Optimum Inducements In Social Menus: The Paradox Of Arrow's Impossibility Theorem

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ABSTRACT

This article develops combinatorial theory. The utility of an individual depends on the nature of the goods and, in essence, on the way they are arranged within the subset of consumption or, in other words, the manner in which they are combined. An individual may transitorily construct a scale of utilities or preferred social menus but it is impossible, or nearly impossible, to construct a social menu. By the use of inducements we can get close to the best possible menu, which will always differ from the best individual menu.

Keywords: combinatorial, menus, social menu, inducements, the impossibility paradox.

INTRODUCTION

Depending on how different goods are combined, different utilities are obtained, one for each combination which, in this article, no attempt is made to measure. Thus, if we suppose that there are n number of goods, different menus may be arranged, some of which will be preferred to others, provided that in no case are there two or more combinations having the same level of utility. Each menu implies a specific combination of goods and the menu preferred by all will be optimum menu.

Within each combination there may be sub-sets that are preferred intensely: we will call these 'hard cores' and 'welds'.

As Arrow's impossibility theorem tells us that no coherent and perfect democratic vote is possible, there cannot be an optimum social menu that is optimum for everyone. Clearly, Arrow's paradox is unanswerable. However, by means of inter-group inducements we can obtain something close to an optimum menu.

INDUCEMENTS

An inducement is any action (for example, a gift) intended to give pleasure in exchange for another action (freely granted) that suits the inducer. In an inducement both the inducer and the induced obtain a benefit. In an auction, the result, i.e. the sale-purchase of the object, is immediate. In an inducement, however, the result, though critical, is indirect/deferred. In an inducement what is sought is a result, which requires perfect knowledge of the tastes of the both the inducer and the induced, and also of the outcome it is intended to achieve.

In this essay, the inducer is an individual or group of individuals that freely (i.e. willingly) concede a subset of goods to be combined and allow the combination of these goods to be carried out by other groups. This concession of combinatory activity should be regarded as an inducement to other groups in order for the latter to consent to the certain specific combinations made by the inducers.

The key concept is that all parties, i.e., all the individuals comprising the group, always consume n goods and that what some concede and others accept is the possibility of combining them. Both before and after the inducement, n number of goods will continue to be consumed by both the inducers and the induced.

The inducement chain constitutes an improvement in utilities – never measurable except in terms of preference – for both inducers and induced. The purpose of the negotiation indicates the desire to improve one's circumstances, in order for which it is essential that some be willing to induce and that the others consent to being induced.

THE LOT AND THE INDIFFERENT LEVIATHAN

The lot is defined as the combination of undifferentiated items that the subject has to hand and can consume. By undifferentiated we mean that their combination is irrelevant; the only relevant aspect is their quantity and nature.

In reality the goods are generated by the subjects on account of two considerations: the first being that a need arises that can be satisfied; and the second, that each combination of goods constitutes a different good. So, for example, the combination of a sweet (pudding), coffee and a smoke is considered as one good; and the combination coffee, a smoke and a sweet is considered another good. In this way the idea of a lot carries no other significance than an assortment of goods of which we know the nature and the quantity.

Utility is always individual. In reality, there is no such thing as social utility; social utility derives in every case from the individual consideration of the person consuming a specific good.

In our scenario the goods will be offered by the public sector, which will undertake two activities: produce and supply the goods, i.e., the lot, never the combination. It will also, and this is most important, supply the information. Using this lot the individuals try to construct their transitive individual menus and, by means of inducements, try to reach the optimum social menu. The public sector, despite its coercive force and the breadth of its knowledge, will act in such a way that the information is complete and neutral.

So, we must differentiate two menus: one being the optimum social menu that is impossible to achieve because of the inescapable impossibility theorem of Arrow and the possible or feasible social menu, which we should be able to achieve by means of the inducement chain. In any event, this latter social menu will be possible and, it may even be said, not remotely removed from the theoretical optimum.

COMBINATORY THEORY

The theory and practice of combining goods in all possible ways is known as combinatory theory. The part of this theory that deals with all possible combinations of n goods without any good being repeated or any being omitted calls these 'ordinary permutations'. This is the precisely what we are dealing with here. The formula used to express this concept of all possible different combinations or permutations is:

$P_n = n!$

If each menu is a specific combination – and provided that no two or more menus create the same utility – the number of total possible menus will be: n! In the case of seven goods for example, we can generate 7.6.5.4.3.2 = 5,040 different menus.

Among the menus themselves we can establish a chain of transitive preference criteria that bears a logical relation to the choice of menu, i.e. a scale or ladder of preferences. What is true of each individual will bear a logical relation to each and every menu. Thus, if menu number two is preferred to menu number 16 and this to menu 300, then the first will be preferred to this last.

Arrow's impossibility theorem shows than in an intergroup social context, such a chain of transitive preferences is unviable.

The optimum menu is that which is preferred by all and all the other menus rank after it according to their distance from the optimum or first menu. An organised collection of menus is at the opposite extreme from the idea of a lot, which is an undifferentiated assortment of goods.

WELDS AND HARD CORES

It is feasible that within combinations there should be subsets that generate special or intense utilities that are much preferred to others. For example, a lover of Havana cigars who has a sweet tooth may prefer among, say, seven goods the specific combination liqueur, sweet, coffee and Havana cigar and none other. In other words this combination is much preferred to any other.

We call 'hard cores' those combinations that are much preferred, much more than 'welds', and are not susceptible to being broken up into other partial combinations. For example, using the same case, our sweet-toothed cigar smoker may be persuaded to start with the sweet and then take the liqueur, but he would under no circumstances reverse the order coffee–Havana. That last subset is what we call a hard core.

For our purposes the similarity between hard cores and welds is that they are both subsets of goods the combination of which is much preferred. The difference is that the combination called hard core is much preferred to the combination described as a weld.

Given that welds are less highly preferred than hard cores, cases may arise in which the subject is prepared to accept breakage, which would never be the case with a hard core.

These combinations are important when it comes to negotiations based on possible inducements. It is possible to induce by means of conceding combinations that are of scant interest to the inducers.

OPTIMUM INDUCEMENTS IN CREATING A SOCIAL MENU

As Arrow demonstrated in 1950 with his impossibility theorem, a perfect vote is not possible. To do this, he used an example. Say there are three desirable programmes, A, B and C, listed in order of transitory preference, and three groups of equal numbers of citizens: 1, 2 and 3. Say that Group 1 prefers A to B and B to C. Group 2 prefers B to C and C to A. And Group 3 prefers C to A and A to B. The situation is contradictory or paradoxical given that A is preferred to B by a majority (Groups 1 and 3), in the same way that B is preferred to C (Groups 1 and 2). Yet there is another majority (Groups 2 and 3), which prefers C to A. The argument is unanswerable.

What we want to do here is arrive at a possible social menu using a system of negotiation based on a chain of inducements. Such inducements are possible thanks to combinatory theory and special cases such as welds and hard cores. Welds and hard cores are an important part of an optimum or near-optimum menu thanks to their intense preferences. If preferences are intense, it means utilities are high. If such special combinations exist, it shows that there are other subsets of combinations that are not as intense and can thus be traded or *conceded.* The inducement thus consists of groups having welds or hard cores being prepared to concede the other goods for them to combine as they wish. Before going any further we should point out that they do not concede the goods themselves but the possibility of combining them, in such a way that at the end of the day each of the subjects will always consume each and every one of the goods available.

If the other groups agree to being induced, it means two things: first, that they can obtain very intense or preferred combinations in exchange for not touching the welds and hard cores of the inducer (in extreme cases, they may go so far as to break some of the welds but not the entire weld and, under no circumstances, the hard cores); second, we would be approaching a situation of acceptance among the groups of a menu which interests everyone but which does not exactly match the individual optimum menu. The outcome would be a *possible* optimum social menu (never a pure optimum social menu).

We can plot the voting sequence in the following manner:

Group 1 prefers combination A to combination B, thus freeing n_1 goods to induce Group 2 to consent to its preferences.

Group 2 prefers combination B to that of C but, by being suitably induced by Group 1, agrees to prefer Group A to B.

Group 3 prefers the combination C to A, but is dissuaded by being induced by Group 2, which concedes n_2 goods to compensate it for preferring Group A to Group C.

For all this to happen there would have to be a rotating force obliging all Groups to negotiate, and each time a round of bartering occurred there would be fewer undifferentiated combinations with which to induce, with the result that we would have a process of *converging inducement.* When the sequence of inducements ended, the menu that is left would be not the optimum social menu but the *feasible* optimum social menu.

CONCLUSION

This study focuses principally on combinations of goods, each combination being considered as a menu and each menu determining a level of utility. There is thus an n! chain of utilities and menus that act like a ladder of transitive preferences. The menu preferred by all, the top rung of the ladder, is the optimum menu.

Can there by a social menu? No, because Arrow's impossibility theorem shows us that this cannot be. However, we can approach a *feasible* optimum menu by means of negotiation among the groups. These negotiations are based on consented inducements in which both the inducers and the induced obtain tangible benefits.

AUTHOR INFORMATION

José Villacís González has a PhD in economic science;degree in political science and sociology;lecturer in economy at the University San Pablo- CEU;author of various books: macroeconomía, microeconomía; política monetaria y política fiscal; El Origen de la Macroeconomía en España; La Teoría de las Disponibilidades en España; La Teoría de las Disponibilidades del Interés y de la Renta;La Máquina, la Superación de Leviathan; Germán Bernacer el Circulo de Alicante.

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