The Impact Of Using The Heuristic Teaching Method On Jordanian Mathematics Students

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ABSTRACT

This study investigates the impact of using the heuristic teaching approach for teaching mathematics to tenth grade students in Jordan. The researchers followed the equivalent pre/post T test two group designs. To achieve the goal of the study, a pre/post- test was constructed to measure student achievement in mathematics. The sample for this study consisted of 142 students; 69 male students and 73 female students from tenth grade at King Abdullah School in Irbid, Jordan for the first semester of the academic year 2011/2012. The subjects of the study were distributed into an experimental group and a control group. The experimental group was taught mathematics using the heuristic approach while the control group was taught mathematics using the traditional method of teaching. The subjects were 34 male students for the experimental group and 35 male students for the control group, while the female students for the experimental and control group were 37 and 36 respectively. Those subjects were distributed into two purposefully selected sections at king Abdullah School in Irbid. Descriptive statistical analyses were used (means and standard deviation) for the pre- and post- tests of students. Comparison statistical methods were used (Two Way ANOVA) analysis of variance to make a comparison between the control and the experimental groups and gender variable. The findings of the study indicated that there were statistically significant differences in the post- test between the control and the experimental groups in favor of the experimental group, and there was no statistically significant difference in the students' achievement due to gender. There was no statistically significant difference due to the interaction between gender and group. The researcher proposes some recommendations to enhance the effect of using heuristic approach in teaching mathematics on students' performance such as conducting further studies in other universities.

Keywords: Heuristic Teaching Method for Jordanian Students

INTRODUCTION

Heuristic methods are rules of thumb for making progress on difficult problems (Polya, 1973). They are general suggestions on strategy that are designed to help when we solve problems (Schoenfeld, 1985). For Bruner (1960) they are methods and strategies that can be helpful in problem solving. In sum, they can be explained as non-rigorous methods of achieving solutions to problems, ideas that have been useful in previous problem solving that we might want to apply when we solve our current problems.

Heuristics have been generally recognized as a crucial component for problem solving (Polya 1973; Schoenfeld, 1985; Rubinstein, 1986; Mayer, 2003). In fact, according to Schoenfeld (1985), heuristics have now become nearly synonymous with mathematical problem solving.

A heuristic is a generalized method used to solve a problem, and an algorithm is a step-by-step method used to solve a problem. We can categorize any problem that needs to be solved by a heuristic as one that deals with what may eventually become an infinite series of steps when broken down completely. Theoretically, if a problem

has an infinite number of steps, then it can never be fully broken down—and so we generalize and in our generalization we create a heuristic. Any problem that can be solved via a finite number of steps and has a numerical solution can be approached as needing an algorithm to generate the solution. Granted, some linear equations have an infinite number of solutions, but the solution of the equation has just a finite number of steps. In teaching both concepts, it is best to generalize the definitions of a heuristic and an algorithm

For arguments sake, we shall say we need a heuristic anytime we solve a problem that is non-numeric. The reasoning is that such problems usually require human intervention, and all human actions/behaviors/thoughts are infinite by nature. Anyway, somewhere along the line we have to make a decision of when we can say that we have provided enough information to solve the problem. Hence, we make a generalization and, by definition, we have created a heuristic.

As mentioned in Emanovský, P., Břehovský, J. (2010) the most commonly used inductive teaching and learning methods are inquiry learning, problem-based learning, project-based learning, case-based teaching, discovery learning and just-in-time teaching (Prince, J. M., Felder, R. M. (2006)). The investigations according to Kopka, J. (2004) is possible to consider as a method of the first category. The inquiry learning means that students are presented with questions to be answered, problems to be solved, or a set of observations to be explained (Bateman, W. (1990)). If the method is implemented effectively, the students should learn to formulate good questions, identify and collect appropriate evidence, present results systematically, analyze and interpret results, formulate conclusions, and evaluate the worth and importance those conclusions“ (Lee, V. S. (2004)). In the heuristic method the child is put in the place of discoverer. It involves finding out by the student by complete self-activity. The teacher is only a passive observer.

Using investigations is one method of teaching involved in the full range of the development of a mathematical theory. Investigations also provide students with insights into what it is like to be a mathematician and to experience of mathematical thinking at work.

Students should be able to investigate certain mathematical situations and consequently to formulate problems and hypotheses. This inductive approach should be completed by validation of the hypotheses, i.e. by return deduction. Clearly, the inductive way is much more time-consuming and difficult for teachers and students than traditional one. On the other hand, it contains very important and worth student activities which are all about making the students more active participants in the learning process – an observation, an investigation, formulation and solving of problems and formulation and validation of hypotheses.

SIGNIFICANCE OF THE STUDY

The idea of teaching heuristics explicitly is so that we can expedite these processes of discovering and identifying heuristics, and apply them in problem solving. Whether it is possible to simply expect the discovery and identification of processes, rather than explicitly teaching the heuristics, is questionable, it definitely requires more time and effort and is a less efficient approach to learning heuristics. However, the heuristics learned through discovery and identification is usually ones that we use most often, since we have a much better understanding and appreciation of them.

The common way of teaching heuristics explicitly is to teach them as if they were mathematical concepts or skills. We teach one heuristic at a time and give students problems that can be solved by that particular heuristic at the end of the lesson. The problem with such an approach is that we have isolated each heuristic from the others, since often the relationships between heuristics and the ideas for these heuristics are not mentioned. Students might treat these heuristics as algorithms; procedures they need to follow when they solve problems without really understanding the ideas behind the heuristics, why and when they should them. In fact, the researcher didn't find any study about using heuristics in teaching mathematics in Jordanian schools so she decided to conduct this study.

STATEMENT OF THE PROBLEM

The teaching of inquiry has a long history in school mathematics (D'Ambrosio & Lester, this volume; Stanic & Kilpatrick, 1988). In the past several decades, there have been significant advances in the understanding of the complex processes involved in problem solving and learning inquiry(Lester, 1994; Schoenfeld, 1992; Silver, 1985). There also has been considerable discussion about teaching mathematics with a focus on inquiry. However, teaching mathematics *through* inquiry is a relatively new idea in the history of problem solving in the mathematics curriculum (Lester, 1994). In fact, because teaching mathematics through inquiry is a rather new conception, it has not been the subject of much research. Although less is known about the actual mechanisms students use to learn and make sense of mathematics through inquiry, there is widespread agreement that teaching through inquiry holds the promise of fostering student learning (Schroeder & Lester, 1989). Many of the ideas typically associated with this approach (e.g., changing the teacher's roles, designing and selecting problems for instruction, collaborative learning, problematizing the curriculum) have been studied extensively, and research-based answers to various frequently asked questions about inquiry instruction are now available.

PURPOSE OF THE STUDY

The purpose of this study is to investigate the impact of using heuristic approach for teaching mathematics on tenth grade students' achievement at king Abdullah School in Irbid first educational directorate during the first semester of the academic year 2011/2012.

QUESTIONS OF THE STUDY

1. Are there statistically significant differences ($\alpha \leq 0, 05$) in the tenth grade students' achievement in mathematics due to the teaching strategy they are exposed to using heuristic approach and traditional way?
2. Are there statistically significant differences ($\alpha \leq 0, 05$) in the tenth grade students' achievement in mathematics due to the gender of the students?

LITERATURE REVIEW

Many studies in the field of didactic of mathematics are devoted to issues related to the use of heuristic approaches to teaching mathematics at all levels of schools (Prince, J. M., Felder, R. M. (2006), Kopka, J. (2007)). These studies demonstrate the necessity of using the activating teaching and learning methods and their considerable positive impact on affectivity of education. The use of this approach for teaching the pupils evidently improves their learning, knowledge and skills. Consequently, pupils are able to better understand the genesis of new concepts, their inclusion into a logical structure and the causes of their definition. These reasons lead to many recommendations to use heuristic approaches for teaching of mathematics much more than was practiced previously. Their presence can be useful to improve mastery of concepts and relations between them and a better understanding of the taught topic.

Despite of traditional deductive approach in mathematics most mathematical theories have both an experimental and inductive character. Their beginnings arise out of tentative searching and speculative trial and error; they gain a deductive character only after their period of investigation. Investigations, as described in Kopka, J. (2004), is a method of teaching and learning mathematics which permits students to enter and penetrate more deeply into the world of mathematics that most other teaching approaches fail to do. If one wishes the students to have experiences of how mathematics evolves, then it should be respected how mathematical theories come into existence, how they develop and how they finally gain their form and nature. Too frequently students are only exposed to mathematics in its final and approved form.

An inquiry approach has been advocated by many mathematical researchers – having the potential to provide for a diversity of learners – in terms of engagement and challenge – by using interesting and authentic tasks (Carter, 2004; Diezmann, 2004; Heibert, 2003; Lovitt, 1999; Schoenfeld, 2002; Stein, Grover & Henningsen, 1996; Sullivan, 2001). In an inquiry mathematics classroom, students are actively engaged in the construction of knowledge (Carter, 2004, p.2). Listening, discussion and explaining mathematical thinking are important characteristics of an inquiry classroom as is a climate of cooperation between the teacher and students, creating a

community of learners. "Inquiry approaches (e.g. open-ended questions, investigations, thought experiments) are advocated in mathematics because they engage students and suit a range of student capabilities. They also provide authentic opportunities for teamwork and co-construction of knowledge" (Diezmann, 2004, p.80).

Many researchers (e.g., Carpenter, Franke, Jacobs, Fennema, &Empson, 1998; Kamii, 1989; Maher & Martino, 1996; Resnick, 1989) have investigated students' mathematical thinking and indicated that young children can explore problem situations and "invent" ways to solve the problems, For example, traditionally, to find the sum 38 + 26, students are expected to add the ones (8 + 6 = 14), and write down 4 for the unit place of the sum and carry over 1 to the ten's place. Carpenter et al. (1998) found that many first-, second-, and third-grade students were able to use the following invented strategies to solve the problem: (1) "Thirty and twenty is fifty and the eight makes fifty eight. Then six more is sixty-four"; (2) "Thirty and twenty is fifty, and eight and six is fourteen. The ten from the fourteen makes sixty, so it is sixty four"; (3) "Thirty-eight plus twenty-six is like forty and twenty-four, which is sixty-four." In their study, Carpenter et al. (1998) found that 65% of the students in their sample had used an invented strategy before standard algorithms were taught. By the end of their study, 88% of their sample had used invented strategies at some point during their first three years of school. They also found that students who used invented strategies before they learned standard algorithms demonstrated better knowledge of base-ten number concepts and were more successful in extending their knowledge to new situations than were students who initially learned standard algorithms.

Recently, some researchers (e.g., Ben-Chaim et al., 1998; Cai, 2000) have also found evidence that middle school students are able to use invented strategies to solve problems. For example, when U.S. and Chinese sixth-grade students were asked to determine if each girl or each boy gets more pizza when seven girls share two pizzas and three boys share one pizza equally, they used eight different correct ways to justify that each boy gets more than each girl (Cai, 2000). Collectively, the aforementioned studies not only demonstrate that students are capable of inventing their own strategies to solve problems, but they also show that it is possible to use the students' invented strategies to enhance their understanding of mathematics. Thus, it seems clear that students in elementary and middle schools are capable of inventing their own strategies to solve problems. However, there are at least two unanswered questions. Schoenfeld (1985) has taken a different approach to teaching heuristics. He argued that "most "general heuristic strategies" are so broadly defined that their definitions are far too vague to serve as a guide to their implementation" (p. 95). He proposed breaking heuristics down to what he called "more precise and usable descriptions of heuristic strategies" or sub strategies. Students not only need to learn these sub strategies, they also need to learn how to break down "general strategies" to sub strategies.

DESIGN AND METHODOLOGY

The researchers stated the procedures that they used to conduct the study. They described the study population, sample, variables, instrument, procedures and the statistical analyses that were used in the study. The population of the study consisted of: All tenth grade students in Irbid the first educational directorate enrolling in the first semester 2011/2012. They form (3420) male and female students. The sample of the study comprised of (142) tenth grade students, 69 male and 73 female students in King Abdullah School at Irbid the first educational directorate and was distributed into four sections, which were selected purposefully. Two control groups and two experimental groups. The participants of the study were divided into two groups, experimental and control: The participants of the experimental group were taught by using heuristic approach for (8) weeks, While the participants of the control group were taught by using the conventional way for the same period. The researcher used two strategies for teaching mathematics: A strategy using heuristic approach for teaching mathematics and the conventional strategy. Then the researcher prepared a test based on the instructional material of the tenth grade math book. In the experimental group the teacher train students on using heuristic method; here the student is induced into discovering the solution of a problem all by himself. In its normal form, the teacher may guide the students to discover by framing them carefully and well-graded manner which will ultimately lead them to the discovery. Questioning has to replace telling in the class room.

The researcher used two strategies for teaching mathematics: A strategy by using heuristic approach and the conventional strategy. Then the researcher designed a test based on the instructional material of the tenth grade mathematics book to collect the data. Validity and reliability were verified. Both groups; the experimental group as

well the control group were taught by their teachers. The subjects in both groups took a pre-test to determine their actual level before starting the experiment, and the same test was administered as a post-test at the end of the experiment to assess subjects' achievement. The time interval between the pre-test and the post-test was (8) weeks; a period long enough to minimize the effect of the pre-test on the results and conclusions of the experiment. To ensure the test reliability, the researcher followed test/retest technique. The researcher applied it to a pilot sample of (25) student who were excluded from the study with a two-week period between the test and the re-test. The reliability of the test was calculated using correlation coefficient and found to be 0.87. The researcher considered this value acceptable for the purposes of the study. The researcher designed a vocabulary test taking into consideration the instructional material. The researcher validated the instrument by submitting it to a jury of two supervisors of mathematics working at the Directorate of Education of Irbid the first, and two tenth grade teachers of mathematics. The instructional material was the tenth grade mathematics textbook which includes eight units, but the researcher covered just three units during the application period. The descriptive methods (means and standard deviation) were used for pre and post tests for English vocabulary test for both the experimental and control groups. Differences statistical method (t-test) and Two-way ANOVA were used to make a comparison between the control and the experimental groups.

FINDINGS OF THE STUDY

The purpose of this study is to investigate the effect of using heuristic approach on tenth grade male and female students' achievement in mathematics at King Abdullah School in Irbid the first educational directorate. The researcher followed the equivalent pre /post test two group designs. Therefore, the means, standard deviations and Two-Way ANOVA analysis of variance were used to analyze data. The results will be displayed based on the questions of the research. To determine if there is a statistically significant difference between the male and the female groups, a t-test for independent samples was conducted. Table 1 shows the results.

Table 1: Means and Standard Deviations of the Achievement of two Groups on the Pretest

Group	Sex	Mean	Std. Deviation	Number
Experimental	Male	54.06	6.23	34
	Female	56.11	8.84	37
	Total	55.13	7.72	71
Control	Male	55.03	8.18	35
	Female	53.33	8.10	36
	Total	54.17	8.13	71
Total	Male	54.55	7.25	69
	Female	54.74	8.54	73
	Total	54.65	7.91	142

Table 1 indicates that the difference between males and females is not statistically significant at α=0.05. Thus, since the difference was not significant, the two groups were assumed equivalent and the sample was divided into two groups, an experimental and a control group. The mean for the experimental group was 55.13 while it was for the control group 54.17, which means that there were nearly the same. The experimental group which was taught using heuristic approach consisted of (34) male students and (37) female students while the control group consisted of (35) male students and (36) female students. To determine if the two groups are equivalent in their level in mathematics, a pretest was conducted and Table 2 presents the results.

Table 2: T-Test Results of the Experimental and the Control Groups on the Pretest

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Group	28.887	1	28.887	.460	.499
Sex	1.111	1	1.111	.018	.894
Group * Sex	124.317	1	124.317	1.978	.162
Error	8672.421	138	62.844		
Corrected Total	8830.394	141			

Table 2 shows that the difference between the achievement of the two groups on the pretest is not statistically significant at α=0.05. Since there is no statistically significant difference between the control and experimental groups on the pretest, it was for the experimental .499 which is more than 0.05 and it was for the experimental .894 which is also above0 .05. So, the groups were assumed equivalent. At the end of the experiment, a t-test for independent samples was conducted to determine if there was any statistically significant difference between the males and the females on the posttest, which may be attributed to gender. Table 3 shows the results.

Table 3: Mean and Standard Deviations of the Achievement of Male and Female Groups on the Posttest

Group	Sex	Mean	Std. Deviation	Number
Experimental	Male	83.41	9.33	34
	Female	86.24	6.50	37
	Total	84.89	8.05	71
Control	Male	76.86	9.13	35
	Female	78.25	11.42	36
	Total	77.56	10.30	71
Total	Male	80.09	9.74	69
	Female	82.30	10.03	73
	Total	81.23	9.92	142

The results show that the treatment had the same effect on male and female students and the difference between their achievements was not statistically significant. Table 3 shows that there is a statistically significant difference at α=0.05 between the achievement of the experimental group and that of the control group on the posttest in favor of the experimental group. This difference indicates that using heuristic approach for teaching mathematics for tenth grade students may have had a positive effect on students' achievement on mathematics. The mean score for the experimental group on the posttest was 84.89 while that of the control group was 77.56. The researcher also conducted a two-way analysis of variance to analyze the posttest achievement scores of the two groups. Table 4 shows the results.

Table 4: Summary of the Two-way Analysis of Variance of the Achievement of the control and the Experimental Groups

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Group	1876.444	1	1876.444	21.971	.000
Sex	158.217	1	158.217	1.853	.176
Group * Sex	18.350	1	18.350	.215	.644
Error	11786.082	138	85.406		
Corrected Total	13866.789	141			

Table 4 shows that there is a statistically significant difference between the experimental group and the control group on the posttest, was significantly better than that of the control group. However, the information indicates that there was no significant difference attributed to the interaction between the treatment and gender. To sum up, the researcher believes that the difference in the achievement of the tenth grade students was attributed to the using heuristic approach for teaching mathematics. The experimental group subjects managed to significantly improve their level in mathematics in a period of 8 weeks. The improvement achieved by the control group subjects, however, was not statistically significant. By comparing the results achieved by the two groups, the researcher reached the conclusion that the improvement achieved by the experimental group may have been attributed to the way he rendered instruction; using heuristic approach. As a result of this experience, the researcher concluded that students were more engaged in learning when they were given a chance to use a heuristic method to learn inquiry in mathematics.

CONCLUSION

This study investigates the impact of using the heuristic teaching approach for teaching mathematics to tenth grade students in Jordan. The researchers followed the equivalent pre/post T test two group designs. To achieve the goal of the study, a pre/post- test was constructed to measure student achievement in mathematics. The findings of the study indicate that there were statistically significant differences in the post- test between the control and the experimental groups in favor of the experimental group, and there was no statistically significant difference

in the students' achievement due to gender. There was no statistically significant difference due to the interaction between gender and group. The researcher proposes using the heuristic approach in teaching mathematics and other studies.

The role of the teacher in the classroom is very important for inspiring and stimulating students. In practice the success of the heuristic method depends on good questioning. The teacher no longer teaches in the traditional sense; the learner no longer listens in the traditional sense but if effect becomes the teacher. Subjective observation suggests that students enjoy participating in heuristic approach activities, and find it significantly easier to express their mathematical solutions in written form. The Heuristic approach is not only effective in math courses, but is a successful means of teaching writing in any course that uses problem-solving and requires critical thinking (science, debate, logic, computer science, etc.). Finally, the heuristic approach gives the math educator a tool that helps students express their mathematical solutions verbally, motivates the students, and is a process that both students and educators will find enjoyable.

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