The Determination Of Optimal Reservation Price Setting For Homeowners

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Abstract

This paper considers the problem of determining the optimal reservation price for a seller of real estate. Although there exists much literature regarding the appraisal of real estate and property valuation, little research has focused on the process through which the seller may set a minimum price that he is willing to accept when bids are received over time. An exploratory and preliminary model is developed from which an optimal reservation price is determined as a function of pertinent cash flows and an ex-ante declared cumulative distribution of bids. Bids arrive according to a Poisson process and are a function of the listing price. A numerical example is provided to illustrate the results.

1. Introduction And Literature Review

This paper considers an interesting problem: The determination of a reservation price for a seller in the real estate market. Although scores of articles exist in the literature regarding the appraisal of real estate and property valuation, little research has focused on the actual determination of a minimum price that a seller should be willing to accept for real estate when bids arrive randomly over time. For example, the use of statistical modeling in the valuation process has been advocated by Smith (1995), and ranges from the application of regression analysis in predicting selling price (Warren, 1993) to the use of the Markov Theorem and coefficient of variation in evaluating alternative units of comparison for comparable sales (Boronico and Moliver, 1997). However, a weak linkage between this literature and the issue of reservation price setting in the real estate market exists, although the general issue of decision analytic approaches to negotiations and reservation prices is well established (Raiffa, 1982; Bazerman, 1994). This paper addresses this omission in the real estate literature by presenting a preliminary mathematical model through which a seller may determine an optimal reservation price. It is assumed that bids arrive according to a Poisson process, and that the cumulative distribution of bids may be approximated through the use of comparable sales. It is also anticipated that the work contained here will develop into a more detailed, pragmatic, and applicable model over time.

Current wisdom in the real estate brokerage industry holds that listing price is a primary attribute in successfully closing a sale. Bolin (1996), however, argues that many brokers overestimate the likelihood that bids will be received at a given listing price. This tendency often results in the property being listed for an excessive period of time, which may be detrimental in obtaining a desirable offer (Rosenthal, 1993). Hence the issue of listing price is of importance. The model presented here assumes that the listing price is ex-ante declared and is used in estimating the cumulative distribution function for offers, perhaps through the use of comparable sales. The inclusion of listing price as a decision variable is left as one of a number of possible extensions to the basic model and forms an important implication for future research.

The approach taken here is that a seller (homeowner) wishes to maximize expected profits from the sale of his home and must trade off expected revenues received upon the sale of the property with the intermediary outgoing cash flows incurred during the marketing period when the house is listed (e.g. mortgage payments). The estimation of this marketing period for a home has been discussed elsewhere. For example, Genesove and Mayer (1997) hypothesize that an inverse relationship exists between owner's equity and time to sale. Tradeoffs between risk and returns is considered by Meyers and Wieand (1996), who prescribe a method allowing homeowners to diversify away risk by treating the home as a part of an efficient portfolio. The objective of profit maximization has also been considered (Tabuchi, 1996; Grenadier, 1996). The motivation for the analysis here is to extend much of this previous work by incorporating a Poisson process into the methodology and assuming that monetary bids are stochastic in nature and governed by a cumulative distribution function related to the listing price of the property being sold.

The paper proceeds as follows: Section 2 presents the model formulation, which is then illustrated with a numerical example in section 3. Brief concluding comments and some implications for future research are presented in section 4.

2. Mathematical Model

We assume that the homeowner's objective is to maximize expected profits on the sale of a piece of property (e.g. home), where bids are received over time according to a Poisson process whose mean (λ) is inversely related to the listing price (L). The homeowner must choose a reservation price (k) above which he/she will accept the first bid that is received. The homeowner must make payments in the form of PITI, that is, principle, interest, taxes, and insurance for the term during which his house is on the market. Clearly there are tradeoffs involved between setting the reservation price and profits achieved through the distribution of bids. For example, a higher reservation price will, on average, yield a higher acceptable bid, but at the added expense of a longer interarrival time between listing the house and receiving a bid that equals or exceeds this increased reservation price. Note that the impact of the listing price is not modeled here but is ex-ante declared. Hence it does not form part of the decision process modeled.

In developing the model, some initial simplifying assumptions are made. We assume that PITI payments are limited to monthly mortgage payments of size M, and assume a zero discount rate. Given listing price L, and corresponding arrival rate $\lambda(L)$, the expected interarrival period between listing the house and receiving an acceptable bid may be shown to equal the following:

$$\frac{1}{\lambda(L)Pr(B \geq k)} \tag{1}$$

where B represents the random monetary value of an incoming bid. The homeowner's objective may be stated as follows:

$$Max : E(B \mid B \geq k) - \frac{1}{\lambda(L)Pr(B \geq k)} - M \tag{2}$$

Assume that the CDF for bids received is given by $F(K) = \Pr(B \leq k)$. Equation (2) may then be rewritten as follows:

$$Max : E(B \mid B \geq k) - \frac{1}{\lambda(L)[1 - F(k)]} - M \tag{3}$$

It is known, from standard statistical theory, that:

$$E(B \mid B \geq k) = \frac{\int_{b \geq K} bf(b)}{1 - F(k)} \tag{4}$$

The following then obtains from the substitution of (4) into (3):

$$Max: \left(\int_{b \geq k} bf(b)db - \frac{1}{\lambda(L)} M \right) - \frac{1}{1 - F(k)} \tag{5}$$

Equation (5) represents the unconstrained objective function, utilized below, from which an optimal reservation price may be determined.

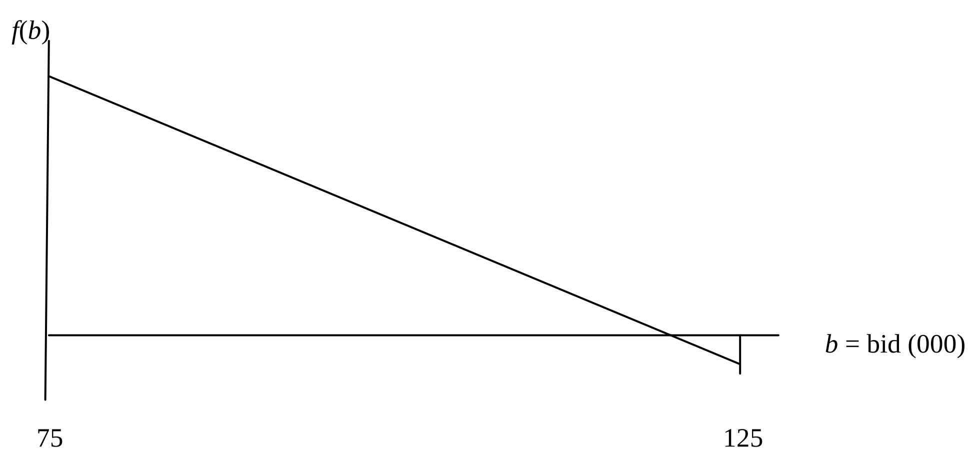
3. Numerical Example

For the purpose of illustrating the model, we incorporate the following parameter values:

$$\lambda = 2/month \quad M = \$750/month \quad Annual\ interest\ rate = .0825$$
$$Remaining\ mortgage\ payments\ at\ listing = 300$$

That is, for the ex-ante declared listing price, bids come in at the mean rate of two per month, the monthly mortgage payment is $750, and there are 300 mortgage payments outstanding at the time of listing. We include the latter figure in order to build in the value of outstanding mortgage payments when calculating overall profits for the homeowner at the time of sale. We also assume that the density function for bids received, for the ex-ante declared listing price, adheres to the following:

Figure 1: Density function for incoming bids



From Figure 1, it is simple to derive the density function:

$$f(b) = .1 - .0008b \tag{6}$$

From this it is straightforward to ascertain the applicable distribution function:

$$F(b) = P(B \le b) = .1b - .0004b^2 - 5.25 \tag{7}$$

Substitution of (6) and (7), together with aforementioned parameter values, into (5) results in:

$$Max : \Pi(k) = \left(\left[264.92 - .05k^2 + .000267k^3\right] - .5\right)\frac{1}{6.25 - .1k + .0004k^2} \tag{8}$$

The solution for *k* would obtain directly from the differentiation of (8) with respect to *k* and the utilization of the calculus, however, if the problem is to incorporate discounting, then the solution is more readily obtainable through the utilization of an Excel spreadsheet. It should again be noted that (8) does not include outstanding mortgage payments to be made at the time of closing (following the sale of the home), although the solution presented below does take this factor, together with discounting, into account. The contour of the objective function is also demonstrated below, in Figure 2.

Optimal Solution:

$$k^* = 111$$

$$E(selling\ \ price \mid k^*) = 115.71$$

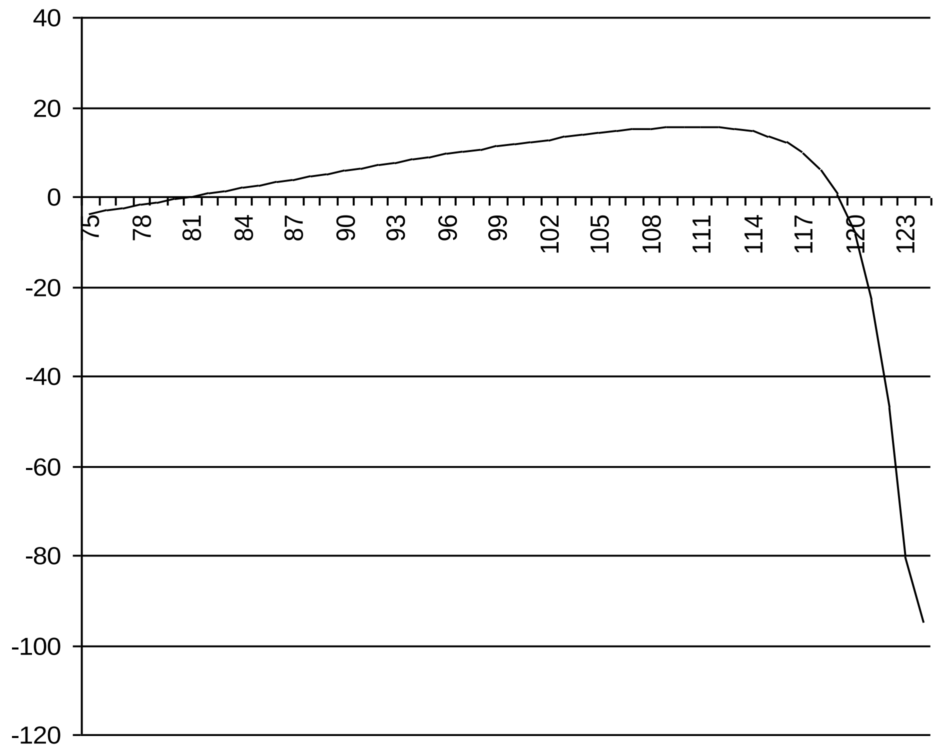
$$P(incoming\ bid \ge k^*) = .0784$$

$$E(number\ of\ \ periods\ until\ \ sale \mid k^*) = 6.38$$

$$\Pi(k^*) = \Pi^* = 15.64$$

The solution indicates that the reservation price should be set at $111,000, that is, the seller should not settle for any offer that does not equal, or exceed, this amount. Given this reservation price, it will take, on average, 6.38 months for an acceptable bid to be received, where the probability that a random bid meets the reservation price is .0738. The expected acceptable bid will be $115,710 with an expected profit (once figuring in the costs associated with outstanding mortgage payments) of $15,640.

Figure 2: Contour for the objective function



IV. Brief Concluding Comments

The mathematical model presented here provides an exploration into a decision process that is encountered by most individuals during their lifetime. The model provides a basis through which a homeowner may ascertain an appropriate reservation price for the sale of a home under the assumption that bids, which are random in nature, follow a Poisson process. It is assumed that the amount offered by a prospective buyer adheres to an ex-ante declared probability distribution. Given this set of assumptions, it is possible to solve for the reservation price that maximizes the expected profit to the seller, for whom this model is designed.

In order to increase the realism of the model, there exist future implications for research. For one, a dynamic economic environment, where the bid arrival rate is impacted on by both the listing price and the anticipated short-term market condition would be desirable. Second, a more accurate assessment of costs associated with house closings and other attributes that might impact on the decision process modeled here would be beneficial. In addition, issues concerning asymmetric information between the buyer and seller, and the incorporation of utility models, or other behavioral models of choice, could be utilized.

In summary, the model discussed here offers insight concerning the merger of functional areas of business, specifically, financial modeling and operations research methodology, in a way that impacts on decision making as it applies to individuals. Despite the prevalence of Operations Research techniques on the organizational level, applications of operations research modeling on a personal level continue to suffer from the lack of familiarity and overall discomfort with the notion of mathematical optimization at this level. The model presented here attempts to bridge this gap by offering a perspective in the area of personal finance and real estate that will assist individuals when selling a tangible asset.

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Notes