beta_1 x_t + u_t$$

If u_t is I(0), then y_t and x_t are co-integrated.

ERROR CORRECTION MODELS

Error correction models are a class of models that provide insight into the long-term relationship between variables in terms of the "impact propensity, long run propensity, and lag distribution for Δy as a distributed lag in Δx."[2] The independent variable is x and the dependent variable is y. An error correction term is computed based on the past values of both x and y. If past values of y are over-estimated, future values will be moved back toward

[2] Wooldridge (2003), page 621.

equilibrium by the error correction factor. In the example of the six-month and three-month Treasury bill rates, the error correction term is computed from the difference of the one period lagged six-month rate and the two-period lagged three-month rate. Thus, if either of the two rates drift too far from the long-term rate, the error correction term shows the tendency of the rates to return to the long-term rate.

If two variables are cointegrated, we can construct a variable, s_t, which is I(0). The resulting error correction equation is

$$\Delta x_t = \alpha^* + \beta^* x_{t-1} + \gamma^* y_t + \gamma^* y_{t-1} + \delta^* s_{t-1} + \varepsilon^*_t \quad [2]$$

where, s_{t-1}, equals $(y_{t-1} - \beta^* x_{t-1})$ and is the error correction term.

We can analyze the short-term effects of the relationship between the two variables. If the value of $\delta < 0$, the error correction term serves to return the process to the long-run value. That is, if $(y_{t-1} > \beta^* x_{t-1})$, the process was above the long-run value in the previous period and has been moved back by the error correction process.

GENERATING THE SIMULATED DATA

We use an Excel spreadsheet and the Excel function Rand() to generate four times series of numbers of 1,000 observations each. Rand() generates a number from zero to one. In order to create a random number series with a value of zero, the random number generated by Rand() is transformed into a zero value function by subtracting 0.50 from each Rand() value, Rand(*)=(Rand()-0.50). This random number generated by Rand() and transformed to a zero value number is used to create an Index value with the following equation:

Index(i,t) = Index(i,t-1) (1+Rand()(Return)+Trend)

Index(i,t) = Index(i,t-1) = 1.0000(1+0.0025+.005)

Index (i,t) is the index value for each period "t" that is calculated from the previous Index (i,t) value plus a randomly generated value with an expected value of zero plus the trend. The trend is a long-run trend added to the random index change in order to create both a random component of the Index plus a trend. Four Indexes are generated using this function with 1001 observations each.

Returns are calculated from each Index (i,t) using the natural logarithm function. Return(i,t) is the natural logarithm of the ratio of Index(i,t) divided by Index (i,t-1).

Return(i,t) = (Index(i,t)) / (Index (i,t-1))

Each return series has 1,000 observations that have both a random component and a trend component. The random component is the value of Rand(*)(Return) that is added to each previous Index (i,t) plus a trend.

ANALYSIS OF THE GENERATED RETURNS

The four return series are analyzed using EViews. Figures 1 to 5 show the probability distribution for each of the four return series. Figure 1 shows the sample statistics and analysis for Return(1,t) which has a mean value of 0.04981 with a standard deviation of 0.005108. The skewness statistic equals -0.022106 and the kurtosis statistic equals 2.8937. The Jarque-Bera statistic to measure normality is 0.55, indicating that the probability distribution for the Return(1,t) is normal. All four Return(I,t) series have expected values and standard deviations that are similar and Jarque-Bera statistics that do not reject normality. That is, all four Return(i,t) series exhibit the probability distribution statistics that one would expect given the method used to construct each of the four Return(I,t) series.

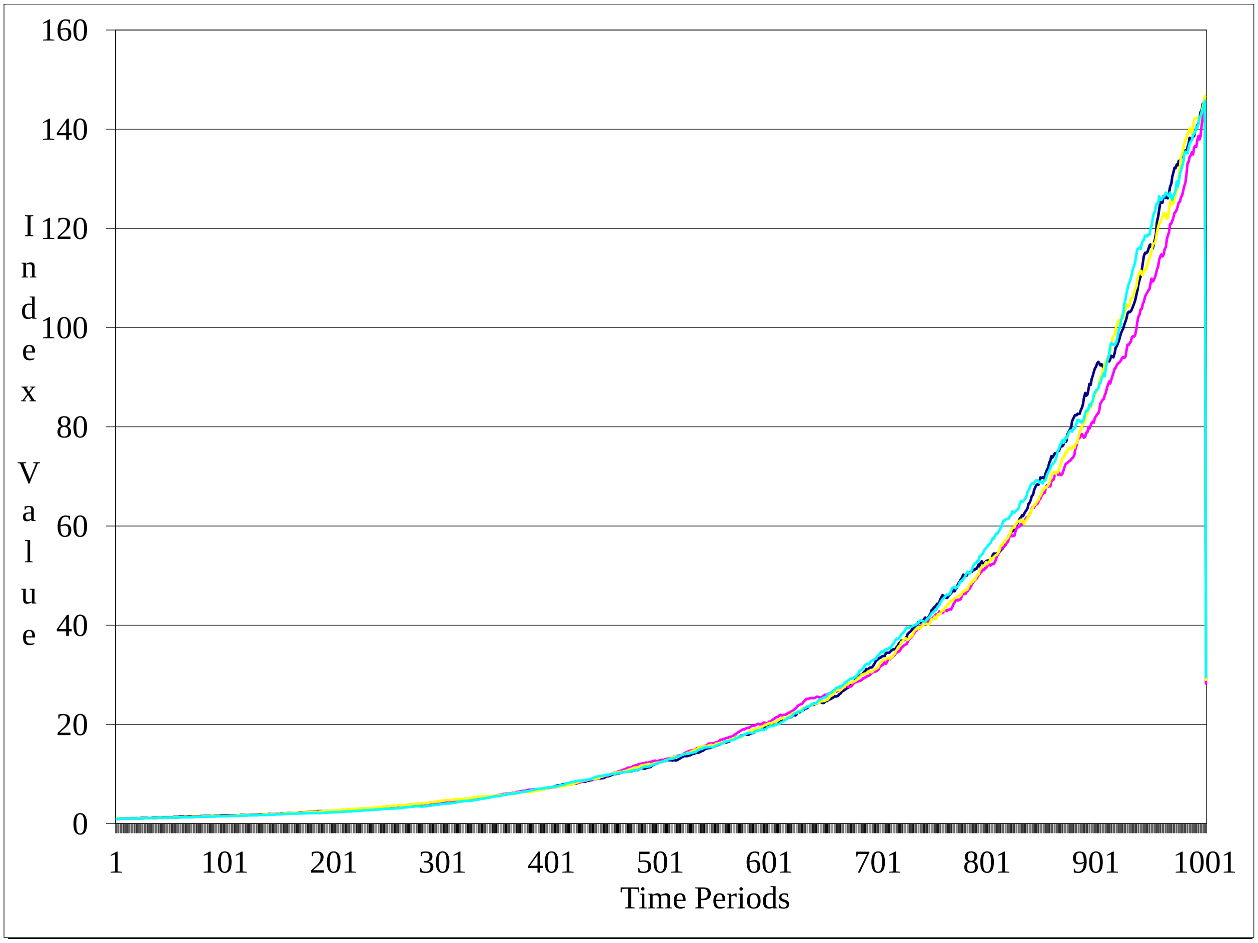
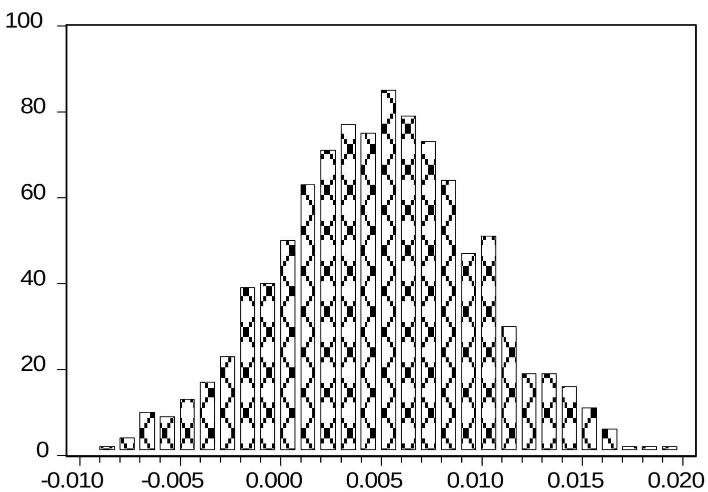


Figure 1: Graph of Four Indexes
Time Series Analysis Simulation



Series: ROR01	
Sample 1 1000	
Observations 1000	
Mean	0.004981
Median	0.005108
Maximum	0.019589
Minimum	-0.009770
Std. Dev.	0.004965
Skewness	-0.022106
Kurtosis	2.893713
Jarque-Bera	0.552149
Probability	0.758757

Figure 2: Summary Statistics for ROR01
Time Series Analysis Simulation

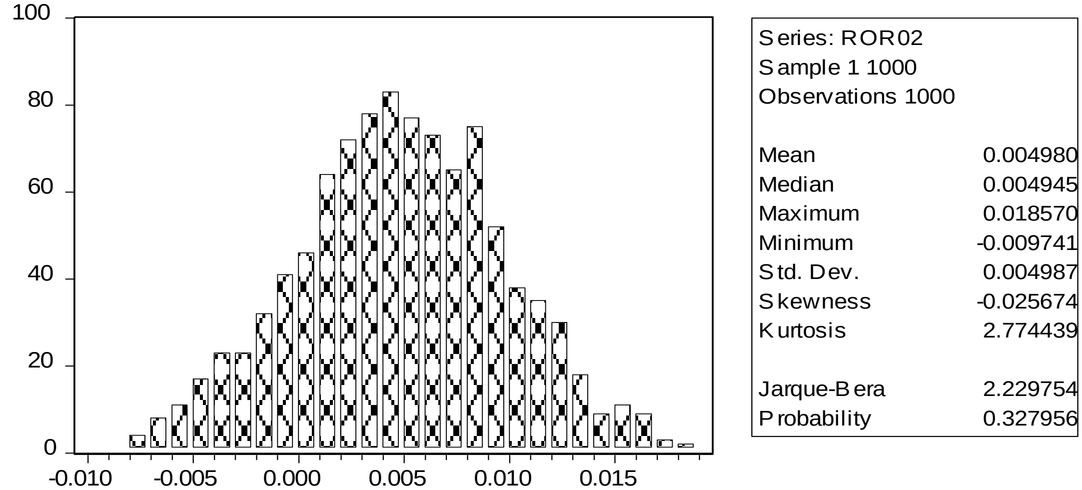


Figure 3: Summary Statistics for ROR02
Time Series Analysis Simulation

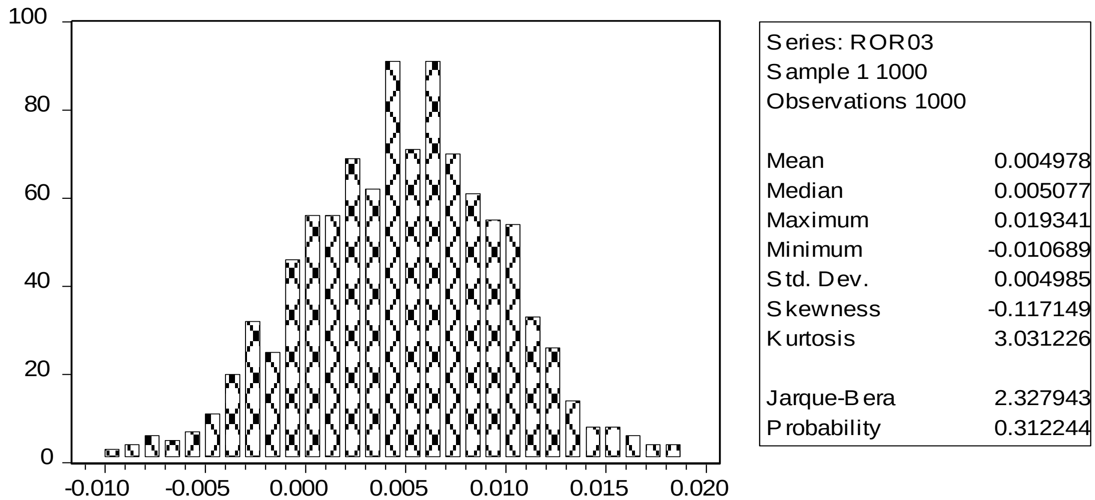


Figure 4: Summary Statistics for ROR04
Time Series Analysis Simulation

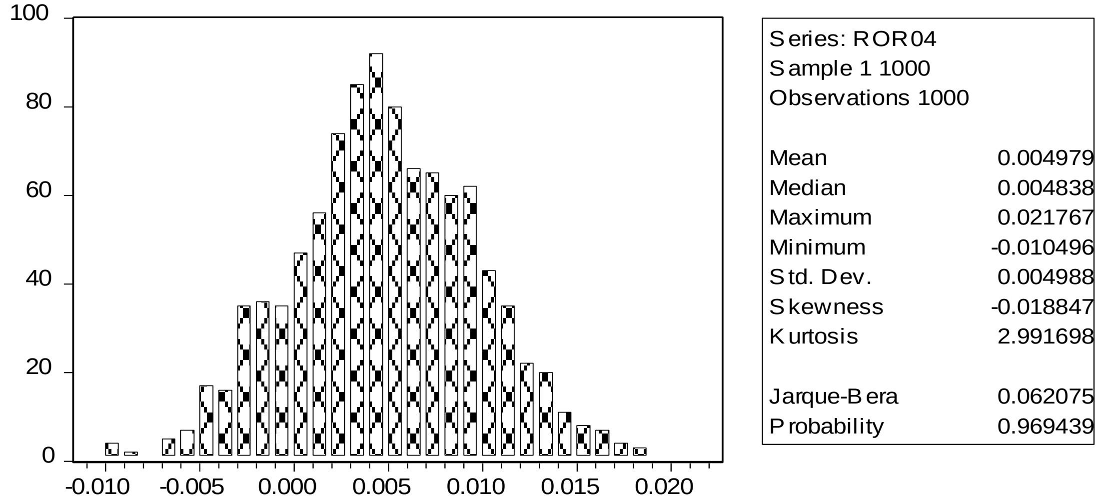


Figure 5: Summary Statistics for ROR04
Time Series Analysis Simulation

Table 2 contains the correlation matrix for the four Return(i,t) series. The four Return(i,t) series are constructed with a short-run random component and a long-run trend component. The correlation coefficients for the four Return(i,t) series reflect the short-run relationship between each of the Return(i,t) series. Thus, we see in Table 1 that the correlation coefficients for the four Return(i,t) series are all low and none are statistically significant.

Table 1: Summary Statistics
Time Series Analysis Simulation

	ROR01	ROR02	ROR03	ROR04
Mean	0.004981	0.004980	0.004978	0.004979
Median	0.005108	0.004945	0.005077	0.004838
Maximum	0.019589	0.018570	0.019341	0.021767
Minimum	-0.009770	-0.009741	-0.010689	-0.010496
Std. Dev.	0.004965	0.004987	0.004985	0.004988
Skewness	-0.022106	-0.025674	-0.117149	-0.018847
Kurtosis	2.893713	2.774439	3.031226	2.991698
Jarque-Bera	0.552149	2.229754	2.327943	0.062075
Probability	0.758757	0.327956	0.312244	0.969439
Observations	1000	1000	1000	1000

Table 2: Correlation Matrix
Time Series Analysis Simulation

ROR01	ROR02	ROR03	ROR04
1.000000	-0.040348	0.001985	0.034449
-0.040348	1.000000	0.023084	-0.039111
0.001985	0.023084	1.000000	0.031080
0.034449	-0.039111	0.031080	1.000000

Generally, the first step in analyzing the relationships between time series is to determine if each Return(i,t) series has a unit root. The Augmented Dickey-Fuller test for a unit root is performed for each of the four Return(i,t) series and the empirical results are detailed in Table 3, Table 4, Table 5, and Table 6 for each simulated return series. For the Return(1,t) series, the ADF test statistic is -14.63 and the critical value for the ADF test statistic is -3.97 which indicates that Return(1,t) series does not have a unit root. None of the four lagged Return(1,t) series variable regression coefficients are statistically significant, but the intercept term is and equals 0.5014. The adjusted R^2 for the regression is 0.4798 and the F-statistic is 152. These results reject the presence of a unit root. That is, Return(1,t) series does not have a unit root which is consistent with the method of creating the Return(i,t) series. The results for all four Return(i,t) series are similar to the results for Return(1,t) series.

Table 3: Unit Root Analysis
Time Series Analysis Simulation (ROR01)

ADF Test Statistic	-14.64202	1% Critical Value*		-3.4397
		5% Critical Value		-2.8649
		10% Critical Value		-2.5685
*MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(ROR01)				
Method: Least Squares				
Date: 11/05/08 Time: 17:44				
Sample(adjusted): 6 1000				
Included observations: 995 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
ROR01(-1)	-1.009307	0.068932	-14.64202	0.0000
D(ROR01(-1))	0.050457	0.061538	0.819932	0.4125
D(ROR01(-2))	0.052299	0.053490	0.977737	0.3284
D(ROR01(-3))	0.050287	0.044008	1.142698	0.2534
D(ROR01(-4))	0.032641	0.031805	1.026271	0.3050
C	0.005031	0.000378	13.31435	0.0000
R-squared	0.479762	Mean dependent var		4.66E-06
Adjusted R-squared	0.477132	S.D. dependent var		0.006887
S.E. of regression	0.004980	Akaike info criterion		-7.760815
Sum squared resid	0.024526	Schwarz criterion		-7.731251
Log likelihood	3867.005	F-statistic		182.4109
Durbin-Watson stat	1.999825	Prob(F-statistic)		0.000000

Table 4: Unit Root Analysis
Time Series Analysis Simulation (ROR02)

ADF Test Statistic	-15.62956	1% Critical Value*		-3.4397
		5% Critical Value		-2.8649
		10% Critical Value		-2.5685
*MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(ROR02)				
Method: Least Squares				
Date: 11/05/08 Time: 17:45				
Sample(adjusted): 6 1000				
Included observations: 995 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
ROR02(-1)	-1.122367	0.071811	-15.62956	0.0000
D(ROR02(-1))	0.145336	0.063859	2.275887	0.0231
D(ROR02(-2))	0.144089	0.054330	2.652124	0.0081
D(ROR02(-3))	0.065803	0.044582	1.476016	0.1403
D(ROR02(-4))	0.078950	0.031802	2.482531	0.0132
C	0.005613	0.000390	14.39128	0.0000
R-squared	0.496699	Mean dependent var		1.23E-05
Adjusted R-squared	0.494154	S.D. dependent var		0.006978
S.E. of regression	0.004963	Akaike info criterion		-7.767596
Sum squared resid	0.024361	Schwarz criterion		-7.738032
Log likelihood	3870.379	F-statistic		195.2052
Durbin-Watson stat	1.999849	Prob(F-statistic)		0.000000

Table 5: Unit Root Analysis
Time Series Analysis Simulation (ROR03)

ADF Test Statistic	-13.85796	1% Critical Value*		-3.4397
		5% Critical Value		-2.8649
		10% Critical Value		-2.5685
*MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(ROR03)				
Method: Least Squares				
Date: 11/05/08 Time: 17:46				
Sample(adjusted): 6 1000				
Included observations: 995 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
ROR03(-1)	-0.977154	0.070512	-13.85796	0.0000
D(ROR03(-1))	-0.030249	0.063229	-0.478401	0.6325
D(ROR03(-2))	-0.006233	0.054924	-0.113485	0.9097
D(ROR03(-3))	-0.007407	0.045141	-0.164077	0.8697
D(ROR03(-4))	-0.003669	0.031785	-0.115440	0.9081
C	0.004858	0.000385	12.63444	0.0000
R-squared	0.504057	Mean dependent var		3.28E-06
Adjusted R-squared	0.501550	S.D. dependent var		0.007089
S.E. of regression	0.005005	Akaike info criterion		-7.750766
Sum squared resid	0.024774	Schwarz criterion		-7.721202
Log likelihood	3862.006	F-statistic		201.0363
Durbin-Watson stat	1.999506	Prob(F-statistic)		0.000000

Table 6: Unit Root Analysis
Time Series Analysis Simulation (ROR04)

ADF Test Statistic	-14.21379	1% Critical Value*		-3.4397
		5% Critical Value		-2.8649
		10% Critical Value		-2.5685
*MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(ROR04)				
Method: Least Squares				
Date: 11/05/08 Time: 17:47				
Sample(adjusted): 6 1000				
Included observations: 995 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
ROR04(-1)	-0.994839	0.069991	-14.21379	0.0000
D(ROR04(-1))	-0.016501	0.062820	-0.262676	0.7929
D(ROR04(-2))	0.004275	0.054997	0.077740	0.9381
D(ROR04(-3))	0.037461	0.045140	0.829900	0.4068
D(ROR04(-4))	0.022649	0.031757	0.713186	0.4759
C	0.004972	0.000383	12.96960	0.0000
R-squared	0.506667	Mean dependent var		3.63E-06
Adjusted R-squared	0.504173	S.D. dependent var		0.007100
S.E. of regression	0.004999	Akaike info criterion		-7.753052
Sum squared resid	0.024717	Schwarz criterion		-7.723487
Log likelihood	3863.143	F-statistic		203.1462
Durbin-Watson stat	1.999818	Prob(F-statistic)		0.000000

The next step in the time-series analysis process is to determine if the four Return(i,t) series Granger cause each other. Table 7 shows the Granger causality statistics for the four Return(i,t) series. There are six combinations of Granger causality between the four Return(i,t) series, such as a determination if Return(1,t) series Granger causes Return(2,t) series and vice versa. In all six cases, Granger causality is rejected, as would be expected since the short-run component for each of the four Return(i,t) series are randomly generated.

Table 7: Granger Causality Tests
Time Series Analysis Simulation

Date: 11/05/08 Time: 17:48			
Sample: 1 1000			
Lags: 2			
Null Hypothesis:	**Obs**	**F-Statistic**	**Probability**
ROR02 does not Granger Cause ROR01	998	1.34586	0.26079
ROR01 does not Granger Cause ROR02		0.62422	0.53589
ROR03 does not Granger Cause ROR01	998	2.82095	0.06003
ROR01 does not Granger Cause ROR03		0.88871	0.41151
ROR04 does not Granger Cause ROR01	998	0.65045	0.52203
ROR01 does not Granger Cause ROR04		2.51855	0.08109
ROR03 does not Granger Cause ROR02	998	0.43306	0.64865
ROR02 does not Granger Cause ROR03		2.51052	0.08174
ROR04 does not Granger Cause ROR02	998	0.39303	0.67511
ROR02 does not Granger Cause ROR04		0.19871	0.81982
ROR04 does not Granger Cause ROR03	998	2.85850	0.05783
ROR03 does not Granger Cause ROR04		0.20629	0.81363

Once one has determined that the four Return(i,t) series are normally distributed with no statistically significant correlation, that the four Return(i,t) series are stationary with no unit roots, and that the four Return(i,t) series do not Granger cause each other, the four Return(i,t) series are tested for cointegration. Cointegration tests determine if the four Return(i,t) series have a long-run relationship that is not random as is the short-run relationship. Given that the four Return(i,t) series are constructed with an equal trend, we expect that the four Return(i,t) series *will* exhibit cointegration, which means that the four Return(i,t) series have a long-run relationship; i.e., the four Return(i,t) series follow the same long-run trend. Table 8 contains the results of the Johansen cointegration test. The test results indicate that there are four cointegrating equations at the 1% level of statistical significance as would be expected by the process by which in indices were constructed.

Table 8: Cointegration Tests
Time Series Analysis Simulation

Sample: 1 1000					
Included observations: 995					
Test assumption: Linear deterministic trend in the data					
Series: ROR01 ROR02 ROR03 ROR04					
Lags interval: 1 to 4					
	Likelihood	**5 Percent**	**1 Percent**	**Hypothesized**	
Eigenvalue	Ratio	Critical Value	Critical Value	No. of CE(s)	
0.211329	774.5075	62.99	70.05	None **	
0.176329	538.2890	42.44	48.45	At most 1 **	
0.163789	345.2743	25.32	30.45	At most 2 **	
0.154760	167.2945	12.25	16.26	At most 3 **	
*(**) denotes rejection of the hypothesis at 5%(1%) significance level					
L.R. test indicates 4 cointegrating equation(s) at 5% significance level					
Unnormalized Cointegrating Coefficients:					
ROR01	ROR02	ROR03	ROR04	@TREND(2)	
-7.236207	12.14660	-2.414050	-0.254757	-8.00E-08	
-0.287789	0.279013	-8.549678	11.99778	-8.95E-07	
-11.87684	-7.897936	1.071641	-0.324156	8.03E-07	
0.004491	-1.859518	-11.12785	-7.478533	3.45E-06	
Normalized Cointegrating Coefficients: 1 Cointegrating Equation(s)					
ROR01	ROR02	ROR03	ROR04	@TREND(2)	C
1.000000	-1.678586	0.333607	0.035206	1.11E-08	0.001565
	(0.23610)	(0.12572)	(0.11973)	(9.3E-07)	
Log likelihood	15225.03				

Table 8: Continued

Normalized Cointegrating Coefficients: 2 Cointegrating Equation(s)					
ROR01	ROR02	ROR03	ROR04	@TREND(2)	C
1.000000	0.000000	69.87104	-98.73838	7.35E-06	0.137262
		(653.897)	(925.581)	(9.3E-05)	
0.000000	1.000000	41.42619	-58.84332	4.37E-06	0.080840
		(389.716)	(551.637)	(5.6E-05)	
Log likelihood	15321.54				
Normalized Cointegrating Coefficients: 3 Cointegrating Equation(s)					
ROR01	ROR02	ROR03	ROR04	@TREND(2)	C
1.000000	0.000000	0.000000	0.072012	-4.85E-08	-0.005315
			(0.11737)	(5.5E-07)	
0.000000	1.000000	0.000000	-0.259132	-1.44E-08	-0.003693
			(0.11115)	(5.2E-07)	
0.000000	0.000000	1.000000	-1.414182	1.06E-07	0.002041
			(0.19130)	(8.9E-07)	
Log likelihood	15410.53				

Table 9 contains the empirical results for the VEC model with an intercept and with an intercept but no trend in the error correction model. This empirical results for this model show that the error correction equation is not statistically significant except in one case, ROR01 and ROR03(-2). The error correction variables are mostly not statistically significant and the signs are random. The adjusted R^2 for the models are 0.003339 or less and the F-statistics are not statistically significant. Table 10 contains the empirical results for the VEC model with a trend in the data and both an intercept and a trend in the error correction model. Given that the four Return(i,t) series are constructed with an intercept and a trend, the model with a trend in the data and a VEC model with both an intercept and a trend would seem to be most appropriate. This empirical results for this model show that the error correction equation is statistically significant but the trend is not statistically significant because the regression model accounts for the long-run trend effect across the four Return(i,t) series. Although the error correction variables are mostly statistically significant, the signs are random. This supports the hypothesis that cointegration is statistically significant but random in effect. The adjusted R^2 for the models are 0.33 or greater and the F-statistics are not statistically significant.

Table 9: Vector Error Correction Regression
Time Series Analysis Simulation

Sample(adjusted): 3 1000				
Included observations: 998 after adjusting endpoints				
Standard errors & t-statistics in parentheses				
	ROR01	**ROR02**	**ROR03**	**ROR04**
ROR01(-1)	0.037473	0.036160	0.045321	0.054493
	(0.03177)	(0.03201)	(0.03184)	(0.03196)
(t-statistic)	(1.17956)	(1.12983)	(1.42337)	(1.70478)
ROR01(-2)	0.006360	-0.007799	0.002631	-0.048527
	(0.03179)	(0.03202)	(0.03186)	(0.03198)
	(0.20009)	(-0.24354)	(0.08259)	(-1.51734)
ROR02(-1)	-0.048239	0.022555	0.057612	0.014743
	(0.03161)	(0.03184)	(0.03168)	(0.03180)
	(-1.52623)	(0.70836)	(1.81870)	(0.46359)
ROR02(-2)	-0.017720	0.007836	0.050514	-0.015419
	(0.03172)	(0.03196)	(0.03179)	(0.03192)
	(-0.55858)	(0.24521)	(1.58878)	(-0.48310)
ROR03(-1)	-0.032753	-0.001593	-0.016305	-0.016277
	(0.03168)	(0.03192)	(0.03175)	(0.03188)
	(-1.03378)	(-0.04992)	(-0.51348)	(-0.51060)
ROR03(-2)	-0.066213	0.030507	0.025281	-0.006665
	(0.03154)	(0.03177)	(0.03161)	(0.03173)
	(-2.09931)	(0.96010)	(0.79976)	(-0.21003)
ROR04(-1)	-0.037516	-0.025320	0.070126	-0.008158
	(0.03161)	(0.03185)	(0.03168)	(0.03181)
	(-1.18675)	(-0.79506)	(2.21334)	(-0.25650)
ROR04(-2)	-0.003718	-0.012148	0.035417	0.025532
	(0.03164)	(0.03188)	(0.03171)	(0.03184)
	(-0.11750)	(-0.38108)	(1.11676)	(0.80195)
C	0.005789	0.004741	0.003626	0.004990
	(0.00046)	(0.00047)	(0.00047)	(0.00047)
	(12.4556)	(10.1258)	(7.78290)	(10.6711)
R-squared	0.011336	0.003520	0.013936	0.006356
Adj. R-squared	0.003339	-0.004540	0.005960	-0.001682
Sum sq. resids	0.024340	0.024703	0.024450	0.024640
S.E. equation	0.004961	0.004998	0.004972	0.004991
F-statistic	1.417471	0.436727	1.747195	0.790793
Log likelihood	3883.974	3876.585	3881.725	3877.850
Akaike AIC	-7.765478	-7.750671	-7.760971	-7.753206
Schwarz SC	-7.721238	-7.706431	-7.716731	-7.708966
Mean dependent	0.004982	0.004991	0.004973	0.004989
S.D. dependent	0.004969	0.004986	0.004987	0.004987
Determinant Residual Covariance		3.63E-19		
Log Likelihood		15523.16		
Akaike Information Criteria		-31.03638		
Schwarz Criteria		-30.85942		

Table 10: VEC Regression
Time Series Analysis Simulation

Date: 11/05/08 Time: 17:52				
Sample(adjusted): 4 1000				
Included observations: 997 after adjusting endpoints				
Standard errors & t-statistics in parentheses				
Cointegrating Eq:	**CointEq1**			
ROR01(-1)	1.000000			
ROR02(-1)	-3.107689			
	(0.56475)			
	(-5.50280)			
ROR03(-1)	0.912382			
	(0.23212)			
	(3.93063)			
ROR04(-1)	-0.294429			
	(0.18021)			
	(-1.63381)			
@TREND(1)	2.80E-07			
	(1.7E-06)			
	(0.16173)			
C	0.007315			
Error Correction:	D(ROR01)	D(ROR02)	D(ROR03)	D(ROR04)
CointEq1	-0.103659	0.292230	-0.065245	-0.004799
	(0.01815)	(0.01644)	(0.01838)	(0.01839)
	(-5.71090)	(17.7705)	(-3.55066)	(-0.26097)
D(ROR01(-1))	-0.564791	-0.168134	0.096734	0.085316
	(0.03221)	(0.02918)	(0.03261)	(0.03263)
	(-17.5362)	(-5.76207)	(2.96677)	(2.61473)
D(ROR01(-2))	-0.281775	-0.104444	0.048403	0.043913
	(0.03050)	(0.02763)	(0.03088)	(0.03090)
	(-9.23802)	(-3.77949)	(1.56749)	(1.42106)
D(ROR02(-1))	-0.271974	-0.024072	-0.085798	0.011477
	(0.04767)	(0.04319)	(0.04826)	(0.04829)
	(-5.70549)	(-0.55737)	(-1.77788)	(0.23765)
D(ROR02(-2))	-0.181455	0.029367	0.016240	0.004272
	(0.03492)	(0.03164)	(0.03536)	(0.03538)
	(-5.19561)	(0.92813)	(0.45932)	(0.12075)
D(ROR03(-1))	0.045638	-0.193440	-0.647836	-0.023562
	(0.03128)	(0.02834)	(0.03167)	(0.03169)
	(1.45908)	(-6.82615)	(-20.4587)	(-0.74355)
D(ROR03(-2))	-0.016325	-0.079441	-0.303542	-0.023596
	(0.02986)	(0.02705)	(0.03023)	(0.03025)
	(-0.54669)	(-2.93644)	(-10.0410)	(-0.77997)

Table 10: Continued

D(ROR04(-1))	-0.039442	0.050380	0.031532	-0.699384
	(0.02978)	(0.02698)	(0.03015)	(0.03017)
	(-1.32425)	(1.86700)	(1.04573)	(-23.1778)
D(ROR04(-2))	-0.017275	0.019816	0.028660	-0.347010
	(0.02951)	(0.02673)	(0.02987)	(0.02989)
	(-0.58546)	(0.74127)	(0.95945)	(-11.6084)
C	1.15E-05	8.67E-06	1.46E-06	1.04E-05
	(0.00018)	(0.00016)	(0.00018)	(0.00018)
	(0.06462)	(0.05373)	(0.00809)	(0.05745)
R-squared	0.338356	0.471541	0.359639	0.360589
Adj. R-squared	0.332323	0.466723	0.353800	0.354758
Sum sq. resids	0.031214	0.025621	0.031991	0.032037
S.E. equation	0.005624	0.005095	0.005693	0.005697
F-statistic	56.08207	97.85510	61.59097	61.84533
Log likelihood	3755.577	3854.005	3743.319	3742.601
Akaike AIC	-7.513695	-7.711144	-7.489105	-7.487665
Schwarz SC	-7.464500	-7.661949	-7.439910	-7.438470
Mean dependent	5.45E-06	9.04E-06	3.63E-07	4.64E-06
S.D. dependent	0.006882	0.006977	0.007082	0.007093
Determinant Residual Covariance		8.18E-19		
Log Likelihood		15102.52		
Akaike Information Criteria		-30.20565		
Schwarz Criteria		-29.98427		

THE VECTOR ERROR CORRECTION MODEL

Vector AutoRegression technique cannot be applied to the four Return(i,t) series because the four Return(i,t) series are cointegrated; that is, the four Return(i,t) series follow the same long-run trend, but the short-run trend is random. There are eight options for running the VEC model. The VEC model can be run with no trend in the VEC but with an intercept included or not. The VEC model can be run with a trend in the VEC and an intercept and/or a trend in the cointegration equation. The vector error correction equation uses lagged deviations for each of the four Return(i,t) series as independent variables for each of the four Return(i,t) series in a regression that also include lagged deviation variables for each of the four Return(i,t) series. Each set of VEC estimated regression includes the cointegrating equation plus a series of deviations from past changes in the four Return(i,t) series with up to two lags, unless more lags are specified. In addition, each VEC analysis can include a trend in the VEC and/or an intercept or a trend for each VEC. Table 10 contains the empirical results for the VEC model with a trend in the data and both an intercept and a trend in the error correction model. Given that the four Return(i,t) series are constructed with an intercept and a trend, the model with a trend in the data and a VEC model with both an intercept and a trend would seem to be most appropriate. The empirical results for this model show that the error correction equation is statistically significant but the trend is not statistically significant because the regression model accounts for the long-run trend effect across the four Return(i,t) series. Although the error correction variables are mostly statistically significant, the signs are random. This supports the hypothesis that cointegration is statistically significant but random in effect. The other three models provide similar results.

SUMMARY AND CONCLUSIONS

In this paper, we generated four Return(i,t) series using Excel that have both a random component and a trend component for each of the four Return(i,t) series. We applied a series of tests for time series analysis – correlation, normality, unit root, Granger causality, cointegration, and vector error correction regressions.

The empirical results are consistent with the method used to create the four Return(i,t) series. Each of the four Return(i,t) series has the same expected value and standard deviation, a low correlation with the other

Return(i,t) series, which reflects the short-run random effect built into the four Return(i,t) series, no unit roots, and cointegration between the four Return(i,t) series, which Return(i,t) series is consistent with the method of constructing the four with a trend. Since the four Return(i,t) series are cointegrated by construction, a vector error correction model is appropriate for analysis of the long-run relationship between each of the four Return(i,t) series. The coinetegration equation is statistically significant as are the error correction variables, but in a random fashion with some of the regression coefficients being positive and some being negative.

In this paper, we show how to use the time series paradigm currently being used to conduct time series analysis. The basis of this analysis is the work in time series analysis done by noble laureate Engle and Granger. We demonstrate each of the steps designed to allow the researcher to determine if a relationship exists between two time series and to define the nature of that relationship.

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