A Macroeconomic Analysis Of Inventory/Sales Ratios

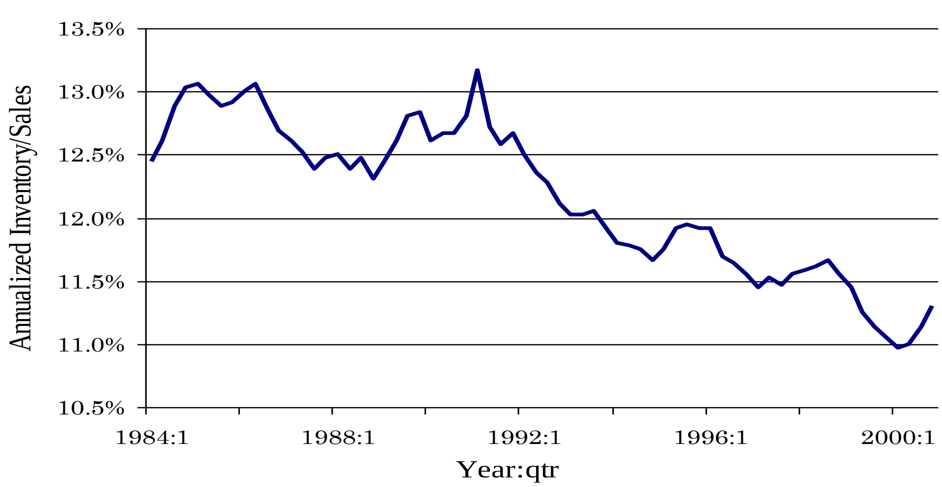
William M. Bassin, Shippensburg University
Michael T. Marsh (E-mail: mtmars@ship.edu), Shippensburg University
Stefanie Walitzer, Shippensburg University

Introduction

Metzler's (1941) research on the relationship between inventory and business cycles initiated serious interest in inventory behavior and its effect on the behavior of firms. A flurry of related research took place in the following two decades. Research of the time clearly demonstrated that, at a macro level, the inventory behaviors are significant features in business cycles. One measure of inventory behavior introduced and analyzed was the inventory-to-sales ratio. We continue to believe that understanding of inventory behavior at both the macro and microeconomic levels is a prerequisite to understanding factors that determine a firm's success, and that analysis of the inventory-to-sales ratio is important component of inventory behavior. The U.S. Department of Commerce and other government and private institutions track this ratio and report regularly. Financial analysts use both a company's trend and its comparative value within a sector to make investment decisions. The data, sources, and explanations can be easily found in both hard copy and electronic formats published by Federal Reserves Banks, U.S. Department of Census, and U.S. Department of Commerce. An in-depth service private source is the Quarterly Ratio Study provided by the Center for Inventory Management. (www.centerforinventorymanagment.org)

Early work in ratio analysis by Feldstein and Auerbach (1976) posed the dilemma that firms use long time periods to adjust production to variations in sales levels even when the range of inventory levels is equivalent to only a few days of production. Blanchard (1983) and Blinder (1981) used inventory-to-sales ratios to reinforced this idea with evidence that production is more variable than sales in most industries. Blinder and Maccini (1991) reported that, even using the sophisticated inventory management techniques developed and implemented in the 1980's, the aggregated ratio of the inventory-to-sales ratio had no distinct trend for a four decade period. Since 1991, however, there has been a marked downward trend in the ratio, which can be clearly seen in Figure 1. This paper examines some of the factors that would account for this decline and proposes a model for inventory-to-sales ratio.

Figure 1: Historical Inventory/Sales Ratio



(Data derived from the Economic and Financial Database at http://www.stls.frb.org)

The Theoretical Model

As developed by Silver and Peterson (1985), the cycle stock inventory (CSI_t) at time t for a firm using an optimal (EOQ) inventory management policy can be calculated as follows:

$$CSI_t = \left(n_t / \sqrt{2}\right) * \sqrt{A_t / r_t} * \sqrt{\mu_t} * \exp\left[-p_t^2 / 8\right] \qquad (1)$$

where n_t = the total number of items in a company's product line (The same physical product with different detailed characteristics such as size of package, flavoring, and the like, or in different locations counts as multiple items.)

A_t = administrative cost per order
r_t = inventory carrying cost per dollar per year
μ_t = mean cost of sales for the individual items in the product line
p_t = the "Pareto" factor (For example, if p is 1.28, the top 50% of the SKU's[1] generate 90% of the cost of sales. If p is 1.64, the top 50% of the SKU's generate 95% of the cost of sales, and so on.)

The cost of sales (CS_t) for a company is

$$CS_t = n_t * \mu_t \qquad (2)$$

It follows that the theoretical ratio of cycle stock inventory to the cost of sales (IS_t) can be expressed as

$$IS_t = CSI_t / CS_t = \left(1/\sqrt{2}\right) * \sqrt{A_t / r_t} * \sqrt{1/\mu_t} * \exp\left[-p_t^2 / 8\right] \qquad (3)$$

(The n_t term drops out, at least temporarily.)

The mean cost of sales for the individual items in the product line, μ_t, can be expressed as the product of μ_{Dt} and μ_{Vt}, where μ_{Dt} is the mean demand in units for the items in the product line, and μ_{Vt} is the mean unit cost for the items in the product line. Accordingly, IS_t can be rewritten as

$$IS_t = \left(1/\sqrt{2}\right) * \sqrt{A_t / \mu_{Vt}} * \sqrt{1/\mu_{Dt}} * \sqrt{1/r_t} * \exp\left[-p_t^2 / 8\right] \qquad (4)$$

and if IS_0 is defined to be the value of the IS ratio in year zero, then

$$IS_0 = \left(1/\sqrt{2}\right) * \sqrt{A_0 / \mu_{v0}} * \sqrt{1/\mu_{d0}} * \sqrt{1/r_0} * \exp\left[-p_0^2 / 8\right] \qquad (5)$$

It follows that the relative value of these two ratios can be defined by the following:

$$\begin{aligned} IS_t / IS_0 &= \frac{(1/\sqrt{2}) * \sqrt{A_t / \mu_{Vt}} * \sqrt{1/\mu_{Dt}} * \sqrt{1/r_t} * \exp(-p_t^2/8)}{(1/\sqrt{2}) * \sqrt{A_0 / \mu_{V0}} * \sqrt{1/\mu_{D0}} * \sqrt{1/r_0} * \exp(-p_0^2/8)} \\ &= \sqrt{\frac{A_t / \mu_{Vt}}{A_0 / \mu_{V0}}} * \sqrt{\frac{r_0}{r_t}} * \sqrt{\frac{\mu_{D0}}{\mu_{Dt}}} * \left[\frac{\exp(-p_t^2/8)}{\exp(-p_0^2/8)}\right] \end{aligned} \qquad (6)$$

Instead of attempting to measure the mean demand, $\mu_{Dt,}$ directly, it is possible, even desirable, to measure these values indirectly. It is intuitive that

[1] Stock keeping unit (SKU) is an identification, usually alphanumeric, of a particular product that allows it to be tracked for inventory purposes. Typically, a SKU is established by the merchant and associated with any purchasable item in a store or catalog.

$$\frac{\mu_{D0} * n_0}{\mu_{Dt} * n_t} = \frac{CGS_0 / IPD_0}{CGS_t / IPD_t} \tag{7}$$

where CGS_t = the cost-of-goods-sold (or sales, if cost of sales data are not available)

IPD_t = the implicit GDP price deflator, or another appropriate price index.

This is because, when the μ_{Dt} is multiplied by n_t, the result is a rough index of the total amount of physical units sold. The ratio of these values from one time period to another is equivalent to a ratio of deflated dollar sales. Therefore,

$$\frac{\mu_{D0}}{\mu_{Dt}} = \left[\frac{CGS_0 / IPD_0}{CGS_t / IPD_t}\right] * \left(\frac{n_t}{n_0}\right) \tag{8}$$

Substituting this expression into equation (6):

$$IS_t / IS_0 = \sqrt{\frac{A_t / \mu_{Vt}}{A_0 / \mu_{V0}}} * \sqrt{\frac{r_0}{r_t}} * \sqrt{\frac{CGS_0 / IPD_0}{CGS_t / IPD_t}} * \sqrt{\frac{n_t}{n_0}} * \left[\frac{\exp(-p_t{}^2 / 8)}{\exp(-p_t{}^2 / 8)}\right] = TR_t \tag{9}$$

where TR_t is the theoretical ratio of the inventory to sales ratios.

An Economic Model of the Values of IS_t

In constructing a model for the values of IS_t, it is reasonable to hold constant, at least for the time being, those variables whose values we cannot now measure. Such variables would certainly include the A_t/μ_{Vt} ratio, as the administrative costs of ordering, or set-ups, are notoriously difficult to measure. However, it is not unreasonable to assume that deflated administrative costs remain constant. The values of p_t are also unavailable at this time. As a result, equation (9) reduces to

$$IS_t / IS_0 = \sqrt{\frac{r_0}{r_t}} * \sqrt{\frac{CGS_0 / IPD_0}{CGS_t / IPD_t}} * \sqrt{\frac{n_t}{n_0}} = TR_t \tag{10}$$

Assume, as a first approximation, that r_t is 15% plus the interest rate on t-bills. Then equation (10) can then be rewritten as

$$IS_t / IS_0 = \sqrt{\frac{(.15 + tb_0)}{(.15 + tb_t)}} * \sqrt{\frac{CGS_0 / IPD_0}{CGS_t / IPD_t}} * \sqrt{\frac{n_t}{n_0}} = TR_t \tag{11}$$

where tb_t is the interest rate on treasury bills at time *t*. Data are available to at least approximate the values of the first two terms of this ratio. Solid data about the ratio of n_t/n_0 is not available, but the model can be examined with different rates of growth to see which yields the best fit to available data.

Multiplying both sides of equation (11) by IS_0, yields the following:

$$IS_t = TR_t * IS_0 \tag{12}$$

Equation (12) is the first independent variable in an economic model that allows a forecast of the values of the actual inventory to sales ratio to be developed. The value of IS_0 can be set equal to the actual ratio for the first time period in which there are data.

Three additional independent variables are proposed for this model. The first relates to the change in sales, above or below expectations. The rationale is that when there is such a change in the sales, inventories are run down below (or overshoot) planned levels.

The second variable relates to corporate profits. Sarte (1999) finds that, as profitability grows, inventories expand *ceteris paribus* because the benefits of additional safety stocks are enhanced, thus adding to total inventory levels. Recall that equation (1) applies only to cycle stocks. Regardless, research indicates that businesses tend to invest internally as profits grow, whether such investments are wise or not, and inventory growth is one such investment. This is consistent with previous analysis by Bils and Kahn (2000).

The third variable is simply a time trend. The model is therefore defined as follows:

$$AIS_t = \beta_1 IS_t + \beta_2 \Delta S_t + \beta_3 P_t + \beta_4 t + \varepsilon_t, \tag{13}$$

where AIS_t = the *actual* ratio of inventory to sales,

IS_t = the *theoretical* ratio of inventory to sales, from equation (12)
ΔS_t = the sales change variable,
P_t = the profitability variable,
t = the time trend variablε_t = an error term , and the
β represents the parameters to be estimated.

Testing the Model with Macroeconomic Data

Figure 2 shows results using quarterly macroeconomic data beginning in 1Q/1984 and ending in 4Q/2000. (U.S. Department of Commerce Seasonally Adjusted Inventory to Sales Ratios; U.S. Department of Commerce Seasonally Adjusted Total Business Inventories; Federal Reserve Board of Governors (H.15 release) 3-Month Treasury Constant Maturity Rate) Business sales data are used, as reliable cost-of-sales data are not available. A range of growth rates in *n* were examined but an assumed growth rate of zero percent gives the best fit. Table 1 shows the regression results for the data. It is noteworthy that all variables are statistically significant, but, as expected, there is no intercept term. The overall fit appears to be representative, with some systematic under-fitting in the early 1990's and some systematic over-fitting immediately thereafter. These patterns could be recession/recovery induced but determining the precise cause is beyond the scope of this paper.

The sales change variable is calculated as the ratio of the change in sales to an eight quarter moving average of changes. The sign of the estimated coefficient for this variable is negative, as might be conjectured intuitively. That is, if sales are accelerating, inventories should shrink relative to sales.

The profits variable is simply deflated corporate profits. The sign of the estimated coefficient is positive, again, as might be hypothesized.

Figure 3 shows the decline in the ratio, with all other factors but time adjusted out. The systematic patterns in the early 1990's, and immediately thereafter again, stand out. Other researchers observed the same patterns of over- and under-fitting and proposed reasonable rationale. King, et al. (1991) suggest the influence of monetary disturbances during that period. Sarte (1999) suggests these variations are a result of JIT methodology popular in the early 1990's. Bils and Kahn (2000) and Ben Salem and Jacques (1996) find anomalies in the inventory/sales ratio during periods of recession. Worthington (1998) and Sarte (1999) pose interesting theories that adoption of positive innovations in technology influenced unexpected variations in the inventory-to-sales ratio. These two anomalies and potential causes, while interesting, does not affect the overall substantial downward trend.

Figure 2: Aggregate Analysis with Zero Annualized Growth Rate in n

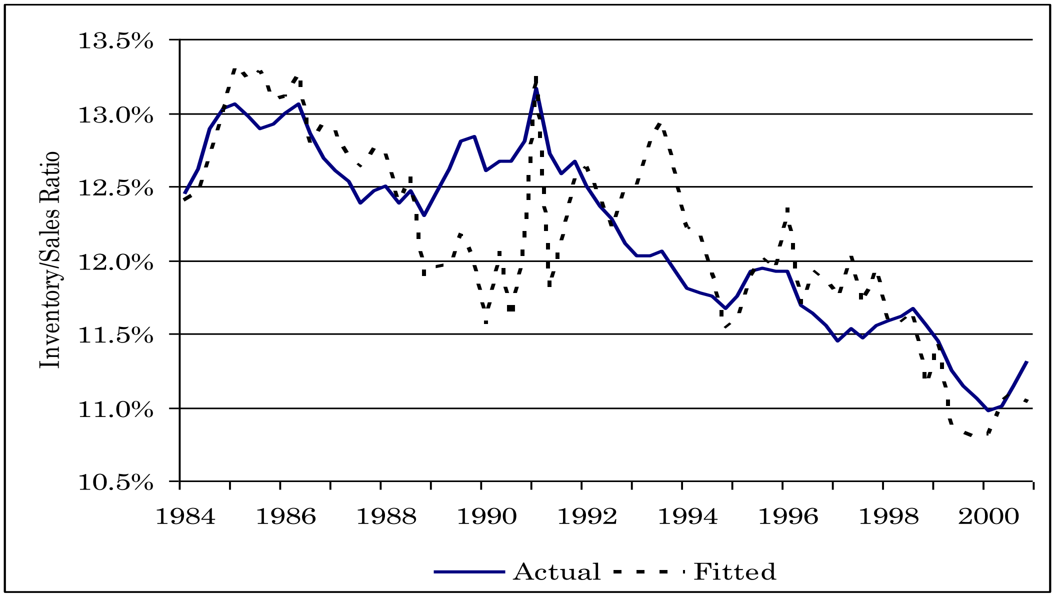


Table 1: Regression Results for Aggregate Analysis with Zero Annualized Growth Rate in n

Variable	*Coeff.*	*Std. Dev.*	*t Stat*
Theoretical Ratio	0.8708	0.0191	45.629
Sales Chg.	-0.2435	0.0471	-5.171
Profits	0.00460%	0.00073%	6.278
Time	-0.0444%	0.0066%	-6.763
Multiple R	74.29%		
R Square	55.18%		
Adj. R Square	51.52%		
Standard Error	0.415%		
Observations	68		

The time variable shows a reduction in the inventory to sales ratio of more than 0.04% per quarter, or reduction of about 0.178% per year.

Figure 3: Aggregate Analysis (regressed on time only)

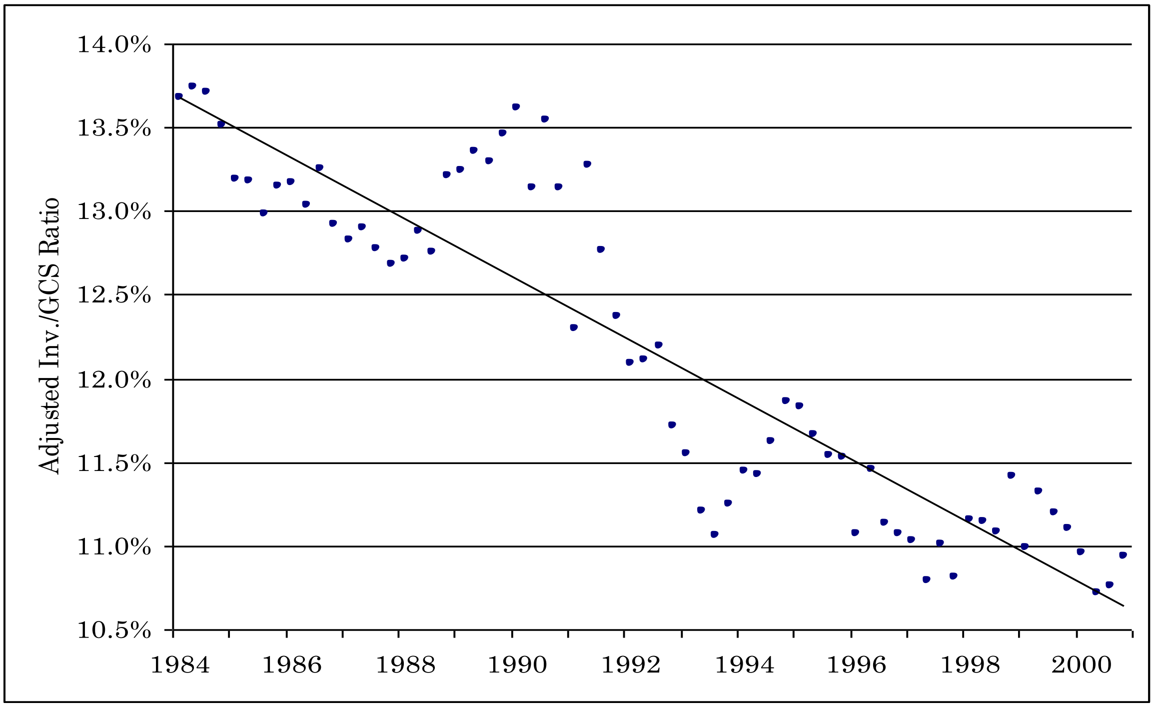


Figure 4 and associated Table 2 show that with an assumed annualized growth rate in n_t of 4%, the fit of the equation, while nearly identical to that in Figure 2, is actually worse. Upon first examination this does not seem reasonable, as the number of sku's has grown over the years. However, preliminary research indicates that companies are reducing the marginal costs of additional product configurations and locations, such that the number of sku's gives the appearance of being constant. At the same time the inventory to sales ratio is declining with time over and above this effect. The regression results for the time variable shows an unreasonably large reduction in the inventory to sales ratio of about 0.440% per year, which is significantly different from that when the assumed growth rate is zero.

It is evident that the average inventory levels (developed under an economic order quantity strategy) are a function of the square rood of annual unit demand. Therefore, if unit demand doubles, average inventory does not double and, by extension, neither do carrying costs. This is an application of economies of scale. However, suppose that demand doubles and the product is produced in two versions (mild and "zesty", for example), simultaneously. Then average inventory for both versions combined will double but no economies of scale will result. A direct result will be an increase in *n*, the number of SKU's. This may be considered as an application of diseconomies of scope, i.e. product diversity.

The high degree of first-order correlation among the regression residuals is evidenced by a Durbin-Watson statistic of 0.480. This does not indicate that there is bias in the parameter estimates but rather that the estimates of the standard errors of the estimates are understated, resulting in overstatement of the t-statistic. However, the estimates of standard error can be corrected by multiplying them by 1.539.[2] As a result, the corrected standard error

[2] As noted in Pindyck and Rubinfeld, Econometric Models & economic Forecasts (1981), the value of the Durbn Watson statistic is approximately equal to 2(1-r), where r is the firs order correlation coefficient among the residuals. Setting this expression equal to 0.480 implies that r is 0.74 resulting in a correction factor or 1.539.

for the time coefficient estimate is 0.0101% = 1.539 * 0.00656%. This leads to a revised t statistic of -0.0444%/0.0101% = -4.394, a highly significant result. This results supports the regression results.

Figure 4: Aggregate Analysis (4% growth rate in n)

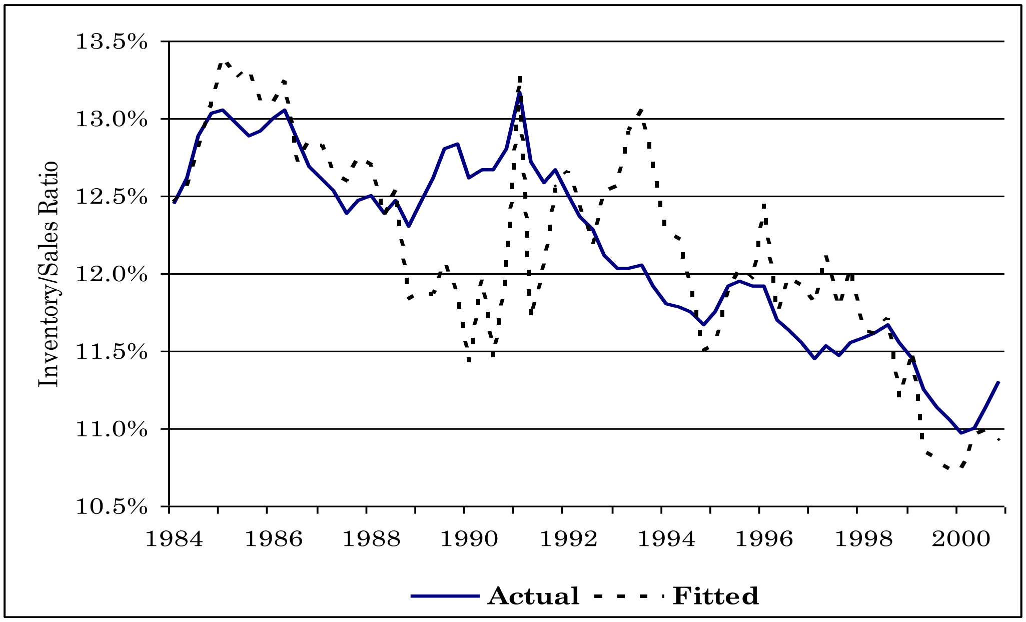


Table 2: Regression Results for Aggregate Analysis with 4% Annualized Growth Rate in n

Variable	***Coeff.***	***Std. Dev.***	***t Stat***
Theoretical Ratio	0.8486	0.0212	40.062
Sales Chg.	-0.2605	0.0535	-4.870
Profits	0.00559%	0.00081%	6.885
Time	-0.1100%	0.0064%	-17.270
Multiple R	65.09%		
R Square	42.37%		
Adj. R Square	38.11%		
Standard Error	0.470%		
Observations	68		

Conclusions and Extensions

In this paper, we have proposed and tested a model for forecasting the inventory-to-sales ratio at a macro level. The model was tested with quarterly data using standard regression methodology.

These general findings emerge from the empirical work:

1. Inventory-to-sales ratios are declining with time even after other determining factors are accounted for.

2. It is difficult, or even impossible, to determine what impact the number of items in a company's product line has simply by observing changes in the inventory to cost-of-goods-sold ratio. This knowledge, or rather lack of, may play an important role in the decisions influenced by analysis of the ratio.

3. It is possible to obtain a reasonably representative regression "fit" with a limited number of assumptions and without a large historical database. The parameter values in the model presented in this paper are all statistically significant and the impact of biases appears to be felt mainly in response to macro-economic factors rather than on company's inventory control policies.

4. These results support research that indicates U.S. businesses are doing an increasingly better job of managing inventories, and as a result, affording economies of scope. Most notable are the observations that, even though the number of sku's is increasing, the number appears to be constant due to efficient management and the consistency in reduction of the inventory to sales ratio, the average reduction being 0.178%.

5. The regressions indicate that zero growth in sku's fits the data best, even though it is well know that the number of sku's for many products is higher than in the past. For example, there are eight varieties of Triscuits ® today where only one existed in 1970. It is unclear, however, whether supply chains are treating a diverse range of product versions or varieties as homogeneous units thus making it difficult for consumers to be confident that the desired version will be available at a particular time.

   Several avenues of future research are suggested by this research:

   1. It would be informative to conduct sensitivity analysis over interest rates, administrative costs, inventorying carrying costs, growth rate, and the Pareto factor to assess the impact of the assumptions on the results.

   2. Try different functional forms of the equations in the model as well as different ways of measuring the sales change and profitability variables.

   3. Additional analysis of the models presented here to include tests for multicollinearity, heteroskedasticity, and serial correlation.
   4. Survey companies to determine changes in the values of n and p.

   5. The authors are currently applying the model to microeconomic data. Preliminary results have been obtained using the inventory, cost-of-sales, and profit data from 37 food processing companies and 19 chemical/pharmaceutical companies. 🕮

References

1. Ben Salem, Melika and J-F Jacques, (1996). About the Stability of the Inventory-Sales Ratio: An empirical Study with US Sectoral Data. *Applied Economic Letters* 3, 467-469.

2. Bils, Mark, and J.A. Kahn, (2000). What Inventory Behavior Tells Us About Business Cycles. *American Economic Review* 90 (3), 458-481.
3. Bivin, David, (1988). The Behavior of Manufacturers' Inventories: 1967-1986, 10 (1), 63-82.
4. Blanchard, O.J., (1983). The Production and Inventory Behavior of the American Automobile Industry. *Journal of Political Economy* 92, 365-400.
5. Blinder, A.S., (1981). Retail Inventory Behavior and Business Fluctuations. *Brookings Papers on Economic Activity*, 443-505.
6. Blinder, A.S. and L.T. Maccini, (1991). Taking Stock: Assessment of Recent Research on Inventories. *Journal of Economic Perspectives* 5 (1), 73-96.
7. Feldstein, Martin and A. Auerbach, (1976). Inventory Behavior in Durable Goods Manufacturing: The Target-Adjustment Model. *Brookings Papers on Economic Activity* 2, 351-396.
8. Hirsch, Albert A., (1996). Has Inventory Management in the U.S. Become More Efficient and Flexible? A Macroeconomic Perspective. *International Journal of Production Economics* 45, 37-46.
9. Kahn, James A., (1987) Inventories and the Volatility of Production. *American Economic Review* 77 (4), 667-679.
10. Metzler, Louis A., (1941). The Nature and Stability of Inventory Cycles. *Review of Economic Studies* 23, 113-29.
11. Sarte, Pierre-Daniel G., (1999) An Empirical Investigation of Fluctuations in Manufacturing Sales and Inventory within a Sticky-Price Framework. *Federal Reserve Bank of Richmond Economic Quarterly* 85 (3), 61-84.
12. Silver, E.A. and Peterson, R., (1985). *Decision Systems for Inventory Management and Production Planning*. John Wiley, New York.
13. Worthington, Paula R., (1998). The Increasing Importance of Retailers' Inventories. *Economic Perspectives* 22 (3), 2-13.

Notes

Notes