

A Multi-Objective Decision-Making Approach For Mutual Fund Portfolio

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ABSTRACT

Investment decision-making problems are generally multi-objective in nature such as minimization of the risk and maximization of the expected return. These problems can be solved efficiently and effectively using multi-objective decision making (MODM) tools such as a lexicographic goal programming (LGP). This paper applies the LGP model for selecting an optimum mutual fund portfolio for an investor, while taking into account specific parameters including risk, return, expense ratio and others. Sensitivity analysis on the assigned weights in a priority structure of the goals identifies all possible solutions for decision-making. The Euclidean distance method is then used, to measure distances of all possible solutions from the identified ideal solution. The optimal solution is determined by the minimum distance between the ideal solution and other possible solutions of the problem. The associated weights with the optimal solution will be the most appropriate weights in a given priority structure. The effectiveness and applicability of the LGP model is demonstrated via a case example from broad categories of mutual funds.

INTRODUCTION

Modern portfolio theory is based on the pioneering works of Markowitz (1952, 1959) and Sharpe (1963). In 1952, Markowitz first laid the foundation of a framework for mean-variance portfolio optimization. He suggested that the portfolio selection problem should be considered as a parametric quadratic programming approach. Several portfolio selection models were previously proposed based on mean-variance formulation. However, the complexity of the models requires training in quadratic programming and its practical application presents a number of problems. First, a large amount of data must be processed, which of course requires performing significant computation. Second, the results are given in the form of alternative optimal portfolios, which require further determination of the best choice. Finally, the size of the optimal portfolio leads to the concept of un-diversifiable market risk, since more securities are held in the portfolio. Sharpe (1963) developed a revised model and summarized the process of portfolio selection based on the assumptions of: (1) making probabilistic estimates of the future performance of securities, (2) analyzing those estimates to determine an efficient set of portfolios, and (3) selecting from that set the portfolio best suited to the investor's performance requirements (Lee and Lerro, 1973).

Sharpe (1967, 1971) and Stone (1973) used linear programming (LP) approach to solving portfolio selection problems and demonstrated that LP models for portfolio selection can provide acceptable results while avoiding the limitations of mean-variance models (Levary and Avery, 1984). Portfolio selection problems typically involve multiple and often conflicting objectives such as the maximization of returns and minimization of risk. As a result of multiple and conflicting objectives, the conventional LP model becomes less adequate to handle mutual fund portfolio selection problems as it was developed to handle a single objective function. However, the complexity of the problems resulting from multiple and conflicting objectives can be handled efficiently and effectively with a multi-objective decision making (MODM) techniques such as a lexicographic goal programming (LGP). For a detailed research survey on LGP, see Lee (1972), Ignizio (1976), Romero (1986, 1991), Tamiz and Jones (1995), Sharma, Alade and Vasishta (1999), and Steuer and Na (2003).

Over the years, LGP has been widely used to solve problems in investment decision-making (Pendaraki et al., 2004). However, there are a few studies that have applied LGP to construct a portfolio from broad categories of mutual funds. Most studies have used conventional LGP to provide the optimal solution and included only a few goals and objectives under the decision-maker's priority structure. In this study we have included additional objectives and constraints to make it a more practical decision-making technique that provides a better solution and may be widely acceptable in a defined priority structure. A better solution also requires a sensitivity analysis on different weights within a given priority structure (Steuer, 1986; Sharma et al., 2004).

The purpose of this paper is to apply the LGP model, in line with previous studies, to construct an optimum mutual fund portfolio for an investor, taking into account specific parameters including risk, return, expense ratio and others. The model can meet the requirements of an individual investor and/or practitioners effectively and efficiently and satisfies a wide spectrum of goals and objectives. We perform sensitivity analysis on the weights in a given priority structure of the goals to find all possible solutions in the decision-making process. We then use the Euclidean distance method to measure distances of all possible solutions from the identified ideal solution. The optimal possible solution is determined based on the minimum distance between the ideal solution and other possible solutions of the problem. The associated weights will be the most appropriate weights in the given priority structure. The effectiveness and applicability of the LGP model is demonstrated via a case example from broad categories of mutual funds.

The remainder of the paper is organized as follows: The second section discusses the review of literature. The third section proposes the mathematical model of the problem. The fourth section demonstrates the model via a case example and presents sensitivity analysis with different weight structures in a priority structure. The fifth section analyses the result obtained from the case example and identifies the most acceptable solution in the decision-making process. The final section presents concluding remarks.

REVIEW OF LITERATURE

Over the years, multiple criteria decision making (MCDM) has attracted many researchers because of its theoretical development and practical applications for solving a wide range of real world problems in business and industry (Hwang and Masud, 1979; Steuer, 1986). The MCDM is concerned with the methods and procedures by which multiple criteria can be formulated into the analytical process. This process can be divided into multi-attribute decision making (MADM) and multi-objective decision making (MODM). The former is often applicable to problems with a small number of alternatives in a probabilistic environment, while the later is generally applied to deterministic problems where the number of feasible alternatives is large. Lexicographic goal programming (LGP) falls in the category of MODM that is broad in scope (Messac, Gupta, and Akbulut, 1996). LGP is one of the most widely used tools for solving MODM problems (Romero, 1991).

GP and its variants have been applied to a wide range of problems (Ijiri, 1965; Ignizio, 1976; Lee, 1972; Romero 1991). Literature is replete with studies using LGP or linear GP for investment decision-making problems (Lee, 1972; Lee and Lerro, 1973; Kumar et al., 1978; Lee and Chesser, 1980; Levary and Avery, 1984; Schniederjans et al., 1992; Sharma et al., 1995; Cooper et al., 1997; Dominiak, 1997; Leung et al., 2001; Pendaraki et al., 2004). Lee and Lerro (1973) developed a LGP portfolio selection model for mutual funds. Kumar et al. (1978) developed a conceptual LGP model for portfolio selection of dual-purpose funds. Lee and Chesser (1980) demonstrated how linear beta coefficient from finance theory reflecting risk in alternative investments could be incorporated into a LGP model. Levary and Avery (1984) also introduced a LGP model representing the investor's priorities and also compared the use of linear programming to GP for the selection of optimal portfolio. Schniederjans et al. (1992) illustrated the use of LGP as an aid to investors planning investment portfolios for themselves. Sharma et al. (1995) presented LGP as an aid for investors or financial planners planning investment portfolios for individuals and/or companies by using beta coefficients and other important parameters. Recently, Pendaraki et al. (2004) has applied LGP on a sample of Greek mutual funds.

GOAL PROGRAMMING MODEL DEVELOPMENT

Goal programming (GP) technique was initially developed to handle multi-criteria situations within the general framework of LP. The essence of the technique is the achievement of the "best possible" solution, which

comes closest to meeting the stated goals given the constraints of the problem. GP was first introduced by Charnes et al. (1955) and Charnes and Cooper (1961). It was later extended by Ijiri (1965), Lee (1972), and Ignizio (1976). The GP model has been described in detail by Lee (1972), Ignizio (1976), Romero (1991), and others. LGP is useful in financial planning because many of financial criteria can be expressed in terms of goals. A general format for a LGP model is:

$$\text{Minimize } [P_1(\bar{d}), \dots, P_k(\bar{d}), \dots, P_K(\bar{d})], \tag{1}$$

Subject to:

$$f_i(\bar{x}) + d_i^- - d_i^+ = b_i, \quad i = 1, 2, \dots, m \tag{2}$$

$$d_i^-, d_i^+, x \geq 0 \quad \text{and} \quad d_i^- \cdot d_i^+ = 0. \tag{3}$$

where $P_k(\bar{d}) = P_k (w_{ik}^- d_{ik}^- + w_{ik}^+ d_{ik}^+)$ and P_k is the k-th priority structure. w_{ik}^- and w_{ik}^+ are the numerical weights associated with the deviational variables d_{ik}^- and d_{ik}^+ respectively at the priority level P_k . \bar{x} is the vector of decision variables. $f_i(\cdot)$ is the i-th goal constraint.

The model approaches the problem of determining the optimum portfolio under a set of goals and constraints imposed by the decision-maker. This is achieved by minimizing a weighted sum of deviations from the target goals. In order to develop a multi-objective goal function, each goal has to be assigned its due weight from decision-maker's point of view.

The Goals

The decision-maker's multiple goals are defined as follows:

1. Utilize total available funds for investment within imposed restrictions.
2. Maximize the portfolio's expected annual return.
3. Minimize the portfolio's risk (Beta and Standard Deviation).
4. Minimize the portfolio's expense ratio.

These target goals may vary from investor to investor, depending on the adjustment of the model to fit different economic environments.

Goal Constraints

The following goal constraints are developed in formulating the general model of mutual fund portfolio selection problem.

Investment

The objective of decision maker is to utilize the total available funds (F). The available funds goal constraint by adding under and over-deviations can be written as:

$$\sum_{j=1}^J X_j + d_1^- - d_1^+ = F \tag{4}$$

where X_j is the amount of money allocated to mutual fund j ($=1, 2, \dots, J$), d_1^- and d_1^+ are underachievement and overachievement of goal constraint in the equation.

Rate of Return

The decision maker’s objective is to get the maximum possible return on the investment. The total annual return (A) goal constraint can be expressed as:

$$\sum_{j=1}^J A_j X_j + d_2^- - d_2^+ = A \tag{5}$$

where A_j is the expected annual rate of return from mutual fund j ($=1,2,\dots,J$).

Risk Willingness

Beta

The portfolio’s beta (β) is called systematic risk and is measured as the sensitivity of a security’s returns to market returns. The composite β of the portfolio can be presented as:

$$\sum_{j=1}^J \beta_j X_j + d_3^- - d_3^+ = \beta_a F \tag{6}$$

where β_j is the measure of risk associated with mutual fund j ($=1,2,\dots,J$) and β_a is an acceptable level of beta.

Standard Deviation

Theoretically, a well-diversified portfolio’s standard deviation is reflected in its systematic risk. However, the existence of some residual or nonsystematic risk is also possible due to fund specific performance. The composite standard deviation (σ) of the portfolio may be limited to a certain maximum value and thereby limiting risk to a certain level. The goal constraint can be expressed as:

$$\sum_{j=1}^J \sigma_j X_j + d_4^- - d_4^+ = \sigma_a F \tag{7}$$

where σ_j is the measure of nonsystematic risk associated with mutual fund j ($=1,2,\dots,J$) and σ_a is an acceptable level of standard deviation.

Diversification

The modern portfolio theory suggests that the allocation of the investor’s capital among various securities can reduce diversifiable risk or nonsystematic risk. In order to reduce risk through diversification, the decision maker may prefer to invest some minimum amount in several different mutual funds, but at the same time establish a maximum amount that can be invested in any particular mutual fund. The diversification goal constraints can be written as:

Large Cap Mutual Funds

The large cap mutual funds (X_l) should be at least a certain percentage (p) of total mutual fund. The goal constraint would be written as:

$$\sum_{l=1}^{l_n} X_{l \in \{1,2,\dots,J\}} + d_5^- - d_5^+ = p \% * F \tag{8}$$

where $\{1,1,2,\dots,l_n\}$ are the large cap mutual funds and $\{1,1,2,\dots,l_n\} \subseteq \{1,2,\dots,J\}$.

Medium Cap Mutual Funds

The medium cap mutual funds (X_m) may not exceed a certain percentage (q) of total mutual funds. The goal constraint can be expressed as:

$$\sum_{m=m_1}^{m_n} X_{m \in \{1,2,\dots,J\}} + d_6^- - d_6^+ = q \% * \sum_{j=1}^J X_j \tag{9}$$

where $\{m_1, m_2, \dots, m_n\}$ are the medium cap mutual funds and $\{m_1, m_2, \dots, m_n\} \subseteq \{1, 2, \dots, J\}$

Small Cap Mutual Funds

The small cap mutual funds (X_s) must be at least a certain percentage (r) of total mutual fund. The goal constraint can be defined as:

$$\sum_{s=s_1}^{s_n} X_{s \in \{1,2,\dots,J\}} + d_7^- - d_7^+ = r \% * \sum_{j=1}^J X_j \tag{10}$$

where $\{s_1, s_2, \dots, s_n\}$ are the small cap mutual funds and $\{s_1, s_2, \dots, s_n\} \subseteq \{1, 2, \dots, J\}$

Foreign Mutual Funds

The foreign mutual funds (X_f) may not exceed a certain percentage (s) of total mutual fund. The goal constraint can be written as:

$$\sum_{f=f_1}^{f_n} X_{f \in \{1,2,\dots,J\}} + d_8^- - d_8^+ = s \% * \sum_{j=1}^J X_j \tag{11}$$

where $\{f_1, f_2, \dots, f_n\}$ are the foreign mutual funds and $\{f_1, f_2, \dots, f_n\} \subseteq \{1, 2, \dots, J\}$

Bond Mutual Funds

The bond mutual funds (X_b) must be at least a certain percentage (t) of total mutual fund. The goal constraint can be written as:

$$\sum_{b=b_1}^{b_n} X_{b \in \{1,2,\dots,J\}} + d_9^- - d_9^+ = t \% * \sum_{j=1}^J X_j \tag{12}$$

where $\{b_1, b_2, \dots, b_n\}$ are the bond mutual funds and $\{b_1, b_2, \dots, b_n\} \subseteq \{1, 2, \dots, J\}$. Similarly, other restrictions for each type of fund may also be considered.

Minimum and Maximum Limits on Investment

Minimum Investment

Generally, each mutual fund company requires a minimum initial investment (M) in each mutual fund. The goal constraint for minimum initial investment can be written as:

$$X_j + d_{10}^- - d_{10}^+ = M_j, j=1,2,\dots,J \tag{13}$$

Maximum Investment

Additionally, no more than a certain percentage (y) of the fund should be invested in a single mutual fund. The goal constraint for maximum investment in a fund can be defined as:

$$X_j + d_{11}^- - d_{11}^+ = y \% * F, j=1,2,\dots,J \tag{14}$$

Annual Expense Ratio

Each mutual fund has an annual expense ratio. The expense ratio (E) goal constraint can be expressed as:

$$\sum_{j=1}^J E_j X_j + d_{12}^- - d_{12}^+ = E_a * F \tag{15}$$

where E_a is acceptable level of expense ratio.

Sensitivity Analysis

The decision-maker defines priorities and weights to obtain optimal solution in the conventional LGP. However, in complex decision-making processes, the desired solution may not be acceptable under the imposed weights within a given priority structure. A better solution can be achieved by performing sensitivity analysis in which a number of different weight structures are allowed within a given priority structure (Steuer, 1986; Sharma et al., 2004).

To determine the appropriate weight structure within the decision-maker’s defined priority structure, we have used K priorities in constructing the model. Within a structure of K priorities, the decision-maker may impose N weight structures which are relevant to the study region. Therefore, N different solutions can be obtained from the problem with N sets of different weight structures.

Let N number of weight structures used to take different sets of solution of the model. N different sets of solution are:

$$\{[X]_j^n\}, j=1,2,3,\dots,J \text{ for each } n = 1,2,3,\dots,N \tag{16}$$

Since in reality, maximum allocation of fund to a particular mutual fund is always desirable for getting maximum return. Thus the ideal solution can be identified as

$$[X]_j^*, j=1,2,3,\dots,J = \text{Maximum } \{[X]_j^n\}, n=1,2,3,\dots,N, \text{ for each } j (=1,2,3,\dots,J). \tag{17}$$

The Euclidean distance $\{D_n\}, n=1,2,3,\dots, N$, of each solution $\{[X]_i^n\}, j=1,2,3,\dots,J$ from the ideal solution $\{[X]_j^*\}, j=1,2,3,\dots,J$ can be presented as

$$D_n = \sqrt{\sum ([X]_j^* - [X]_j^n)^2}, n = 1,2,3, \dots, N \tag{18}$$

In practice, the ideal solution may not be achieved (Cohon, 1978). The solution closest to the ideal solution, will be the best possible solution and the associate weight structure in the priority structure will be the most appropriate weight structure in the decision-making context.

To illustrate the LGP model developed in the previous section, the following case example is considered to demonstrate the usefulness of the model.

CASE EXAMPLE

Mutual fund portfolio refers to a collection of tradable securities such as stocks, bonds, and money market funds. Fund managers select the basket of securities so as to achieve a good rate of return with the least risk exposure possible. However, Investors’ desire to select an efficient portfolio of mutual funds requires further education to diversify investment capital across a broad range of mutual funds. Consider an investor that has one million dollars to invest in mutual funds. The objective of investor is maximum possible return on his investment with the least risk

exposure possible, and diversify fund among twenty-five no-load mutual funds. The relevant financial data for the problem is given in Table 1.

Table 1: Summary Of Data

Mutual Fund	Fund Type	Annual Return (%)	Expense Ratio (%)	Beta	Standard Dev.	Min. Initial Investment (\$)
X ₁	Large	13.6	1.4	0.96	0.15	1,000
X ₂	Large	19.7	1.7	1.29	0.24	1,000
X ₃	Large	18.4	1.5	1.51	0.27	1,000
X ₄	Large	14.0	0.9	0.88	0.18	1,000
X ₅	Large	9.0	1.1	1.01	0.20	10,000
X ₆	Medium	9.5	1.0	0.66	0.14	2,500
X ₇	Medium	34.9	1.8	1.32	0.26	1,000
X ₈	Medium	27.2	1.0	0.91	0.39	2,500
X ₉	Medium	25.2	1.4	1.05	0.21	2,500
X ₁₀	Medium	36.1	1.0	1.35	0.39	2,500
X ₁₁	Small	24.6	1.6	0.89	0.18	1,000
X ₁₂	Small	35.6	1.6	1.63	0.40	1,000
X ₁₃	Small	15.1	1.5	1.30	0.22	2,000
X ₁₄	Small	9.8	1.1	0.88	0.16	1,000
X ₁₅	Small	6.2	0.3	1.0	0.28	3,000
X ₁₆	Foreign	24.5	2.8	0.94	0.30	1,500
X ₁₇	Foreign	23.7	1.6	1.08	0.28	1,000
X ₁₈	Foreign	24.6	2.1	1.51	0.26	1,000
X ₁₉	Foreign	32.8	2.3	0.93	0.30	1,000
X ₂₀	Foreign	31.9	1.7	1.17	0.36	2,500
X ₂₁	Bond	10.09	0.6	0.95	0.13	1,500
X ₂₂	Bond	11.02	0.5	0.90	0.12	2,000
X ₂₃	Bond	8.5	0.7	0.85	0.14	2,500
X ₂₄	Bond	7.0	0.6	0.70	0.10	2,000
X ₂₅	Bond	9.0	0.8	0.90	0.11	2,000

Using the data in Table 1, the LGP model constraints for the mutual fund portfolio problem are formulated as follows:

Constraints

(i) The goal constraint for total investment in various mutual funds can be written as:

$$\sum_{j=1}^{25} X_j + d_1^- - d_1^+ = \$1,000,000 \tag{19}$$

(ii) The goal constraint for annual rate of return from the investment is given below:

$$\sum_{j=1}^{25} A_j X_j + d_2^- - d_2^+ = 215,000 \tag{20}$$

(iii) The goal constraint for the portfolio's beta can be expressed as:

$$\sum_{j=1}^{25} \beta_j X_j + d_3^- - d_3^+ = 1150,000 \tag{21}$$

(iv) The goal constraint for standard deviation is given below:

$$\sum_{j=1}^{25} \sigma_j X_j + d_4^- - d_4^+ = 267,500 \tag{22}$$

(v) The lower limit for initial investment (M) in each mutual fund can be written as:

$$X_j + d_{j+4}^- - d_{j+4}^+ = M_j \quad (j=1,2,\dots,25) \tag{23}$$

(vi) The upper limit for investment in each mutual fund can be expressed as:

$$X_j + d_{j+29}^- - d_{j+29}^+ = 80,000 \quad (j=1,\dots,5) \tag{24}$$

$$X_j + d_{j+29}^- - d_{j+29}^+ = 100,000 \quad (j=6,\dots,10) \tag{25}$$

$$X_j + d_{j+29}^- - d_{j+29}^+ = 70,000 \quad (j=11,\dots,15) \tag{26}$$

$$X_j + d_{j+29}^- - d_{j+29}^+ = 50,000 \quad (j=16,\dots,20) \tag{27}$$

$$X_j + d_{j+29}^- - d_{j+29}^+ = 50,000 \quad (j=21,\dots,25) \tag{28}$$

(vii) The Large-cap mutual funds must be at least 30% of total investment.

$$\sum_{j=1}^5 X_j + d_{55}^- - d_{55}^+ = 300,000 \tag{29}$$

(viii) The Medium-cap mutual funds may not exceed 40% of total mutual funds.

$$\sum_{j=6}^{10} X_j + d_{56}^- - d_{56}^+ = .40 \sum_{j=1}^{25} X_j \tag{30}$$

(ix) The Small-cap mutual funds must be at least 20% of total mutual funds.

$$\sum_{j=11}^{15} X_j + d_{57}^- - d_{57}^+ = .20 \sum_{j=1}^{25} X_j \tag{31}$$

(x) The Foreign mutual funds may not exceed 10% of total mutual funds.

$$\sum_{j=16}^{20} X_j + d_{58}^- - d_{58}^+ = .10 \sum_{j=1}^{25} X_j \tag{32}$$

(xi) The Bond mutual funds may not exceed 15% of total mutual funds.

$$\sum_{j=20}^{25} X_j + d_{59}^- - d_{59}^+ = .15 \sum_{j=1}^{25} X_j \tag{33}$$

(xii) The annual expense ratio may not exceed 1.5% of total investment.

$$\sum_{j=1}^{25} E_j X_j + d_{60}^- - d_{60}^+ = 15,000 \tag{34}$$

The Priority Structure

The priority structure for the model can be established by assigning each goal to a priority level, thereby ranking the goals in order of importance to the decision-maker. When more than one goal seems to be equally important for the achievement of their goals levels then they may be included at the same priority level, where numerical weights represent the relative importance of the goals at the same priority level. In this study, four priority levels are addressed to include the goal constraints according to their importance of achievement in the model. The priority structure in order of decreasing priority ranking of the goals can be defined in Table 2.

Table 2: Priority Structure

Priority	Description	Deviations
P_1	Utilize available funds and satisfy restrictions on Investment	$w_1^+ d_1^+ + w_{j+4}^- d_{j+4}^- + w_{j+29}^+ d_{j+29}^+ + w_{55}^- d_{55}^- + w_{56}^+ d_{56}^+ + w_{57}^- d_{57}^- + w_{58}^+ d_{58}^+ + w_{59}^+ d_{59}^+, j=1,\dots,25$
P_2	Maximize the portfolio's expected annual return	$w_2 d_2^-$
P_3	Minimize the portfolio's risk (Beta and Standard Deviation)	$w_3^+ d_3^+ + w_4^+ d_4^+$
P_4	Minimize the portfolio's expense ratio	$w_{60}^+ d_{60}^+$

RESULTS ANALYSIS

In the problem, four priorities and six different weight structures were considered. A sensitivity analysis on the weights in the given priority structure of goals is performed to obtain all feasible solutions. Therefore, six feasible solutions can be achieved from the analysis. In Table 3, solutions corresponding to six Runs are displayed and the calculated minimum distance of each solution set from the ideal solution set is also presented. The minimum distance from the solutions to the ideal solution corresponds to Runs 3 and 5. These two Runs have equal distance of 143941. In this situation, any Run out of these two Runs with the corresponding weight structure would be the most appropriate to make the investment decision.

CONCLUDING REMARKS

This study demonstrates a LGP model to construct a mutual fund portfolio that allows incorporating multiple goals such as expected annual return, portfolio beta, standard deviation, and expense ratio. The results of the model indicate that the investor can achieve his fundamental objectives of constructing an efficient portfolio, meeting multiples investment goals. All feasible solutions have been considered using sensitivity analysis on the weight structures in the priority structure of the goals. The methodology applied in this paper is similar in approach to previous studies; however, it presents a different weight structure within a priority structure including additional goals and objectives. Additionally, the Euclidean distance function is used to measure distances of all possible solutions from the ideal solution. The model developed is flexible enough to accommodate other situation-specific constraints as may be defined by the goals and objectives of the problems. Performance of the model depends on the appropriate weights in a priority structure. The solution of the problem indicates that all four goals in the given priority structure are fully achieved.

The model presented in this paper has been devised in a way that it may help the decision-maker find the proper weight structure within a priority structure to achieve desired goals, and may be used as an analytical tool for both financial advisors and investors.

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Table 3: Priorities And Corresponding Solution

Run	Weight Structure	Results			Distance
1	$P_1(w_1^+ = w_{j+4}^+ = w_{j+29}^+ = w_{27}^+ = w_{55}^+ = w_{56}^+ = w_{57}^+ = w_{58}^+ = w_{59}^+ = 1)$ $P_2(w_2^+ = 1)$ $P_3(w_3^+ = w_4^+ = 1)$ $P_4(w_{60}^+ = 1)$	$X_1=80,000$ $X_2=1,000$ $X_3=80,000$ $X_4=80,000$ $X_5=59,000$ $X_6=100,000$ $X_7=100,000$ $X_8=87,500$ $X_9=2,500$	$X_{10}=100,000$ $X_{11}=70,000$ $X_{12}=70,000$ $X_{13}=2,000$ $X_{14}=1,000$ $X_{15}=57,000$ $X_{16}=1,500$ $X_{17}=50,000$ $X_{18}=45,000$	$X_{19}=1,000$ $X_{20}=2,500$ $X_{21}=1,500$ $X_{22}=2,000$ $X_{23}=2,500$ $X_{24}=2,000$ $X_{25}=2,000$	146972.8
2	$P_1(w_1^+ = w_{j+4}^+ = w_{j+29}^+ = w_{27}^+ = w_{56}^+ = w_{58}^+ = w_{59}^+ = 1, w_{55}^+ = w_{57}^+ = 1.5)$ $P_2(w_2^+ = 1)$ $P_3(w_3^+ = w_4^+ = 1)$ $P_4(w_{60}^+ = 1)$	$X_1=80,000$ $X_2=80,000$ $X_3=80,000$ $X_4=80,000$ $X_5=10,000$ $X_6=100,000$ $X_7=100,000$ $X_8=2,895$ $X_9=2,500$	$X_{10}=100,000$ $X_{11}=1,000$ $X_{12}=70,000$ $X_{13}=2,000$ $X_{14}=70,000$ $X_{15}=57,000$ $X_{16}=1,500$ $X_{17}=1,000$ $X_{18}=46,500$	$X_{19}=1,000$ $X_{20}=50,000$ $X_{21}=1,500$ $X_{22}=2,000$ $X_{23}=50,000$ $X_{24}=9,105$ $X_{25}=2,000$	156155.7
3	$P_1(w_1^+ = w_{j+4}^+ = w_{j+29}^+ = w_{27}^+ = w_{55}^+ = w_{56}^+ = w_{57}^+ = w_{58}^+ = w_{59}^+ = 1)$ $P_2(w_2^+ = 2)$ $P_3(w_3^+ = 1.5, w_4^+ = 1)$ $P_4(w_{60}^+ = 1.5)$	$X_1=80,000$ $X_2=50,000$ $X_3=80,000$ $X_4=80,000$ $X_5=10,000$ $X_6=100,000$ $X_7=100,000$ $X_8=87,500$ $X_9=2,500$	$X_{10}=100,000$ $X_{11}=70,000$ $X_{12}=70,000$ $X_{13}=2,000$ $X_{14}=1,000$ $X_{15}=57,000$ $X_{16}=1,500$ $X_{17}=50,000$ $X_{18}=45,000$	$X_{19}=1,000$ $X_{20}=2,500$ $X_{21}=1,500$ $X_{22}=2,000$ $X_{23}=2,500$ $X_{24}=2,000$ $X_{25}=2,000$	143941.0
4	$P_1(w_1^+ = w_{j+4}^+ = w_{j+29}^+ = w_{27}^+ = w_{55}^+ = w_{56}^+ = w_{57}^+ = w_{58}^+ = w_{59}^+ = 1)$ $P_2(w_2^+ = 1)$ $P_3(w_3^+ = w_4^+ = .5)$ $P_4(w_{60}^+ = 1)$	$X_1=59,000$ $X_2=80,000$ $X_3=80,000$ $X_4=1,000$ $X_5=10,000$ $X_6=100,000$ $X_7=100,000$ $X_8=2,500$ $X_9=2,500$	$X_{10}=100,000$ $X_{11}=55,000$ $X_{12}=70,000$ $X_{13}=2,000$ $X_{14}=70,000$ $X_{15}=3,000$ $X_{16}=45,500$ $X_{17}=1,000$ $X_{18}=50,000$	$X_{19}=1,000$ $X_{20}=2,500$ $X_{21}=50,000$ $X_{22}=20,000$ $X_{23}=2,500$ $X_{24}=38,500$ $X_{25}=2,000$	154795.0
5	$P_1(w_1^+ = w_{j+4}^+ = w_{j+29}^+ = w_{27}^+ = w_{55}^+ = w_{56}^+ = w_{57}^+ = w_{58}^+ = w_{59}^+ = 1)$ $P_2(w_2^+ = .5)$ $P_3(w_3^+ = w_4^+ = .5)$ $P_4(w_{60}^+ = 1)$	$X_1=80,000$ $X_2=50,000$ $X_3=80,000$ $X_4=80,000$ $X_5=10,000$ $X_6=100,000$ $X_7=100,000$ $X_8=87,500$ $X_9=100,000$	$X_{10}=100,000$ $X_{11}=70,000$ $X_{12}=70,000$ $X_{13}=2,000$ $X_{14}=1,000$ $X_{15}=57,000$ $X_{16}=1,500$ $X_{17}=50,000$ $X_{18}=45,000$	$X_{19}=1,000$ $X_{20}=2,500$ $X_{21}=1,500$ $X_{22}=2,000$ $X_{23}=2,500$ $X_{24}=2,000$ $X_{25}=2,000$	143941.0
6	$P_1(w_1^+ = w_{j+4}^+ = w_{j+29}^+ = w_{27}^+ = w_{56}^+ = w_{57}^+ = w_{58}^+ = w_{59}^+ = 1, w_{55}^+ = 1.5)$ $P_2(w_2^+ = 1.5)$ $P_3(w_3^+ = w_4^+ = 1)$ $P_4(w_{60}^+ = .5)$	$X_1=80,000$ $X_2=80,000$ $X_3=80,000$ $X_4=80,000$ $X_5=10,000$ $X_6=100,000$ $X_7=100,000$ $X_8=2,500$ $X_9=2,500$	$X_{10}=100,000$ $X_{11}=1,000$ $X_{12}=70,000$ $X_{13}=2,000$ $X_{14}=70,000$ $X_{15}=57,000$ $X_{16}=1,500$ $X_{17}=1,000$ $X_{18}=46,500$	$X_{19}=1,000$ $X_{20}=50,000$ $X_{21}=1,500$ $X_{22}=2,000$ $X_{23}=50,000$ $X_{24}=9,500$ $X_{25}=2,000$	156296.2

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